The Bi-Objective Location-Routing Problem Based on Simultaneous Pickup and Delivery with Soft Time Window

Elham Jelodari Mamaghani, Mostafa Setak

Abstract
The location-routing problem, while being new, is the most significant research field in location problems; features of vehicle-routing problem have been simultaneously considered along with the original problem for achieving high-quality integrated distribution systems, in addition to the global optimum. Contribution to the existing research presents the bi-objective multi-depot capacitated location-routing problem based on simultaneous pickup and delivery with soft time window (BOCLRPSPDSTW). Reasonable grounds that exhorted authors to get involved in this area and whence arising simultaneous pickup and delivery based on time window are the two main characteristics of logistic management that have been used separately in most of the location routing problem in spite of their various real-life applications with each other. Furthermore, market world competition circumstances always compel distribution managers to try creating a distribution system layout along with the lowest total system cost and enhancing service levels for providing customers' satisfaction, such that they can make the perpetuity of the distribution systems possible in the competition. Accordingly, to achieve the main goal within the demonstrative bi-objective mixed-integer linear programming model for BOCLRPSPDSTW, this study addresses the minimization of summation of all problem costs and minimization of maximum summation of delivery times and service times for meeting customer service level with respect to simultaneous pickup and delivery with soft time windows. Since this type of problem is NP-hard, NSGAII and NRGA are proposed to attain the Pareto frontier for the given problem. To validate the performance of the proposed algorithms in terms of solution quality and diversity levels, various problems are carried out, and their efficiency based on some comparison metrics is compared.

Key words: Location-routing problem with time window, Location-routing problem, Simultaneous pickup and delivery, Mixed integer linear programming, Bi-objective location-routing problem.

1. Introduction
Within developing and competitive world throughout the future, paying attention to logistic prospects is one of the principle indicators of a company’s success. Undeniably, these centers deal with strategic, tactical, and operational decisions. Among them, two decisions, namely location and routing, which are strategic and operational decisions, play a crucial role. Location of depots, allocation of customers to each depot, and transportation policies for determining routes for establishing depots to all of the devoted customers are imperative sides with which managers cope. For a long time, researchers have separately considered the drawn problems with regard to their complex combination. An important question, which comes to mind of operation managers, is why Location problem and Routing have to be combined with each other. Salhi and Rand (1989) proved that considering location and routing problem separately obtains a sub-optimal solution. Hence, for having global optimum and efficient distribution system, both of the problems have to be combined with each other. Nagy and Salhi (2007) presented an exhaustive survey of LRP s proceeding to 2006. They suggested a classified method and considered a variety of the location-routing problem from the different aspects. According to the article, the obvious difference between location-routing problem (LRP) and location allocation problem (LAP) is that the latter one considers the occasion in the facility is located and assumes the straight-line, whereas the former involves a visit of customers through tours and tries to find the optimal facility location and route design at the same time.

Various traits have been described in LRP papers, but one of the important specifics that is not used significantly is simultaneous pickup and delivery problem. This is the linkage point of LRP with reverse logistic. In comparison to LRP, which presumes that all customers have just delivery demand, customers can actually have pickup and delivery demands in the real world. In their view, both demands should be met at the same time and all cumulative pickups should be returned to the depots. By taking this kind of demand structure, LRP with simultaneous pickup and delivery (LRPSPD)
opens a new field in LRP research. The main cause of simultaneous pickup and delivery is reducing cost and using it in reverse logistics. In addition, in the prior item, every customer is intended to be served in time window framework. To meet this problem, time windows are occupied in supply chain and distribution corporations. In the last decades, the latter research field in the VRP is tended to give acceptable service for all customers in each customer time range (Wang and Chen, 2012). One of the reputable functions of LRPSPD is in the beverage industry (e.g., distributing beer, juice, etc. and collecting empty bottles for reusing) and the grocery store chains that have been remarked in most of the relation papers (Karaoglan et al., 2012). In most real cases of supply chains, service level of customers tremendously depends on the minimizing maximum delivering and serving times; the role of the second objective in the proposed model meets the important one: having useful position in the supply chains and obtaining the most quota of the markets in the competitive business arena. Without a doubt, the outstanding effect of the second objective definition is on the time-sensitive systems such as food delivery, military services, and healthcare (Ghaffari-nasab et al., 2013). The imperative job is in the healthcare systems which necessarily revolve around the restoration of some drugs, predominantly expired medications, and switching devices.

Consequently, there is an occasion to attract all attentions and do research in the bi-objective LRP alongside the simultaneous pickup and delivery with soft time windows. Considering the mentioned areas, contribution of the article is introduced as modeling and solving of the multi-objective capacitated location-routing problem based on simultaneous pickup and delivery with soft time window and multi depot (BOCLRPSPDSTW). The proposed model encompasses efficiency (in the sense of cost minimization) and effectiveness (in connection with customer service level) of the distribution systems for leading customer satisfaction. Furthermore, two metaheuristic algorithms, NSGAII and NRGA, are advised for solving this model. Comparing these methods is placed in an important section of this paper.

the current paper is organized as follows. Brief surveys of the LRP: LRP with simultaneous pickup and delivery and LRP based on time window are expressed in section 2. In Section 3, BOCLRPSPDSTW mathematical model is presented. Section 4 contains a detailed execution of NSGAII and NRGA for solving the BOCLRPSPDSTW. Computational results obtained by applying the presented solution scheme to a series of test problem instances are reported in Section 5. Section 6 is dedicated to the final and concluding point of this paper.

2. Literature Review

Location routing problem can be categorized from a variety of outlooks. The prevalent classifications seen in most of the research studies are made up of PLRP (Periodic LRP), LRP with hub idea, LRPTW(LRP with time window), and LRPSPD(LRP based on simultaneous pickup and delivery). The last two criteria, regardless of the first two, are discussed in the current article in detail. Nevertheless, the worthwhile research papers have been done in the first two fields. Prodhon and Prins (2008) used memetic algorithm with population management for solving periodic location routing problem. Their goal was to consider multiple decision levels concurrently. A large variable neighborhood search was proposed by (Pirkwieser and Raidl, 2010). Prodhon (2011) demonstrated mathematical model and hybrid evolutionary algorithm for solving this model. The algorithm is the combination of the extended local search and Clarke and Wright algorithm. One of the studies in the hub LRP was done by (Setak and Karimi, 2013). Their study is about incomplete networks in hub LRP grounds, related to urban transportation. There is a lot of literature review about the LRP which is written from diverse features like mathematical model and demonstrating exact algorithms and metaheuristic ones. Albareda-sambola et al. (2005), Prins et al. (2006), Mariankis et al. (2008), and Duhamel et al. (2010) are some of the related researchers. Whereas the BOCLRPSPDSTW is not mentioned in the literature in advance, outlining the related works with this problem would be of priority. The BOCLRPSPDSTW consists of two sub problems: the facility location problem (FLP) and the vehicle routing problem with simultaneous pickup and delivery (VRPSPD) based on soft time windows. The previous one is composed of two sub problems: VRPSPD and VRPSTW. FLP, VRPSPD, and VRPSTW are among the research subjects that have been studied over the decades. (Parragh et al., 2008) and (Smith et al., 2009) are the survey articles in the VRPSPD and FLP, in that order. Concerning VRPSPDSTW, the keen readers can refer to (Wang and Chen, 2012) research. LRPSPD was considered by (Moshieov, 1994) for the first time. His research is based on travelling salesman location problem with pickup and delivery. The author’s contemplated customer demands are stochastic variables used for solving the presented model; heuristic approach is founded on customers rankings. LRPSPD is also the expansion of the LRP in terms of each customer demand which is composed of pickup and delivery problems simultaneously. To many, LRP (MMLRP) introduced by (Nagy and Salhi, 1998) is the general form of LRPSPD. In this problem, numerous customers desire to send products to another; in addition, streams between depots are acceptable. The prominent studies in LRPSPD were conducted by (Karaoglan et al., 2011), (Karaoglan et al., 2012); they are labeled as the leading and inspirational sources for the current authors. In the first paper, a new model from a new vision was suggested for the LRPSPD based on arc routing problem. In the advised model, the number of vehicles is not considered. For solving this NP-hard problem, exact and branch-and-cut algorithms have been used. Besides the arc presented model in the first
paper, in the last demonstrated article, node-based model was depicted. To solve the large-sized LRPSPD, a two-phase heuristic algorithm was derived from simulated annealing; for tp-SA initial solution, the heuristic algorithm was developed.

In perusing literature of LRPTW, one of the salient works is related to (Nikbaksh and Zegordi, 2010). They presented 4-index non-linear 2-layer model for the LRP with soft time windows. For solving this model, heuristic approach based on Or-opt has been utilized. Numbers of multi-objective LRP works are as follows: Lin and Kwok (2006) offered multi-objective metaheuristic algorithm for LRP with multiple uses of vehicles. Tabu search and simulated annealing were used for solving the problem. At last, their performance was compared. The multi-objective model of (Caballero et al., 2007) was based on four-objective that tabu search metaheuristic algorithm has applied for real instances. In addition to the usual cost of objective function, there are social objectives (social rejection by towns on the vehicle routes, maximum risk as a fairness scale, and the adverse effect on the plant closing towns). Hassan-Pour et al. (2009) offered a new bi-objective mathematical programming model for a stochastic location-routing problem (MO-LRP) by an SA algorithm with genetic operators. Demonstrating a new model for bi-objective LRP was done by (Tavakkoli moghaddam et al., 2012). The first objective, as all single objectives, is cost; the second objective function maximizes the whole demand served. This objective function reveals that responses to all customers are not done. Concerning the solution method, scatter search was used for solving the model. Bi-objective model of (Ghaffari-nasab et al., 2012) had stochastic time variable in order to convince the objective functions. Each customer simultaneously in the time window outline depots for fulfilling the pickup and delivery demands of each customer. Concerning BOCLRSPDSTW, the goal is to determine the location of potential depots, vehicle routes from depots for fulfilling the pickup and delivery demands of each customer simultaneously in the time window outline in order to convince the objective functions.

3.1. Problem assumption

- The mentioned problem is defined in a completed, directed graph where customers and depots are placed in their nodes
- Customer demands are deterministic and composed of pickup and delivery simultaneously
- All of the vehicles are homogeneous and capacitated
- At any node, total load of vehicle cannot surpass its capacity
- Total pickup and total delivery of the assigning customer to each establishing depot do not exceed its capacity

3.2. Parameters and variables of the proposed model

Let $G=(N,A)$ be the directed weighted completed network, where $A=\{(i,j)|i,j\in EN\}$ is the edge between two nodes, travelling cost between $i,j$ is $c_{ij}$. $N=C\cup D$ where $C$ and $D$ indicate costumers and depots, respectively. $d_i$ and $p_j$ are delivery and pickup of customer $j$. $Q_k$ is capacity of each depots, $V$ is capacity of vehicles, $O_i$ is fixed cost of depot establishing, $F$ is fixed cost of vehicle employing, $s_i$ is service time for $jth$ costumer, $t_{ij}$ is travelling time between $i,j$, and $u$ and $z$ are correspondingly lower and upper bounds of time window for $jth$ costumer, and eventually $u_j,\beta_j$ depict penalties for arriving after the upper bound and before the lower bound of time window. The subsequent binary variables are: $x_{ij}=1$ if vehicle travels directly from $i$ to $j$; $z_i=1$ if depot $i$ is opened; $y_{ij}=1$ if costumer $j$ is allocated to depot $i$, at will be the arriving time to $jth$ node, $SD_j$ is delivery load on vehicle before having serviced customer $j$, $SP_j$ is pickup load on vehicle after having serviced customer $j$. Violation of the time window for the upper and lower bounds is $E_p, L_p$, respectively. The mathematical programming for BOCLRPSPDSTW can be expressed as follows:

$$
\min \sum_{jEN} \sum_{jED} c_{ij} x_{ij} + \sum_{iED} \sum_{jEC} Fx_{ij} + \sum_{iED} O_i Z_i + \sum_{jEC} E_{i} \alpha_i + \sum_{jEC} L_{i} \beta_j
$$

$$
\text{Min} \ \max \left( \sum_{jEN} \sum_{jED} x_{ij} + \sum_{nEN} \sum_{jEC,n=} x_{nj} s_j \right)
$$

St:

$$
\sum_{jEN} x_{ij} = 1 \quad (\forall i \in C, i \neq j)
$$

$$
\sum_{jEN} x_{ji} - \sum_{jEN} x_{ij} = 0 \quad (\forall i \in N, i \neq j)
$$

$$
\sum_{kED} y_{ik} = 1 \quad (\forall i \in C)
$$

$$
x_{ik} \leq y_{ik} \quad (\forall i \in C, k \in D)
$$

83
\[ x_{ki} \leq y_{ik} \quad (\forall i \in C, k \in D) \] (7)
\[ x_{ij} + y_{ik} + \sum_{m \in D, m \neq k} y_{jm} \leq 2 \quad (\forall i, j \in C, k \in D, i \neq j) \] (8)
\[ \sum_{i \in C} d_i y_{ik} \leq Q_k z_k \quad (\forall k \in D) \] (9)
\[ \sum_{i \in C} p_t y_{ik} \leq Q_k z_k \quad (\forall k \in D) \] (10)
\[ SD_j - SD_j + V x_{jj} + (V - d_j - d_{j'}) x_{j'j} \leq (V - d_j) \quad (\forall j, j' \in C, j \neq j') \] (11)
\[ SP_i - SP_j + V x_{jj} + (V - p_j - p_{j'}) x_{j'j} \leq (V - p_{j'}) \quad (\forall j, j' \in C, j \neq j') \] (12)
\[ SD_j - d_j + SP_j \leq V \quad (\forall j \in C) \] (13)
\[ SD_j \geq d_j + \sum_{i \in C, i \neq j} d_i x_{ij} \quad (\forall j \in C) \] (14)
\[ SP_i \geq p_j + \sum_{j' \in C, j' \neq j} p_{j'} x_{j'j} \quad (\forall j \in C) \] (15)
\[ SD_j \leq V - (V - d_j) \sum_{i \in D} x_{ij} \quad (\forall j \in C) \] (16)
\[ SP_i \leq V - (V - p_j) \sum_{i \in D} x_{ij} \quad (\forall j \in C) \] (17)
\[ at_j + s_j + t_{jj} - T(1 - x_{jj}) \leq at_{j'} \quad (\forall j \in N, j' \in C, j \neq j') \] (18)
\[ E_j \geq at_j - u_j \quad (\forall j \in C) \] (19)
\[ L_j \geq l_j - at_j \quad (\forall j \in C) \] (20)
\[ L_j \geq 0 \quad (\forall j \in C) \] (21)
\[ E_j \geq 0 \quad (\forall j \in C) \] (22)
\[ x_{ij} \in \{0, 1\} \quad (\forall i, j \in N) \] (23)
\[ y_{ji} \in \{0, 1\} \quad (\forall j \in C, i \in D) \] (24)
\[ z_i \in \{0, 1\} \quad (\forall i \in D) \] (25)
\[ at_i = 0 \quad (\forall i \in D) \] (26)
\[ at_j \geq 0 \quad (\forall j \in C) \] (27)

The first objective function minimizes summation of transportation cost, establishment of fixed cost of depot, and vehicles fixed cost. The last two parts of it present penalty costs due to the violation of the upper and lower bounds of time window. The role of the second objective is minimizing maximum summation of travelling time and service time between two nodes. Constraint (3) shows that each customer has to be visited just once. The forth describes a number of arcs in which entering to the node and removing it are alike. Equation (5) is used for allocation of just one depot to each customer. Constraints (6) - (8) forbid all of the unauthorized routes between depot and customer and also between the customers. In fact, the eighth equation shows that arc between two customers exists when both of them are allocated to the same depot. Constraints (9), (10) show that the total delivery related to a depot and the total pickup of a depot must be less than depot capacity. Equations (11), (12) are sub tour eliminations. Full amount load on any arc does not exceed total load on vehicle; it is presented in equation (13). Constraint (14) explains that delivery load on vehicle before serving jth customer should be larger than the consumer delivery and delivery of the next customers, which are connected with each other. In constraint (15), the previous state is adopted to the pickup, but it describes the pickup after serving the jth customer due to the pickup’s additional variable definition. Regarding equations (16),(17), they indicate relations between the additional variables and vehicle capacity in the last customer and the first customer for delivery and pickup’s additional variables, respectively. Constraint (18) is a special constraint for the time window as it explains the relation between receiving time for every node and the previous time of it. T is a sufficiently large number. Equations (19),(20) specify penalties constraints. The rest of the constraints are zero, one, and integer constraints.

The second objective function in the presented model is of non-linear type. In MILP, it has to be linearized. The following model causes a linearization corresponding to the previous model except altering objective function (2) and adding another constraint, thanks to the linearization:

Objective function (1):
\[ \min TT \] (28)
St:
\[ \sum_{i \in N} \sum_{j \in N} t_{ij} x_{ij} + \sum_{n \in N} \sum_{j \in C, n \neq j} x_{nj} s_j \leq TT \] (29)
\[ (3)-(26) \] (30)
\[ TT \geq 0 \] (31)

4. Methods for Solving BOLRPSPDSTW

In the current section, the main approaches for solving the suggested multi-objective model are presented. Nagy and Salhi (2007) demonstrated that LRP is composed of two NP-hard sub problems: location and routing; as a result, their combination will be surely of NP-hard kind. Since the description model is NP-hard, a favorable solution to ε-constraint for large-sized problem is not achieved. Hence, metaheuristics are necessary to obtain solutions for these problems. In the existing research, NSGA-II and NPGA are the reputable approaches for solving multi-objective problems applied.

4.1. Solution representation

A suitable representation for solving NP-hard problems is important. Without a doubt, solution structure besides the coding methods should satisfy all constraints. Definitions of two matrices, A and P, and the use of coding approaches can obtain this aim of the article problem. Structure of the solution is like figure 1.
Additionally, the structure is a linear matrix with one row, and the number of columns is equal to the number of customers. Every element of this matrix indicates which customer is devoted to each open depot. It is of significance to mention that the initial solution used by the algorithm is generated randomly. In P matrix, there is one row and column points for the customer serving priority. If a customer cannot be served with respect to satisfying constraint in this priority, another customer has to be served from this depot.

4.1.1. Using operators

To solve the presented model by using proposed algorithms, two-crossover and two-mutation operators for each matrix are applied.

Crossover operator

An important part of evolution in the nature depends on chromosomes and parent election methods for their combination in the right way. Each solution chromosome is composed of two matrices (A, P). When crossover is done on parent’s chromosomes, in fact, this operation is done on each matrix (A and P). However, by paying attention to shape and structure of matrices, different operators can be used. Crossover operator used by matrix A is uniform and for P is a single-point crossover, but this is the kind used in permutation structures.

Because of the permutation trait of P, it is necessary to consider the justified action when it operates. This sufficient action prevents gene duplication. For using this method, at first, the crossover point is selected. The genes before this point are transferred directly; to compose the rest of the chromosome in the first individual, all of the second parent’s genes are considered and compared among the selected gens. If there are not repetitive ones, the next gene is set in the first individual. Concerning the second offspring, polar action is operated. This operation is shown in figures 2.a and 2.b. According to the second parent’s genes, the rest of the first offspring will be 5, 2, and this matter about the second offspring with respect to the first parent will be 3, 2.

Mutation over A matrix

For Mutation over A matrix, in the first step, it is necessary to create a random matrix with the same size of A. In the next one, the elements, whose amount is less than mutation rate, are found. Their amount with the random number generated in the [1, D] is changed. Suppose that over 1 to 7 positions of parents’ mutation occur. In this state, according to produced randomized depots, new depots would be replaced by mutation elements. These kinds of mutation are depicted in figure 3.

Mutation over P matrix

Due to the permutation specification of this matrix, there are two steps available:
1) Randomly choose two elements (two customers) from a row.

![Fig. 1. A, P using matrix in solution structure](image1)

![Fig. 2.a. the crossover point in p matrix](image2)

![Fig. 2.b. the first three gens of offspring’s point in p matrix](image3)

![Fig. 3. Mutation over A matrix](image4)
2) One of the swap, insertion, and reversion operators is chosen randomly and operates on the two-selected customer in the previous step. It is possible for each of these operators to have similar actions to another. So, all of these states have to be emitted.

5. Computational Study

In this section, the result of applying NSGA-II and NRGA to BOCLRPSPDSTW is done and shown. The algorithms have been coded in MATLAB R 2010b and run on a laptop with an Intel Core i5 CPU (2.27 GHZ) and 4 GB memory. In order to evaluate performance of the proposed algorithms, there is no benchmark problem for the presenting model. Nonetheless, there is a benchmark for LRPSPD (Karaoglan et al., 2012), without considering time window concept. For this reason, the current authors had to make samples with their own instances for the time window part. Of course, it is essential to mention that the fundamental dataset of the research paper of (Karaoglan et al., 2012) is based on different methods: that of Nagi and Salhi and Angellelli and mansini’s approaches, which are mentioned in their papers extensively; the parts of the pickup and delivery data cited in this paper are related to Angellelli and Mansini’s approach. Time windows should be adjusted to the customers with respect to their conditions.

5.1. Comparison metrics

Overall, in contrast to single objective, diversity of pareto solutions and also their convergence are important factors in the multi-objective problems (Deb et al., 2002). The following four comparison metrics are applied.

5.1.1 The number of Pareto solutions (NPS)

This metric shows the number of pareto optimal solutions found by each algorithm (Schaffer, 1985).

5.1.2 Diversification metric (DM or diversity)

The diversification of metric measures spread the solution set and is defined by (Zitzler, 1999).

\[
D = \sqrt{\sum_{i=1}^{n} \max(|x_i - y_i|)}
\]

Where the parenthesis statement indicates Euclidean distance between the nondominated solutions \(x_i, y_i\).

5.1.3. Mean Ideal distance (MID)

Proximity of answers from ideal point that is equal to (0, 0) on the pareto front appraisal. The next equation shows that algorithm efficiency is high if this scale is the least (Rahmati et al., 2012).

\[
MID = \frac{1}{N_{os}} \sum_{i=1}^{N_{os}} c_i \text{ where } c_i = \frac{\sum_{j=1}^{m} f_{ij}}{\sqrt{\sum_{j=1}^{m} f_{ij}}}
\]

5.1.4. Time

Time is the most important and main scale in comparing the two algorithms.

5.2. Parameter setting

Parameter tuning may affect computational results of quality. For having accurate comparison between two algorithms, it is necessary to consider both of them in the same situation. One of them is to have the same solution. Due to this fact, regarding the number of iteration, nlt, paying attention to the number of population, npop, is logical. In the initial experiments, different combinations of parameters in NSGA-II and NRGA were considered and tested on the set of test problem samples. Table 1 specifies ranges of parameter used.

Table 1

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Parameter</th>
<th>Range</th>
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<tbody>
<tr>
<td>NSGA-II &amp; NRGA</td>
<td>npop</td>
<td>50-150</td>
</tr>
<tr>
<td></td>
<td>pc</td>
<td>0.3-0.7</td>
</tr>
<tr>
<td></td>
<td>pm</td>
<td>0.1-0.3</td>
</tr>
<tr>
<td></td>
<td>muterate</td>
<td>0.1-0.3</td>
</tr>
</tbody>
</table>

RSM is an approach, which is utilized in this study for parameter tuning. (MID/Diversity) is used for determining the surface parameter. Considering two important scales simultaneously will be useful, thanks to the effect of two scales instead of just one of it. In the recent parameters, the number of iteration appoints tuning stopping criteria. The tuning parameters are as follows:

Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
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<tr>
<td>npop_nasgaII, nrga</td>
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<tr>
<td>pc_nasgaII, nrga</td>
<td>0.3, 0.3</td>
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<tr>
<td>pm_nasgaII, nrga</td>
<td>0.3, 0.1</td>
</tr>
<tr>
<td>muterate_nasgaII, nrga</td>
<td>0.1358432, 0.1928898</td>
</tr>
<tr>
<td>itr_nasgaII, nrga</td>
<td>400, 400</td>
</tr>
</tbody>
</table>

5.3. NSGAII and NRGA applied in the proposed instances

To compare the performances of NSGAII and NRGA, forty-six data instances are applied for the comparison. C and D depict the number of customer and depots, sequentially. \(C = \{5, 10, 20, 30, 50, 70, 85, 100, 130\}; D = \{2, 3, 5, 10\}.\) For eliminating the effect of the problem size, RPD can be a suitable alternative while it demonstrates what the distance from the best answer is. It is palpable that the smaller RPD is preferred. Each problem is solved by both of the algorithms for a number of times. The best-earned amount of all executed solutions, \(Best_{sol}\) and best result in each algorithm execution, \(Alg_{sol}\) are required for calculating this criterion.

\[
RPD = \frac{|Best_{sol} - Alg_{sol}|}{Best_{sol}} \times 100 \quad 0 \leq RPD \leq 100
\]
Tables 3, 4, 5 are boded rudimentary information, NRGA_soft time window, and NSGAII_soft time window, respectively. The premier algorithm on every scale is evident in every table.

<table>
<thead>
<tr>
<th>NO. Problem</th>
<th>C</th>
<th>D</th>
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<td>10</td>
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Average
Table 5
NSGAIISTW data for 46 instances based on RPD for specified scales

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Average: 17.5878 | 9.153285 | 2753624 | 19.7335

Evaluations of the two algorithms are depicted in figure 4 which lightly proves the excellent one in each scale.
In accordance to tables 4, 5 and figure 4, the average performances of both algorithms in each scale can be comprehended. Table 6 contains these averages in each scale for both algorithms established upon RPD.

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How these algorithms work in every scale is that in MID, NSGAII is preferable to NRGA; in the rest of the scales, NRGA is better than the second one. For enriching comparison statistical analysis, F-test has been done by Minitab16 statistical software. In the statistical analysis, if p-value is smaller than 5%, null hypothesis ($H_0$) is not accepted. Hypothesis refusal demonstrates salient difference between performance evaluation criteria of the algorithms, and vice versa. Variance analysis output in table 7 is presented.

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Table 7 displays Time scale; $P_0$ is accepted in others. Accordingly, there is no significant difference between the algorithms and in the MID; Diversity and NOS algorithms are competitive and reactionary to each other. Both Figures 6 and 7 are Pareto optimal curves for NRGA and NSGAII, respectively.

![Fig. 5. NRGA Pareto Curve](image1)

![Fig. 6. NSGAII Pareto Curve](image2)

As it is clear from figures 5 and 6, there is an obvious conflict in the most points between two objective functions of the problem (total cost and maximum summation of delivery time and service time). But, the important and noteworthy point of the pareto optimal curves is that in some points, there is no complete conflict. In fact, when the decision maker wants to minimize time, cost maximization is not mandatory in some cases, because in these events that time minimization is not salient, cost raise will not occur. Chiefly, conflict occurs when time decreasing is high and sensible. In other words, if time has a main role in the decision, surely, choosing a solution with low service time and delivery time is logical even if it incurs high cost. For instance, in critical cases like in medical, food distribution, and military cases, the key role is to take over the role of time minimization, although spending much more cost has to be obligatory. In multi-objective problem, there is no priority between objectives, the same as in the above figures is depicted; it is required to define metrics in order to compare the algorithms.

One of the obvious results of the soft time window which is caused by increasing penalty cost for some customers, amount of $E_j$, $L_j$ is decreased and arriving time for these customer sets in the time window. This outcome can be due to the greater impact of the penalties on the values of the objective function of the soft time windows.

5. Conclusion

In this paper, BOCLRPSPDSTW was presented intensely out of the presented mathematical model and also solutions derived from metaheuristic algorithms. This paper dealt with the problem for the first time, since it can be used for the diverse real-world cases in the distribution networks, particularly for the reverse logistics and time-sensitive cases like healthcare, military service, the food distribution. In most of these real conditions, the primary goal of the supplier is to meet all customer needs, or at least the overall time and cost. Bi-objective mathematical programming models were applied to the problem formulation; besides, two algorithms were presented for solving problem. Whereas dealing with this problem was done for the first time, there were not any benchmark set and obtained solutions by metaheuristic algorithms. For this reason, problem sets that were made with respect to the literature and authors were tested by both of NSGAII and NRGA; at last, comparing each algorithm’s solution based on four metrics was done. Pareto non-dominated solution curve evidently implied the nature of the bi-objective problem due to the relative conflicting total cost function and maximizing the summation of delivery time and service time. This research can provide a new opportunity in the real world application, especially with fuzzy or statistic parameters. The noticable, yet interesting point of this paper is reflecting on the special state of period aspect with $(p=1)$ day of service. Periodic attribute to several serving days is one of the practical position, rarely received attention by researchers.

References


Problem Based on Simultaneous Pickup and Delivery with Soft Time Window.

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