Journal of Industrial Engineering 4 (2009) 1- 11

Building a Multi-Objective Model for Multi-Product Multi-Period Production Planning with Controllable Processing Times: A Real Case Problem

Mir Bahadorgholi Aryanezhad [a], Mehdi Karimi-Nasab [a,*], Seyed Mohammad Taghi Fatemi Ghomi [b]

[a] *Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*

[b] *Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran*

Received 25 Feb., 2009; Revised 27 Mar., 2009; Accepted 1 Apr., 2009

Abstract

Model building is a fragile and complex process especially in the context of real cases. Each real case problem has its own characteristics with new concepts and conditions. A correct model should have some essential characteristics such as: being compatible with real conditions, being of sufficient accuracy, being logically traceable and etc. This paper discusses how to build an efficient model for a real case production planning problem. This process is reinforced by providing the proofs confirming the special characteristics of the final model such as proving its NP-Completeness. Also, the extremes of both objective functions – production smoothing versus cost minimizing – are calculated analytically. Finally, the case study and its solution methods are discussed briefly.

Keywords: Multi-objective; Single machine; Production planning; Multi-Product; Multi-Period; Controllable processing time; Pareto-optimal solutions.

1. Introduction

Production managers usually seek to use an ideal production plan for their plants with some of critical features such as:
1- Clearance
2- Flexibility and robustness
3- Cost saving
4- Monotonicity
5- Reliability
6- Customer focused, and
7- Feasibility

Certainly, there are many other parameters that can be listed above but we think that they are not as general and important as those above-mentioned features. On the other hand, there are lots of conditions in production plans belonging to the other extreme of desirability for production managers. Clearance refers to the degree of vagueness of the production plan and the amount of total numbers needed to construct a comprehensive plan. Of course, clearance itself is a fuzzy concept in some Respects. Generally, more important data in a plan makes it clearer and more comprehensive.

Flexibility is, of course, a factor of importance. This refers to how a production plan could be adapted in emergency working changes such as the increase of customer demand. On the other hand, robustness means how insensitive a production plan is to the changing parameters and conditions of the environment. There are also some differences and similarities between these 2 features in terms of modeling process and in the use of the final plan. Up to now, cost saving has been the most desired feature of all production plans. All cost optimizing models in production planning are of almost less complexity than the models of costs depending on other variables such as production volume, or demand, etc. Most managers prefer to experience fewer changes in their working environment. To this end, some production managers desire to produce in a monotone rate in different periods [1]. This amounts to characteristic no 4. One pitfall of common production models is that they do not take account of this requirement, and ,consequently, their optimal solutions are characterized by lots of

*Corresponding author E-mail: mehdikariminasab@yahoo.com

variations in batch size of different periods [3], [22]. In most cases, these variations make managers not accept the production plan [2].

Reliability is another feature most production mangers seek to include in their plans. This refers to how many factors, constraints, etc and in which manner are considered to obtain the production plan. This characteristic of the plans is a bit fuzzy and vague to evaluate too. When a production plan is called "customer focused", it means that it is obtained while considering customer's need among other parameters. Some of the well-known production philosophies such as JIT are concerned with this critical feature in the production plans as far as possible.

Feasibility of production plan is obtained only when all working constraints are met. Producing an arbitrary product type will result in many other constraints such as governmental constraints, customers' willing, etc.

As this paper focuses on modeling a real case problem in Iran, related topics are discussed in the rest of the work.

In recent years, various models have been proposed to solve this problem in order to satisfy managers. Most existing models seek a way to determine an ideal production level around which the variations of batch sizes are as small as possible in the form of a narrow band as depicted in Fig. 1 [23]. This band is usually labeled as the ideal production band. Some models in the literature attempt to impose some dummy objectives on the classic batch sizing models [3], [8]. Due to conflicts between different goals of an existing model, a number of solution methods, such as goal programming, game theory, etc. have been proposed in the literature [23]. Some researchers tried to obtain the narrowest ideal band as far as possible. A few studies are aimed at forcing their models to obtain an ideal production band limited to the maximum production capacity [25]. Fig. 1 shows an example of the ideal level and ideal production band. The dashed line in the ideal band is the ideal production level. In Fig. 1, the forecasted demand (D_t), and smoothed batch size (x_t*) of periods t = 1, 2, ..., 12 are plotted. It is worth noting that a fundamental assumption is that values of demands in each period is given or forecasted by a good method. The upper dot line indicates the maximum production capacity. Another fundamental assumption in the literature is that the maximum production capacity of all periods is constant. One concern in the current literature is the relation between JIT and production smoothing approaches. For example, it is asked whether to use production smoothing models via production in the framework of JIT. If so, how should this be done so that it is not far from the philosophy of JIT [24]? In quick prompt, we can say that JIT requires that demands of different periods be forecasted in a reasonable interval, such as week/ day/ hour, or to be taken from customers [14]. Now if demand data vary strongly from one period to another period, or at least one piece of data exceeds the maximum production capacity, how should the production manager

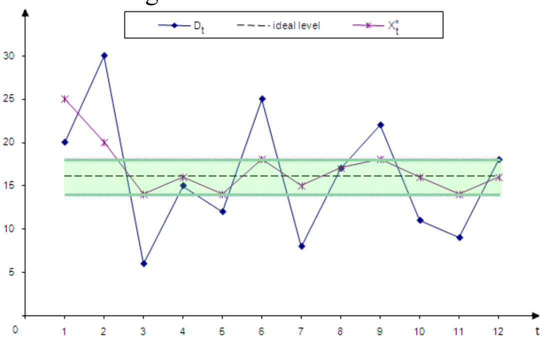


Fig. 1. Ideal production band

Plan? For example, consider the data set of 200, 300, 800, 15, 2, 350, 238, 645, 700, 46, 582, and 212 for demands of 12 planning periods by the maximum production capacity of 550. Sometimes the maximum production capacity is forced by total available time. As we know, a JIT framework requires the production to have few variations in market demands [14]. To solve this dilemma and to comply to the philosophy of JIT, some techniques can be wielded. For example, a part of customers' demand can be met in maximum one period delay. In the literature, there are two main approaches that can be summarized as follows: (*a*) if the demand of a period is less than the maximum production capacity, that period should produce as JIT dictates, else the customer is to be lost, or (*b*) the difference between demand and maximum production capacity can be produced in the adjacent periods [4]. Some reasons for using JIT are high shortage and low holding costs. The high shortage costs are due to the importance / attentions paid to customers. So the above-mentioned approaches depend on holding / shortage / delay /… costs considered in the model [19]. Furthermore, factories producing in the framework of JIT should not be worried about using the proposed model. These problems make us use a mathematical model instead of decision making based on simple conditions which consider different scenarios of production planning. Most of the times, the constructed model is so complex and full of many integer variables that even finding a feasible solution becomes so critical [7]. Yavuz and Tufekci [24] extended the previous research work on the batch production smoothing problem and introduced a bounded dynamic programming approach for medium sized problems. Minner [17] provided a comparison of simple heuristics for multi-product dynamic demand lot-sizing with limited warehouse capacity and proposed a heuristic for single-item uncapacitated lot-sizing that successively improves an initial lot-for-lot schedule by combining replenishments according to a cost savings-based priority rule to the multi-item capacitated problem. Zhou *et al.* [26] presented a general time-varying demand inventory lot-sizing model with waiting-time-dependent backlogging and a lot-size-dependent replenishment cost.

McMullen [15] presented a technique which addresses a JIT production-scheduling problem where two objectives were as (*a*) minimization of setups between different products and (*b*) optimization of schedule flexibility, and employed an efficient frontier approach to address the situation, where the most desirable sequences in terms of both objectives are found. Kovacs *et al.* [12] presented a novel mathematical programming approach to the single-machine capacitated lot-sizing and scheduling problem with sequence-dependent setup times and setup costs. Biskup and Jahnke [6] analyzed the problem of assigning a common due date to a set of jobs and scheduling them on a single machine such that the processing times of the jobs are assumed to be controllable. Jaber *et al.* [11] extended the classical economic manufacture / order quantity (EMQ/EOQ) model and the lot sizing problem with learning and forgetting by including the entropy cost concept. Merce and Fontan [16] proposed two heuristics based on an iterative procedure that is an MIP-based algorithm within a rolling horizon framework. Haral *et al.* [10] investigated bicriteria scheduling with nontraditional requirements using an experimental approach and a random keys genetic algorithm to find Pareto-optimal solutions. Bylka and Rempala [5] considered the problem of finding the optimal schedule of production runs of single machine to meet all discrete multi-product demands which are required at discrete points of time. Mollick [18] investigated physical data from the Japanese vehicle industry, covering monthly observations from 1985:1 to 1994:12, across nine goods, ranging from bicycles to large buses and concluded that the production smoothing model of inventories depends on a convex short-run cost function and adjustment costs that induce firms to maintain inventories for dampening the effects of demand fluctuations. Schaller [20] considered the problem of scheduling on a single machine when family setup times exist and showed that in most of the cases the total tardiness is minimized as well. Tang [21] provided a brief presentation of simulated annealing techniques and their application in lot sizing problems.

2. The Proposed Model

In this section, the proposed model is presented. We are seeking a model that: (I) determines the volume of production for each product type in each planning period, (II) considers the following conditions that are stated by the customers, working conditions, desires of the managers, etc. Now each condition of the case problem will be explained in mathematical language gradually. The schedule of the machine should be determined such that by considering controllable processing times, the production time becomes less than available time. Surely this feature increases the complexity concerned with analyzing the model. Because of imperfect knowledge about the trade-off between cost and time, their relation is assumed linear as shown in Fig 2. There are two critical points in Fig. 2 as (P_n, $C(P_n)$) and (P_c, $C(P_c)$) corresponding to (normal process time, cost of normal process time) and (crash process time, cost of crash process time) respectively.

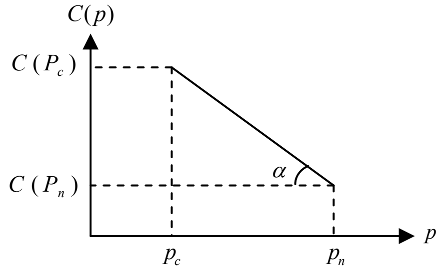


Fig. 2. The linear trade-off between cost and time

Before presenting the proposed model, it is worthy formulating the relation between cost and time of process. As it is clear from Fig. 2, the linear relation between points (P_n, $C(P_n)$) and (P_c, $C(P_c)$) could be written as (1) for $P_c \le P \le P_n$:

$$C(p)-C(p_n)=\frac{C(p_c)-C(p_n)}{p_c-p_n}(p-p_n) \tag{1}$$

Because the line slope is assumed to be non-positive and for simplicity the absolute value of the line slope is shown with *CS* as (2), then after some of simplifications from (1) and (2) the followings are obtained:

$$CS=\left|\frac{C(p_c)-C(p_n)}{p_c-p_n}\right| \tag{2}$$

$$C(P)=-CS.P+(CS.P_n+C(P_n)) \qquad (P_c \le P \le P_n) \tag{3}$$

As it is clear $CS.P_n + C(P_n)$ is always constant for every $P_c \le P \le P_n$, then from now the term $k = CS.P_n + C(P_n)$ is called as the fixed cost of processing and (3) could be re-written as (5) below:

$$k = CS.P_n + C(P_n) \tag{4}$$

$$C(P) = -CS.P + k \qquad (P_c \le P \le P_n) \tag{5}$$

Now the proposed model is constructed according to the following assumptions:

There are some of independent products that should be produced during finite number of planning periods.

There are two main objectives as (1) obtaining a smoothed production plan as possible and (2) minimizing the total cost of the corresponding production plan.

- There is a single machine operating on all products.
- The machine could process each product in an interval of time by trade off with cost such that the more process time, the less process cost linearly as Fig. 2. The optimal process time of machine on each product type should be the same in all planning periods as manager desires so. Because of preventive maintenance works, the available time in each planning period could be different from the others.

- Each product type has its own demand in every planning period that is a finite, deterministic and integer value.
- There is no initial stock of inventory for each product type.
- Products should be produced in a way that there would be no lost sales until the end of the last planning period, but backorder is allowed in every period except the last period. In the other words, the sum of the amounts produced during all periods should not be less than the sum of demands over all of periods.
- Unit shortage cost and unit inventory holding cost are deterministic values that could vary from a period to another period.
- Setup consists of some activities for each product type such as adjusting the machine speed, changing the tool, brushing and cleaning the machine, etc. And cost of changing the setup is so high. So for each product type, only one setup time is needed regardless of the amount of the given product type produced. Consequently all of amounts that should be produced in a period of each product type are produced consecutively once the corresponding setup is done as well.
- Pre-emption is not allowed and amount of each product type that should be produced in every planning period is to be a non-negative integer value.

After defining some of parameters and variables in Table 1, the proposed model is presented as Model 1.

Table 1
Parameters and variables of the model

Parameters	
T	total number of planning periods
t	index of planning period
n	number of product types
i	index of product type
$d_{i,t}$	demand of product type i in period t
AT_t	available time in period t
P_i^n	normal processing time on product type i
P_i^c	crashed processing time on product i
S_i	setup time of machine for product type i
SC_i	setup cost of machine for product type i
$\pi_{i,t}$	unit shortage cost
$h_{i,t}$	unit inventory holding cost
CS_i	cost slope of processing product i on machine
k_i	fixed cost of processing product i on machine
Variables	
$B_{i,t}$	amount of shortage for product type i in period t
$I_{i,t}$	amount of inventory for product i in period t
$y_{i,t}$	$\begin{cases} 1 & \text{if product type } i \text{ in period } t \text{ faces shortage} \\ 0 & \text{else} \end{cases}$
$r_{i,t}$	$\begin{cases} 1 & \text{if product } i \text{ is produced in period } t \\ 0 & \text{else} \end{cases}$
P_i	processing time of product type i
$x_{i,t}$	amount of product type i that should be produced in period t

Model 1

$$\min \quad Z_1 = \sum_{i=1}^{n}\sum_{t=1}^{T-1}(x_{i,t+1} - x_{i,t})^2 \tag{6}$$

$$\min \quad Z_2 = \sum_{t=1}^{T}\left\{\sum_{i=1}^{n} sc_i r_{i,t} + \sum_{i=1}^{n}(k_i - cs_i p_i)x_{i,t} + \sum_{i=1}^{n}\pi_{i,t}B_{i,t} + \sum_{i=1}^{n}h_{i,t}I_{i,t}\right\} \tag{7}$$

s.t.

$$\sum_{i=1}^{n}(s_i r_{i,t} + x_{i,t}p_i) \le AT_t \qquad (\forall \quad t) \tag{8}$$

$$\sum_{t=1}^{T} x_{i,t} \ge \sum_{t=1}^{T} d_{i,t} \qquad (\forall \quad i) \tag{9}$$

$$p_i^c \le p_i \le p_i^n \qquad (\forall \quad i) \tag{10}$$

$$x_{i,t} \le M.r_{i,t} \qquad (\forall\, i,t) \tag{11}$$

$$x_{i,t} \ge 1 - M.(1 - r_{i,t}) \qquad (\forall\, i,t) \tag{12}$$

$$x_{i,t} + I_{i,t-1} - d_{i,t} \ge -M.y_{i,t} \qquad (\forall\, i,t) \tag{13}$$

$$x_{i,t} + I_{i,t-1} - d_{i,t} \le M(1 - y_{i,t}) \qquad (\forall\, i,t) \tag{14}$$

$$I_{i,t} = \left(x_{i,t} + I_{i,t-1} - d_{i,t}\right)(1 - y_{i,t}) \qquad (\forall\, i,t) \tag{15}$$

$$B_{i,t} = \left(d_{i,t} - x_{i,t} - I_{i,t-1}\right) y_{i,t} \qquad (\forall\, i,t) \tag{16}$$

$$y_{i,t}\ , r_{i,t} \in \{0,1\} \qquad (\forall\, i,t) \tag{17}$$

$$x_{i,t}\ , B_{i,t}\ , I_{i,t} \in Z^+ \cup \{0\} \qquad (\forall\, i,t) \tag{18}$$

Objective (6) and is seeking a smoothed production plan while objective (7) is aiming at minimizing the total cost of the production plan, consisting of setup cost, production cost, shortage and inventory holding cost as well. Constraints (8), (9), (10) and (18) are checking the feasibility of the production plan, since they construct the feasible solution space while constraints (11) to (17) aim to determine some of penalty values in Z_2. Constraint (8) is satisfied when in every period t the sum of setup time and production time are less than available time. Constraint (9) ensures that in every feasible production plan, no lost sales could occur but backorder could. Constraint (10) states that production time of each product type should be in its permissible interval. Constraints (11) and (12) are intended to determine whether in period t product type i needs setup or not. Constraints (13) through (16) determine the amount of inventory or shortage for product type i in period t. Constraints (17) and (18) state the type of decision variables.

2.1. Model Modification

In this section, Model 1 is modified for purpose of easier analysis. These modifications do not violate basic assumptions of Model 1. As it could be simply shown the feasible solution space of both models are the same, but Model 2 has less constraints and variables. This is because the omitted variables and constraints were not independent. In more details, by omitting two linear constraints (13), (14), a new linear constraint (19) is appended, and by omitting two nonlinear constraints (15), (16), a new simple nonlinear constraint (20) is added. Then, $n \times T$ number of binary variables ($y_{i,t}$) are omitted from the original Model 1. So Model 2 is more efficient regardless of what solution method chosen.

Model 2

$$\min \quad Z_1 = \sum_{i=1}^{n}\sum_{t=1}^{T-1}(x_{i,t+1} - x_{i,t})^2 \tag{19}$$

$$\min \quad Z_2 = \sum_{t=1}^{T}\left\{\sum_{i=1}^{n} sc_i r_{i,t} + \sum_{i=1}^{n}(k_i - cs_i p_i)x_{i,t} + \sum_{i=1}^{n}\pi_{i,t}B_{i,t} + \sum_{i=1}^{n} h_{i,t}I_{i,t}\right\} \tag{20}$$

s.t.

$$\sum_{i=1}^{n}(s_i r_{i,t} + x_{i,t}p_i) \le AT_t \qquad (\forall \quad t) \tag{21}$$

$$\sum_{t=1}^{T} x_{i,t} \ge \sum_{t=1}^{T} d_{i,t} \qquad (\forall \quad i) \tag{22}$$

$$p_i^c \le p_i \le p_i^n \qquad (\forall \quad i) \tag{23}$$

$$x_{i,t} \ge 1 - M.(1 - r_{i,t}) \qquad (\forall\, i,t) \tag{24}$$

$$x_{i,t} \le M.r_{i,t} \qquad (\forall\, i,t) \tag{25}$$

$$I_{i,t} - B_{i,t} = \left(x_{i,t} + I_{i,t-1} - d_{i,t}\right) \qquad (\forall\, i,t) \tag{26}$$

$$I_{i,t}.B_{i,t} = 0 \qquad (\forall\, i,t) \tag{27}$$

$$r_{i,t} \in \{0,1\} \qquad (\forall\, i,t) \tag{28}$$

$$x_{i,t}\ , B_{i,t}\ , I_{i,t} \in Z^+ \cup \{0\} \qquad (\forall\, i,t) \tag{29}$$

On the other hand, the followings are correct for recursive relation between $I_{i,t}$ and $I_{i,t-1}$,:

$I_{i,1}$-$B_{i,1}$=$I_{i,0}$+$x_{i,1}$-$d_{i,1}$

$I_{i,2}$-$B_{i,2}$=$I_{i,0}$+$x_{i,1}$+$x_{i,2}$-$d_{i,1}$-$d_{i,2}$

$\vdots$

$$I_{i,t} - B_{i,t} = I_{i,0} + \sum_{q=1}^{t}(x_{i,q} - d_{i,q}) \tag{30}$$

Where $I_{i,0}$ is the initial stock of inventory for product type i. Moreover, as previously mentioned in problem assumptions, should $I_{i,0}$ be zero for each product type i. Then (22) shows the above-mentioned assumption as well:

$$I_{i,t} - B_{i,t} = \sum_{q=1}^{t}(x_{i,q} - d_{i,q}) \tag{31}$$

Then re-writing (7) as below and considering (22) simultaneously, the followings are obtained:

$$Z_2 = \sum_{t=1}^{T}\sum_{i=1}^{n}\{sc_i r_{i,t} + (k_i - cs_i p_i)x_{i,t} + \pi_{i,t}B_{i,t} + h_{i,t}I_{i,t}\}$$

$$Z_2 = \sum_{t=1}^{T}\sum_{i=1}^{n}\left\{sc_i r_{i,t} + (k_i - cs_i p_i)x_{i,t} + \pi_{i,t}B_{i,t} + h_{i,t}(B_{i,t} + \sum_{q=1}^{t}(x_{i,q} - d_{i,q}))\right\}$$

$$Z_2 = \sum_{t=1}^{T}\sum_{i=1}^{n}\left\{sc_i r_{i,t} + (k_i - cs_i p_i)x_{i,t} + (\pi_{i,t} + h_{i,t})B_{i,t} + h_{i,t}\sum_{q=1}^{t}(x_{i,q} - d_{i,q})\right\} \tag{32}$$

As (23) shows, $I_{i,t}$ could be omitted from Z_2. Furthermore, again by using $y_{i,t}$, it is possible to omit $I_{i,t}$ from constraints such that (19) and (20) could be replaced by (24) and (25) easily.

$$B_{i,t} \le \sum_{q=1}^{t}(x_{i,q} - d_{i,q}) + M.y_{i,t} \qquad \forall\, i\,; t = 1,\dots,T-1 \tag{33}$$

$$B_{i,t} \ge \sum_{q=1}^{t}(d_{i,q} - x_{i,q}) - M.(1 - y_{i,t}) \qquad \forall\, i\,; t = 1,\dots,T-1 \tag{34}$$

In this way, Model 3 is more efficient than Model 2. This is because two simple linear relations (24) and (25) have been appended and both the linear relation (19) and the nonlinear relation (20) have been omitted. Encountering shortage of product type i in the planning period t, Z_2 will select $\sum_{q=1}^{t}(d_{i,q} - x_{i,q})$ as $B_{i,t}$ by $y_{i,t}$=1. This behavior comes from the minimum seeking characteristic of Z_2. After all of the above, the number of required $y_{i,t}$ in Model 3 is less than needed $I_{i,t}$ in Model 2 as n. As it has been shown in lemma 2, this fact originates from (9) that only back order is allowed not lost sales. So $B_{i,T}$ = 0 and $y_{i,T}$ = 0 are always true without solving the model. Then, (7) is replaced with (26).

Model 3

$$\min \quad Z_1 = \sum_{i=1}^{n}\sum_{t=1}^{T-1}(x_{i,t+1} - x_{i,t})^2 \tag{35}$$

$$\min \quad Z_2 = \sum_{t=1}^{T}\sum_{i=1}^{n}\left\{sc_i r_{i,t} + (k_i - cs_i p_i)x_{i,t} + h_{i,t}\sum_{q=1}^{t}(x_{i,q} - d_{i,q})\right\} + \sum_{t=1}^{T-1}\sum_{i=1}^{n}(\pi_{i,t} + h_{i,t})B_{i,t} \tag{36}$$

s.t.

$$\sum_{i=1}^{n}(s_i r_{i,t} + x_{i,t} p_i) \le AT_t \qquad (\forall \quad t) \tag{37}$$

$$\sum_{t=1}^{T} x_{i,t} \ge \sum_{t=1}^{T} d_{i,t} \qquad (\forall \quad i) \tag{38}$$

$$p_i^c \le p_i \le p_i^n \qquad (\forall \quad i) \tag{39}$$

$$x_{i,t} \ge 1 - M.(1 - r_{i,t}) \qquad (\forall\, i,t) \tag{40}$$

$$x_{i,t} \le M.r_{i,t} \qquad (\forall\, i,t) \tag{41}$$

$$B_{i,t} \le \sum_{q=1}^{t}(x_{i,q} - d_{i,q}) + M.y_{i,t} \qquad \forall\, i\,;\, t = 1,...,T-1 \tag{42}$$

$$B_{i,t} \ge \sum_{q=1}^{t}(d_{i,q} - x_{i,q}) - M.(1 - y_{i,t}) \qquad \forall\, i\,;\, t = 1,...,T-1 \tag{43}$$

$$r_{i,t}, y_{i,t} \in \{0,1\} \qquad (\forall\, i,t) \tag{44}$$

$$x_{i,t}\ , B_{i,t} \in Z^+ \cup \{0\} \qquad (\forall\, i,t) \tag{45}$$

Lemma 1. Possible values of Z_1 are between $Z1_{PIS}$ and $Z1_{NIS}$ (i.e. $Z1_{PIS} \le Z_1 \le Z1_{NIS}$) and these limits are considered as its positive and negative ideal values respectively:

$$Z1_{PIS} = 0 \tag{46}$$

$$Z1_{NIS} = \left((AT_1)^2 + 2\sum_{t=2}^{T-1}(AT_t)^2 + (AT_T)^2\right).\left(\sum_{i=1}^{n}(\frac{1}{P_i^c})^2\right) \tag{47}$$

Proof. It is obvious that Z_1 is a positive combination of some quadratic terms. Thus, it will never be lower than zero. Given the minimum seeking characteristic of Z_1, zero would be as its positive ideal value. Then (27) is always true. On the other hand, by re-writing (8) as below, it is easy to show that a potential upper bound of each $x_{i,}$ is obtained when other non-negative variables s_i, $r_{i,t}$ and $x_{k,t}$ ($k \neq i$) are simultaneously zero. Then (29) is such an upper bound for each $x_{i,t}$:

$$(s_1 r_{1,t} + x_{1,t} p_1) + ... + (s_n r_{n,t} + x_{n,t} p_n) \le AT_t \qquad (\forall \quad t) \tag{48}$$

$$x_{i,t} \le \left(\frac{AT_t}{p_i^c}\right) \qquad (\forall \quad i,t) \tag{49}$$

Then, obviously relations (30) and (31) are correct for two adjacent planning periods t and t+1:

$$0 \le x_{i,t} \le \frac{AT_t}{p_i^c} \tag{50}$$

$$0 \le x_{i,t+1} \le \frac{AT_{t+1}}{p_i^c} \tag{51}$$

Then, relation (32) is obtained by squaring both sides of relations (30) and (31) and adding each side to the other one. In the same way (33) is made of multiplying each side of (30) and (31) to other one too.

$$(x_{i,t})^2 + (x_{i,t+1})^2 \le \left(\frac{AT_t}{P_i^c}\right)^2 + \left(\frac{AT_{t+1}}{P_i^c}\right)^2 \tag{52}$$

$$x_{i,t}x_{i,t+1} < \left(\frac{AT_t}{P_i^c}\right)\left(\frac{AT_{t+1}}{P_i^c}\right) \tag{53}$$

Now the followings are obtained simply from (32) and (33):

$$(x_{i,t})^2 + (x_{i,t+1})^2 - 2x_{i,t}x_{i,t+1} \le \frac{(AT_t)^2 + (AT_{t+1})^2}{(P_i^c)^2}$$

$$(x_{i,t+1} - x_{i,t})^2 \le \frac{(AT_t)^2 + (AT_{t+1})^2}{(P_i^c)^2}$$

$$\sum_{i=1}^{n}\sum_{t=1}^{T-1}(x_{i,t+1} - x_{i,t})^2 \le \sum_{i=1}^{n}\sum_{t=1}^{T-1}\frac{(AT_t)^2 + (AT_{t+1})^2}{(P_i^c)^2} = \sum_{i=1}^{n}(\frac{1}{P_i^c})^2\sum_{t=1}^{T-1}\left((AT_t)^2 + (AT_{t+1})^2\right)$$

$$Z_1 \le \left((AT_1)^2 + 2\sum_{t=2}^{T-1}(AT_t)^2 + (AT_T)^2\right).\sum_{i=1}^{n}(\frac{1}{P_i^c})^2$$

$$Z_1 \le Z1_{NIS} = \left((AT_1)^2 + 2\sum_{t=2}^{T-1}(AT_t)^2 + (AT_T)^2\right).\sum_{i=1}^{n}(\frac{1}{P_i^c})^2 \tag{54}$$

Because (28) is always true for every feasible solution, then the right hand side of (28) would be as the negative ideal value of Z_1. Now the proof is complete □.

Lemma 2. The lower and upper bounds of Z_2 are (34) and (35) respectively, known as $Z2_{PIS}$ and $Z2_{NIS}$. In the other words, $Z2_{PIS} \le Z_2 \le Z2_{NIS}$ always holds true for every feasible solution.

$$Z2_{PIS} = \sum_{i=1}^{n} sc_i + \sum_{i=1}^{n}((k_i - cs_i p_i^n)\sum_{t=1}^{T} d_{i,t}) \tag{55}$$

$$Z2_{NIS} = T.\sum_{i=1}^{n} sc_i + (\sum_{t=1}^{T} AT_t).\sum_{i=1}^{n}(\frac{k_i - cs_i p_i^c}{p_i^c}) + \sum_{i=1}^{n}\sum_{t=1}^{T-1}\pi_{i,t}.\left(\sum_{q=1}^{t} d_{i,q}\right) + \sum_{t=1}^{T}\sum_{i=1}^{n} h_{i,t}\sum_{q=1}^{t}(\frac{AT_q}{p_i^c} - d_{i,q}) \quad (56)$$

Proof. As it could be seen in (7), Z_2 is the sum of some nonnegative variables with nonnegative coefficients respectively. So it could be considered as the total sum of setup cost, production cost, shortage and inventory holding costs as well. Then it is clear that the term of setup cost will be maximum if there will be one setup for each product in every planning period (*i.e.* $r_{i,t}=1$) as:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{n} sc_i r_{i,t} \le \sum_{t=1}^{T}\sum_{i=1}^{n} sc_i = T.\sum_{i=1}^{n} sc_i \quad (57)$$

On the contrary, this term will be minimum when there will be only one setup during all planning periods as:

$$\min \sum_{t=1}^{T}\sum_{i=1}^{n} sc_i r_{i,t} \ge \sum_{i=1}^{n} sc_i \quad (58)$$

The term of production cost will be maximized if all products are processed in crash time. Each $x_{i,}$ should be maximized as far as possible and as previously proved (39) is always true:

$$x_{i,t} \le \left(\frac{AT_t}{p_i}\right) \qquad (\forall \quad i,t) \quad (59)$$

Consequently the following is correct for the term of production cost in Z_2:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{n}(k_i - cs_i p_i)x_{i,t} \le \sum_{t=1}^{T}\sum_{i=1}^{n}(k_i - cs_i p_i^c)\frac{AT_t}{p_i^c} = (\sum_{t=1}^{T} AT_t).\sum_{i=1}^{n}(\frac{k_i - cs_i p_i^c}{p_i^c}) \quad (60)$$

On the contrary, the term of production cost will be minimum if each $x_{i,}$ is minimized so that (9) is satisfied and all of products are produced in normal time as:

$$\min \sum_{i=1}^{n}\sum_{t=1}^{T}(k_i - cs_i p_i)x_{i,t} = \sum_{i=1}^{n}((k_i - cs_i p_i)\sum_{t=1}^{T} x_{i,t}) \ge \sum_{i=1}^{n}((k_i - cs_i p_i^n)\sum_{t=1}^{T} d_{i,t}) \quad (61)$$

The term of shortage cost in Z_2 is maximized if there are T-1 periods of shortage that are not corresponding to the minimum of (unit shortage cost times demand) or ($\pi_{i,t}.d_{i,t}$) for each product type as well. Another fact emanating from (8) is that only back order is allowed not lost sales for each product type. Consequently there should be no shortage in the last planning period. Therefore (40) will result from considering (22) and (30) simultaneously:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{n}\pi_{i,t}B_{i,t} = \sum_{i=1}^{n}\sum_{t=1}^{T-1}\pi_{i,t}.(I_{i,t} - \sum_{q=1}^{t}(x_{i,q} - d_{i,q})) \le \sum_{i=1}^{n}\sum_{t=1}^{T-1}\pi_{i,t}.\sum_{q=1}^{t}(d_{i,q} - x_{i,q})$$

$$\max \sum_{t=1}^{T}\sum_{i=1}^{n}\pi_{i,t}B_{i,t} \le \sum_{i=1}^{n}\sum_{t=1}^{T-1}\pi_{i,t}.\left(\sum_{q=1}^{t} d_{i,q}\right) \quad (62)$$

In (40), it is assumed that π is sorted shortage penalties for every product (i) among t=1 to T periods ascending. So $\pi_{,}$ is the smallest for each i=1 to n.
On the other hand, the term of shortage cost in Z_2 is minimized when there is no shortage in every planning period for each product type (*i.e.* $y_{i,t}$= 0 for each i, and t). Then (41) will be held:

$$\min \sum_{t=1}^{T}\sum_{i=1}^{n}\pi_{i,t}B_{i,t} \ge 0 \quad (63)$$

Similarly the term of inventory holding cost in Z_2 is maximized if there are more products than what needed in each planning period for each product type. Consequently there should be no shortage in all periods for each product type (*i.e.* $B_{i,t}$=0 $\forall$ i, t). On the other hand, given (22) and (39), an upper bound is obtained as (42):

$$\max \sum_{t=1}^{T}\sum_{i=1}^{n} h_{i,t}I_{i,t} = \sum_{t=1}^{T}\sum_{i=1}^{n} h_{i,t}(B_{i,t} + \sum_{q=1}^{t}(x_{i,q} - d_{i,q})) \le \sum_{t=1}^{T}\sum_{i=1}^{n} h_{i,t}\sum_{q=1}^{t}(x_{i,q} - d_{i,q})$$

$$\max \sum_{t=1}^{T}\sum_{i=1}^{n} h_{i,t}I_{i,t} \le \sum_{t=1}^{T}\sum_{i=1}^{n} h_{i,t}\sum_{q=1}^{t}(\frac{AT_q}{p_i^c} - d_{i,q}) \quad (64)$$

Similarly, the term of inventory holding cost in Z_2 could be minimized if there is no excess of product for every planning period and each product type. So (43) will always hold true:

$$\min \sum_{t=1}^{T}\sum_{i=1}^{n} h_{i,t}I_{i,t} \ge 0 \quad (65)$$

Now it is worthy noting that the meaning of positiveness and negativeness in PIS and NIS come from the type of the original objective function. In another words, when the original objective function is to find a minimum value over the feasible solution space, its minimum value or every value less than it, would be considered as its positive ideal value and vice versa. On the contrary, the maximum value of the above-mentioned objective function or every value higher than that, would be considered as its negative ideal value. PIS and NIS are solution vectors corresponding to positive ideal value and negative ideal value respectively.
After all, the positive ideal value of Z_2, known as $Z2_{PIS}$, could be elaborated as (32) by considering (37), (39), (41) and (43) as well:

$$Z2_{PIS} = \sum_{i=1}^{n} sc_i + \sum_{i=1}^{n}((k_i - cs_i p_i^n)\sum_{t=1}^{T} d_{i,t}) \quad (66)$$

And finally the negative ideal value of Z_2, known as $Z2_{NIS}$, could be elaborated as (33) by considering (36), (38), (40) and (42) simultaneously:

$$
\begin{aligned}
Z2_{NIS} = {} & T.\sum_{i=1}^{n} sc_i + (\sum_{t=1}^{T} AT_t).\sum_{i=1}^{n}(\frac{k_i - cs_i p_i^c}{p_i^c}) + \\
& \sum_{i=1}^{n}\sum_{t=1}^{T-1}\pi_{i,t}.\left(\sum_{q=1}^{t} d_{i,q}\right) + \sum_{t=1}^{T}\sum_{i=1}^{n} h_{i,t}\sum_{q=1}^{t}(\frac{AT_q}{p_i^c} - d_{i,q})
\end{aligned}
\tag{67}
$$

Now the proof is complete.

3. Problem Complexity

In this section, the classification of the problem in terms of complexity is considered briefly. Before proving the degree of complexity of the problem, the concept of space dimensionality is given as simple as possible, because this concept is used in the proof of the following theorems. Informally, space dimensionality refers to how large an instance of a problem is.

Definition1. Space dimensionality in every decision/optimization problem is determined by the number of independent variables provided the state/solution space is not null.

Theorem1. Appending some of the new decision variables and/or new constraints to every decision/optimization problem causes a new problem whose complexity is not less than the initial model provided that the number of space dimensionality in the new problem is not less than the initial problem.

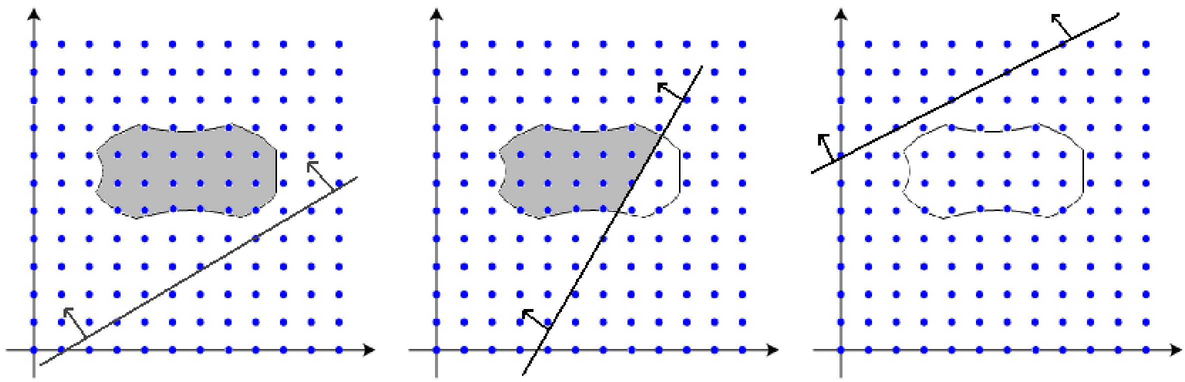
Proof. Regarding definition 1, augmenting new decision variable to any problem will not reduce space dimensionality. Also, appending new constraints to every problem results in one of the following cases:

*Case*1. The new constraint is valid for every feasible solution of the initial problem, then such a constrain does not reduce space dimensionality of the new problem, thus, this case confirms the theorem (Fig. 3 (*a*)),

*Case*2. The new constraint cuts a part of state/solution space such that space dimensionality of the new problem is not less than that of the old problem. So in this case, the theorem is correct because every arbitrary algorithm should decide or move in state/solution space consisting of a new constraint. Surely this means that the needed time for the new problem is not less than that needed in the previous problem (Fig. 3 (*b*)), and

*Case*3. The new constraint is in stark conflict with some of the old constraints and demolishes the state/solution space, thus according to the claim of the theorem; such a constraint is not considered (Fig. 3 (*c*)).

Finally given the above reasons, appending some new variables and new constraints to a problem simultaneously will make a new problem which will not be of less degree of difficulty. Now the proof is complete □.



(*a*) (*b*) (*c*)

Fig. 3. Different cases of adding a new constraint to a problem

Theorem 2. Augmenting a new objective to an ordinary optimization problem results in a new problem whose complexity is not less than the initial problem.

Proof. There are two main cases when a new objective is augmented to an arbitrary problem:

Case 1. The new objective approaches its optimal value when the other one does so. In other words, there is no conflict between the two objectives in the new problem. In this case, solving the new problem is equivalent to solving the single objective problem. So in this case the theorem is proved.

Case 2. The new objective contradicts the other objective. In this case, improving each objective results in the decline of the other one. So, it becomes more complex to search over feasible solution space (FSS) so that both objectives are near their own optimal values as far as possible. (Of course, in this case we are looking for a set of Pareto-optimal solutions instead of an overall best solution). Then in this case, the new problem is not of less complexity and perhaps would be of more complexity. So this case confirms the theorem. Now the proof is complete □. Now, formal presentations of the corresponding decision problems SPMP and MPMP-CP are given.

Single Product, Multi Period Production Planning Problem (SPMP)

Instance: Number of $T \in Z^+$ periods for each period t, $1 \le t \le T$, a demand $d_t \in Z^+ \cup \{0\}$, a production capacity $AT_t \in Z^+ \cup \{0\}$, a production setup cost $sc_t \in Z^+ \cup \{0\}$, an incremental production cost coefficient $p_t \in Z^+ \cup \{0\}$, and an inventory coefficient $h_t \in Z^+ \cup \{0\}$, and an overall bound $Best \in Z^+$.

Question: Do there exist production amounts $x_t \in Z^+ \cup \{0\}$, and associated inventory levels $I_t = \sum(x_t - d_t)$, $1 \le t \le T$, such that all $x_t \le AT_t$, all $I_t \ge 0$, and $\sum(p_t.x_t + h_t.I_t + sc_t.r_t) \le Best$? (Where r_t is a binary variable determining whether $x_t > 0$ as $r_t = 1$ or not as $r_t = 0$).

Garey and Johnson reported that SPMP is solvable in pseudo-polynomial time, but remains NP-Complete even if all demands are equal, all setup cost are equal, and all inventory costs are zero [9]. If all capacities are equal, the problem can be solved in polynomial time.

Multi Product, Multi Period Production Planning Problem with Controllable Processing Times (MPMP-CP)

Instance: Number of $T \in Z^+$ periods and number of n products, for each period t, $1 \le t \le T$, each product i, $1 \le i \le n$, with demand $d_{i,t} \in Z^+ \cup \{0\}$, a production capacity $AT_t \in Z^+ \cup \{0\}$, extremes of process time as P_i^c and P_i^n, a production setup time s_i, a production setup cost $sc_i \in Z^+ \cup \{0\}$, and unit production cost $(k_i - cs_i.p_i)$, an inventory holding coefficient $h_{i,t} \in Z^+ \cup \{0\}$, a shortage coefficient $\pi_{i,t} \in Z^+ \cup \{0\}$, and an overall bounds $\{Best_1, Best_2\} \in R^+$.

Question: Do there exist production amounts $x_{i,t} \in Z^+ \cup \{0\}$, and production processing times p_i, $P_i^c \le p_i \le P_i^n$ and associated inventory levels $I_{i,t}$ and shortage levels $B_{i,t}$, for $1 \le t \le T$, $1 \le i \le n$, and such that all $\sum(s_i.r_{i,t} + x_{i,t}.p_i) \le AT_t$, and $\sum x_{i,t} \ge \sum d_{i,t}$ all $\{I_{i,t}, B_{i,t}\} \ge 0$, and $\sum\sum(x_{i,t+1} - x_{i,t})^2 \le Best_1$ and $\sum\sum((k_i - cs_i.p_i).x_{i,t} + h_{i,t}.I_{i,t} + sc_i.r_{i,t} + B_{i,t}.\pi_{i,t}) \le Best_2$ simultaneously? (Where $r_{i,t}$ is a binary variable determining whether $x_{i,t} > 0$ as $r_{i,t} = 1$ or not as $r_{i,t} = 0$).

Theorem 3. The problem MPMP- CP is NP-Complete.

Proof. It could be observed that: (1) MPMP-CP take account of more variables than SPMP, (2) MPMP-CP has more constraints than SPMP, and (3) MPMP-CP has more objectives than SPMP. Then according to theorems 1 and 2, complexity of MPMP-CP is not less than SPMP. Since SPMP is proven to be NP-Complete, MPMP-CP is NP-Complete too. Furthermore SPMP could be considered as a special case of MPMP-CP. On the other hand, if a special case of a problem is proved to be NP-Complete, then the more general problem is NP-Complete too [13]. So MPMP-CP is surely NP-Complete. Now the proof is complete □.

4. Case Study

In this section, parameters gathered from a real case study in Iran are presented. The plant under study produces different types of a communication control instrument in Iranian police vehicles under the license of two international companies. Demands of each product type ($d_{i,t}$), unit shortage and unit inventory holding costs ($\pi_{i,t}$, $h_{i,t}$) are obtained from market research of the case and shown in Table 2 and Table 3 respectively. Intervals of processing time / cost and setup times/costs are asked from production experts, shown in Table 4. Finally available time of each planning period (AT_t) is obtained by subtracting needed maintenance hours from potential available time of the corresponding planning period, shown in Table 2.

Before running an arbitrary solution method, and for these set of parameters, $Z1_{PIS}$, $Z1_{NIS}$, $Z2_{PIS}$ and $Z2_{NIS}$ are computed according to (28), (29), (35) and (36) respectively as $Z1_{PIS}=0$, $Z1_{NIS}=925579.60$, $Z2_{PIS}=105044$, $Z2_{NIS}=1651899$. All of computations are run on a Pentium IV, 2.53 GHz CPU, 256 MB RAM.

5. Conclusions and Future Research

As it was observed in this paper, the proposed model takes account of different categories of costs, parameters, constraints, and etc, involved in a common production planning. It seems that the model could be developed for many other environments such as parallel machine, flow shop, job shop, open shop, cellular manufacturing systems, etc. Also, the current paper could be extended for dealing with stochastic demands and indefinite cost coefficients. It was assumed that the available time in each period is determined by subtracting the maintenance time from potential working time. But in real world, there are other cases involving uncertain needed maintenance time. Another potential future research is when there are some of dependent products among under consideration products. After all, the paper could be extended to consider infinite planning horizon.

It should be noted that a suitable solution method should be used that at least have the following advantages:

1. The natural conflict of objectives should be observed in the corresponding plot of objectives as Fig. 4, so that improving Z_1 results in the decline of Z_2.
2. Due to model complexity, the solution method should be efficient in both average solution quality and the average CPU time. This could be evaluated via comparison with well-known algorithms such as Multi-Objective Genetic Algorithm (MOGA), Niched Pareto Genetic Algorithm (NPGA), Non-dominated Sorting Genetic Algorithm (NSGA), Sub-Population Genetic Algorithm (SPGA), Strength Pareto Evolutionary Algorithm (SPEA) and etc in average run time and average solution quality.
3. Also, we are better off if the average solution quality is measured by a unique index over FSS. This is because

there is usually a natural conflict between two pairs of objectives in multi-objective problems.

4. If the solution method covers the solutions only in FSS, it has the chance of outperforming most common algorithms proposed in the literature for multi-objective problems.

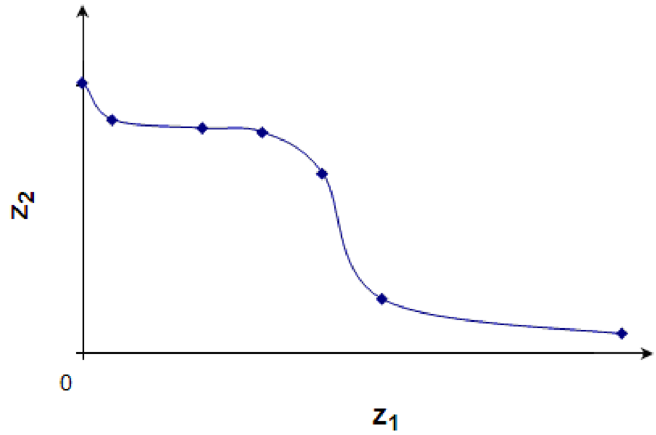


Fig. 4. A Pareto-optimal frontier

Table 2
Demands of products ($d_{i,t}$) and available times (AT_t)

product (i)	planning periods (t)											
	1	2	3	4	5	6	7	8	9	10	11	12
A	15	0	0	33	6	12	30	45	28	18	15	20
B	30	30	46	35	32	15	18	12	6	23	4	0
C	5	18	4	42	47	21	38	26	17	5	5	5
AT_t	5904	588	582	594	576	588	594	570	588	582	594	564

Table 3
Shortage and holding costs of products ($\pi_{i,t}$, $h_{i,t}$)

products	planning periods (t)											
(i)	1	2	3	4	5	6	7	8	9	10	11	12
A	(3,1)	(4,1)	(5,2)	(7,1)	(7,1)	(7,1)	(8,1)	(8,2)	(8,3)	(6,3)	(6,1)	(∞,2)
B	(4,2)	(4,2)	(4,2)	(4,2)	(4,1)	(4,0)	(5,1)	(7,2)	(7,3)	(7,4)	(7,5)	(∞,3)
C	(9,2)	(7,2)	(3,1)	(5,2)	(3,2)	(3,2)	(3,1)	(9,3)	(6,2)	(7,3)	(5,2)	(∞,1)

Table 4
Time/cost of process and setup

product (i)	p_i^n	$C_i(p_i^n)$	p_i^C	$C_i(p_i^C)$	CS_j	k_i	S_i	SC_i
A	11	311	7	530	54.75	913.25	21	2
B	12	33	4	285	31.50	411.00	11	1
C	16	119	5	390	24.64	513.18	18	3

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