Introducing a Relationship for Estimation of the Sediment Transport Rate through Rockfill Structures

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\textbf{ABSTRACT}

Rockfill dams are accounted as useful measures in watershed management projects. However, the sediment transport through the rockfill materials, could impact their lifetime duration. Previously, researches have been conducted to describe the various hydraulically issues of the flow passing through these structures. However, there is no general relationship to estimate the sediment transport rate passing through the of rockfill structures. In this research, by using several sets of data provided from three various sources, we introduced a more general relationship to estimate the sediment transport passing through the rock materials. We compared the predicted results from the new equation with those estimated from the other equations. We found that the accuracy of the new relationships is much more than previously introduced equations. Using more measured data, we found that the mean relative error of the predicted results is \%21 showing the reasonable accuracy of the present relationship.

\textbf{Keywords}
None Darcy flow, Reynolds number, Rockfill structures, Sediment transport rate

\textbf{1. Introduction}

There are many hydraulic investigations on the previous rockfill structures but since these structures are used to convey a large amount of sediment particles, results of most of them are not favorable in real conditions. Designing of such rock embankments has two important issues. First, because of the flow regime deviation from Darcy law and existence of the sediment particles inside the flow, the flow is complicated and there is some uncertainty. Second, threshold level of the sediment motion through the mentioned pore system is dependent upon different variables and could not be estimated easily (Chapokpour and Amiri 2012). Necessity of the critical hydraulic gradient for sediment motion through the pore system was shown in their research. They concluded by depicting and intersecting the transport rate versus pore velocity (Fig. 1) that by an increase in the media Reynolds number (represented by pore velocity) the transport rate increases and a zero transport level is presented by an especial value of Reynolds number.

Normally, existence of sediment particles adds more resistance to the flow which is called skin friction in sediment transport literature but in alluvial flow motion through rock pore system, sometimes clogging process happens which imposes additional resistance against flow direction (Chapokpour et al. 2013).
Nazemi (2011) noted that despite the valuable findings, available research results have some limitations to be used in the real field condition including:

1- Modifications are needed to the previous formulations to make them more applicable,
2- Rock particle sizes and Reynolds numbers were small and must be larger to generalize the results for the real field conditions,
3- More tests must be performed to enable more acceptable analyses.

Also, deposition and erosion of the upstream and downstream faces of detention rockfill dams puts the dam stability at risk and affects the dam safety directly (Samani and Emadi 2003).


Based on Duboy’s approach in open channel flow, Sakthivadivel developed a sediment transport function. This approach assumes a linear velocity change between superimposed sliding layers. In this assumption, it is noted that the initiation of the sediment motion would happen by receiving a shear stress more than the critical level in the upper layer, i.e. the outer layer having a velocity equal to the seepage velocity and the innermost layer being stationary (Joy et al. 1991).

Considering N-1 moving layers of sediments and by assuming the linear distribution of the sediment layers’ velocity, Sakthivadivel (1972) introduced Eq. (1) to estimate the sediment transport rate inside the pore system in laminar flow:

$$q_s = K'_{sak} \rho_s d_s \left( \frac{i}{i_c} \right) \left( \frac{i}{i_c} \right)$$

Where, $i$ is the hydraulic gradient (dimensionless), $i_c$ is the critical hydraulic gradient (dimensionless), $q_s$ is the sediment transport rate (kg/m.s), $\rho_s$ is the density of the sediment particles (kg/m3) and $K'_{sak}$ is a constant coefficient.

Cunningham et al. (1987) conducted some laboratory tests on river bed material in a flume. The length of the flume was 7.6 m and the flow was passed through and over the media. He investigated the clogging
process by sedimentation inside and over the media. Four sediment concentration variables were used by continues sediment feeding mood to find the variations of the material permeability during the time. For each test, flow discharge passing the media was measured in time intervals during 5days and then using dimensional analysis, Eq. (2) was introduced.

\[
\frac{Q_t}{Q_0} = f \left( \frac{V R_h}{\vartheta}, C \frac{t_c}{d_s}, \frac{R_h}{H} \right)
\]  

(2)

Where \(Q_t\) is the magnitude of the initial discharge passing the bed material (lit/s), \(Q_0\) is the discharge in time t after sediment feeding (lit/s), \(V\) is the average flow velocity in flume (m/s), \(R_h\) is the hydraulic radius (m), \(\vartheta\) is the kinematic viscosity of the fluid (m²/s), \(C\) is the weighted concentration of the sediment (dimensionless), \(t_c\) is the thickness of the sediment layer over the material (m) and \(H\) is water height over the material (m).

Results showed that reduction of the permeability (represented by \(\frac{Q_t}{Q_0}\)) had a favorable correlation with \(\frac{V R_h}{\vartheta}\) and \(\frac{t_c}{d_s}\) but it had an unfavorable correlation with other two non-dimensional parameters.

Joy et al. (1991) conducted a series of experiments by applying turbulent sediment contained flow through large porous media. They accomplished the experiments considering non-linear flow conditions. One of the most important limitations of the research work validation was the lack of experimental and theoretical knowledge for doing the comparisons. Their results were compared with the limited existing data and the model which was developed by Sakthivadivel for linear flow conditions. It was not valid because of the difference in flow regimes of the experiments. Sakthivadivel model was also altered to make it applicable for the test conditions. Evaluation of Joy’s results suggested an alternative transport model using suitable dimensionless parameters. The model was shown to be superior to the other models until 1991 and most of the future investigators did their researches by focusing on their non-dimensional parameters.

Joy et al. (1991) accomplished a research on a laboratory flume with variable slopes and a confined porous media in the turbulent sediment contained flow regime. They used three different sediment particle sizes and three different porous media particle sizes to run 40 tests. They also limited the Reynolds numbers in porous media in the range of 180–940.

They used dimensional analysis and derived the following formula for sediment discharge.

\[
q_s = 26.2 (R_e)^{-1.23} (\lambda_d)^{0.54} (S_p)^{-1.39}
\]  

(3)

Where:

\[
q_s = \frac{n_d}{\rho_s \lambda d B_b} 
\]  

(4)

\[
R_e = \frac{V d_n \vartheta}{\rho \vartheta} 
\]  

(5)

\[
\lambda_d = \frac{d}{d_s} 
\]  

(6)

\[
S_p = \tan (\varphi - \theta) 
\]  

(7)

In which: \(q_s\), \(R_e\), \(d\), \(d_s\), \(\lambda_d\), \(S_p\), \(\varphi\), \(\theta\) and \(n\) are dimensionless sediment discharge, Reynolds Number in porous media (dimensionless), porous media particle size (m), sediment particle size (m), ratio of the porous media particle diameter to sediment particle diameter (dimension less), dimensionless parameter of slope, angle of repose of sediment in porous media, angle of bed slope, kinematic viscosity of water (m²/s) and porosity of rock media (dimensionless), respectively.
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Emadi et al. (2003) presented an equation to estimate sediment transport rate through rockfill dams as:

$$q_s = A \cdot \rho_s \cdot d_s \cdot \left( \frac{Q - Q_c}{Q_c} \right)^B$$  \hspace{1cm} (8)

In which the terms of the equation are as follows:

$$Q_c = E \cdot h \cdot i_c^{0.52}$$  \hspace{1cm} (9)

$$E = \left( \frac{\beta (n, d)}{27} \right)^{0.52}$$  \hspace{1cm} (10)

Where $h$, $Q_c$, $q_s$ are the average water depth upstream of the dam (m), critical water discharge per unit width of the dam (liter/m.s) and sediment discharge per unit width through the rockfill dam (kg/m.s), respectively.

They arranged 55 tests to find out $A$ and $B$ values. They also performed some experiments for the critical hydraulic gradient and tried to do experiments for the transport rate with identical material sizes. They calibrated the equation using test results and a regression analysis (Correlation Coefficient = 95%) and determined the coefficients as:

$$q_s = 0.046 \rho_s \cdot d_s \cdot \left( \frac{Q - Q_c}{Q_c} \right)^{0.62}$$  \hspace{1cm} (11)

In their experiments, the pore Reynolds number was limited to the range of 1165-3990 and the bulk Reynolds number (Reynolds number of the flow through dam body) was limited to the range of 478-1835.

Mousavi et al. (2010) designed their tests to develop two equations for predicting the critical hydraulic gradient and sediment discharge through a rockfill dam. They used sakthivadivel’s model of critical hydraulic gradient for laminar flow by applying some modifications to make it suitable for the turbulent flow in rockfill dam. They carried out 18 laboratory tests in a fixed bed slope flume in which rockfill and sediment particle sizes were in the range of 3-4.5 cm and 0.15-0.36 mm, respectively. Reynolds number of their tests was also limited to the range of 3174-9667. They conducted 36 laboratory tests using the same material and set up as the critical hydraulic gradient tests to calibrate and validate the proposed formula in the subject of sediment transport rate.

Their sediment discharge equation is presented in the following form.

$$q_s = 0.0725(R_e)^{-0.3517} (\lambda_d)^{-0.1346} (S_p)^{0.258}$$  \hspace{1cm} (12)

They assumed that the bed slope parameter is equal to the hydraulic gradient ($S_p = i$).

Nazemi (2011) performed a wide series of laboratory tests (144 tests) including three different bed slopes (0.001, 0.005 and 0.01), three different sediment particle sizes (0.425mm, 0.6mm and 0.85mm), two different rock particle sizes (5 and 12) cm, two different standard deviations for rock particles (1and2) and four different water discharges (5.8 up to 30.7, liter per second). He calibrated his modified relationship and introduced Eq. (13).

$$q_s = 41.64(R_e)^{-0.992} (\lambda_d)^{0.32} (i - i_c)^{0.556}$$  \hspace{1cm} (13)

Where, $i$ is the hydraulic gradient over the rockfill dam and $i_c$ is the critical gradient for initiation of sediment motion through the media.

Generally, real conditions of rockfill dams and rock embankments show invalidation of the assumed condition by Sakthivadivel. With these situations hydraulic gradients tend to be much higher and the pores are somewhat larger.

This in turn leads to the situation of non-linear (called None-Darcy) flow, i.e. the resulting seepage velocity is no longer proportional to the gradient with power of 1 and Darcy's law is invalid. Deviations from Darcy's law typically begin at a media
where Reynolds number \( R_m = \frac{V_d m}{n \theta} \); \( \theta \): fluid viscosity, \( n \): media porosity, \( d_m \): media size), is somewhat between 1 and 10. Inclusion of the porosity in the expression for Reynolds number essentially converts the seepage velocity to the interstitial velocity (Bear, 1972).

Most of the above mentioned investigators have focused on the basic relationship presented by Sakthivadive (1972) and Joy (1991). They tried to calibrate the coefficients with experimental data for both liner and none-linear conditions. But a general relationship which could cover all the available data with better estimation accuracy is not presented yet. Nevertheless, the results of each investigator show some limitations such as rock particle sizes and ignorance of turbulent flow inertia and ignorance of real field condition which should be generalized.

2. Laboratory and experimental conditions of data series

In this research it was tried to collect all the experimental data from 3 previous studies by knowing the fact that all were performed in fully developed turbulent conditions. A short description of the experimental tests is illustrated in the Table 1. Total number of experiments was 208. It should also be noted that all of the above mentioned tests were performed in free water surface conditions and were extracted from laboratory flume, not from the pressurized permeable media. In all data sets water profiles were measured during the operation using image processing from piezometers.

3. Results and discussion

To conclude a suitable equation from previously presented literature such as the equation to estimate sediment discharge through rockfill embankments with acceptable accuracy subjected to sediment transport cases, some modifications were done to Joy’s basic equation and then using available laboratory data, the results of the new relationship was compared to the results provided from previously introduced relationships.

It is worth mentioning that because of some other factors which could not be considered, there is some uncertainty in sediment transport cases. It is important that the rate of the mentioned uncertainty in the case of sediment transport through rockfill media would be higher than other sediment transport cases. Therefore, as mentioned by sediment transport engineers, any equation that could estimate the subject with an accuracy of \( \pm 40\% \) would be reliable.

Table 1. Experimental characteristics of data series

<table>
<thead>
<tr>
<th>No.</th>
<th>Data index</th>
<th>Investigator name and research date</th>
<th>Number of Media diameter (mm)</th>
<th>( d_{50} ) of Medium diameter (mm)</th>
<th>( d_{50} ) of sediment diameter (mm)</th>
<th>Range of Media Reynolds number</th>
<th>Medium length, width and height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SC-1</td>
<td>Samani and Emadi (2003)</td>
<td>2</td>
<td>14.5 and 21</td>
<td>0.256, 0.363 and 0.512</td>
<td>1100-4000</td>
<td>0.6, 0.3, 0.3</td>
</tr>
<tr>
<td>2</td>
<td>SC-2</td>
<td>Mousavi (2011)</td>
<td>2</td>
<td>30 and 45</td>
<td>0.15, 0.27 and 0.36</td>
<td>7000-13000</td>
<td>0.3, 0.6, 0.7</td>
</tr>
<tr>
<td>3</td>
<td>SC-3</td>
<td>Nazemi (2011)</td>
<td>2</td>
<td>50 and 120</td>
<td>0.425, 0.6 and 0.85</td>
<td>2000-20000</td>
<td>0.78, 0.6, 0.58</td>
</tr>
</tbody>
</table>
The most famous relationship for sediment discharge through rockfill media is Joy’s equation. Previously Samani and Emadi (2003), Mousavi et al. (2010) and Nazemi (2011) have done some corrections to develop a new relationship for None Darcy flow through rockfill media by changing the form of the equation. But unfortunately they did not generalize their research to other investigator’s data.

In the present study, other dimensionless parameters ($\lambda_1$ and $i$) are added to the Joy’s basic equation in the form of Eq. (14).

$$q^* = k(\lambda_1)^a(\lambda_2)^b(R_e)^c(i)^d(S_p)^e$$  \hspace{1cm} (14)

Dimensionless parameters of the above equation are presented in the following equations.

$$q^* = \frac{n q_s}{\rho_s d_s v_p}$$  \hspace{1cm} (15)

$$\lambda_1 = \frac{(d_m - \delta)}{d_s}, \lambda_2 = \frac{l}{(d_m - \delta)}$$  \hspace{1cm} (16)

$$R_e = \frac{V_m d}{u s}$$  \hspace{1cm} (17)

$$i = \frac{\Delta h}{l}$$  \hspace{1cm} (18)

$$S_p = \tan(\phi - \theta)$$  \hspace{1cm} (19)

Where, $\delta$ is standard deviation of media rocks (m), $\Delta h$ is total head over the media (m) and $l$ is media length (m). Other terms of Eq. (14) were identified previously.

The coefficients and exponents of the proposed relationship were calibrated with experimental data series and the final relationship is illustrated in Eq. (20). $q^* = 0.0057 (\lambda_1)^{0.567} (\lambda_2)^{-2.124} (R_e)^0.637 (i)^{-0.105} (S_p)^{-0.07}$  \hspace{1cm} (20)

A comparison was done between present study and two recently introduced relationships (Nazemi 2011 and Mousavi et al. 2011). As illustrated in Fig. 2, if all the available data is participated in estimation of the sediment discharge, Mousavi’s equation underestimates the transport rate with relative average error of 61% and Nazemi’s equation overestimates the subject with relative average error of 212%. Despite the generation of favorable none-dimensional parameters by Joy, unfortunately because of the lack of data series in extraction of his equation, the equation has a low accuracy of estimation than all of the previous equations.

In this research, the obtained equation is calibrated and validated with results of 208 laboratory tests from three different sources. As it is shown in Fig. 2 the present study (Eq. (20)) has the best estimation accuracy and all of the predicted data are in good agreement with the observed data.

![Fig 2. A comparison between previously introduced relationships and the present study](image-url)
The mean relative error of presented equation is 21%. As mentioned previously, this accuracy is valuable in sediment transport.

Some of the randomly selected data series were not participated in equation development for the validity test of the extracted equation. In these data series, the observed dimensionless sediment discharges versus predicted values are depicted in Fig. 3. As it is shown, data points are around the line of agreement and the magnitude of R square is about 0.92 which verifies the accuracy of the equation.

4. Conclusion

The introduced relationships to determine sediment transport rate through the porous media of rockfill dams are limited to apply for a wide range of conditions because of their limitations. By using three available sets of laboratory data, and based on the equation provided by Joy et al. (1991), we introduced a new relationship to determine the sediment transport rate. We compared the predicted results of the new relationship with those provided from the other relationships and we find that the new relationship has a much better agreement with observed data than the others. Moreover, using more observed data, we find that the mean relative error of the predicted results by the new relationship is 21%, indicates that it can be used to estimate the sediment transport rate with reasonable accuracy.

References

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