Influence of Slope and the Number of Steps on Energy Dissipation in Stepped Spillway Using Numerical Model

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ABSTRACT

Recently the stepped spillways have been used as an appropriate solution for energy dissipation. In the present study, Siahbisheh dam spillway is simulated by using Computational Fluid Dynamic (CFD), in which the Mixture method and Reynolds Stresses Model (RSM) turbulence model is used. In the first modeling series the overall steps slope is constant. The number of the steps is increased to 80, in which the step height and length is decreased to 0.51 and 1.53 m, respectively. In another case, by reducing the number of the steps to 30, step height and length increased to 1.36 and 3.9 m, respectively. In the next modeling series, the steppes slope was changed. By reducing the number of the steps in to 30, spillway slope increased to 33$^\circ$ and the height of the steps increased to 1.36 (step length was 2.1 m). Then by increasing the number of the steps to 80, spillway slope decreased to about 14$^\circ$ and the height of the steps decreased to 0.51m (step length was 2.1 m). Results analysis and comparison of the increasing the energy dissipation to 60 steppes case shows that by decreasing the number of the steppes into 30, the energy dissipation decreases. This trend is expected to continue by increasing the steppes number into 80, but it is not. It seems that there is no fixed pattern between the number of the steppes and energy dissipation percentage. It can be said the increasing the number of the steppes and decreasing the slope up to a certain value increases the energy dissipation, and this number would be optimal under certain conditions. Finally it seems that the effect of the slope is considerable, so that the case in which larger number of steppes with a mild slope is closer to the optimum condition of maximum energy dissipation.

Keywords

Stepped spillway; Energy dissipation; Numerical method; CFD

1. Introduction

In recent decades with the appearance of roller compacted concrete (RCC) technique in dam construction, tendency towards construction of stepped chute has increased. RCC dams are now constructed in such a way that the downstream body is stepped to dissipate the energy of the flood passing over them. Reduction of the stilling basin dimensions at dam downstream to reduce to significant energy dissipation along the chute. Overviews of spillways with particular focus on stepped chutes are given by Chanson (1994); Vischer and Hager (1998) and Minor (2000).

The aim was to investigate the scale effects in modeling skimming flow down stepped spillways, the inception of air entrainment, and air concentration and velocity distributions.

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Risk of cavitation due to excessive negative pressure is omitted by applying stepped spillways which substantially decrease flow speed and increase considerable air entrainment into the flow. Geoffrey et al. (1999) implemented a comprehensive study on the effects of step geometry on flow regime, the amount of dissipation, equilibrium conditions downstream a stepped spillway and the scale effect on the results.

They set up two series of models of 1:0.6 slope with different step heights. Their results show that the percentage of energy dissipation is independent of step height and this result does not depend on flow equilibrium conditions. Equilibrium momentum flux is achieved 30 to 50 meters far from the spillway crest. Percentage of energy dissipation for the stepped spillway under the equilibrium conditions at the height of 58 meters below dam crest, decreases from 60% for \( h_c = 0.3 \) to 54% for \( h_c = 3.5 \). Tabara (2005) numerically simulated the flow over these spillways with different combinations of the steppes. Judi et al. (2011) numerically investigated the flow profile in the stepped spillways of RCC dams by considering roughness and steppes dimension variations. Chamani and Rajaratnam (1999) studied velocity profiles and air concentration in large experimental simulation in two different slopes and in a range of flow intensities for the values of \( dc/h = 0.7-4.4 \).

They mentioned relative energy dissipation over these spillways in the range of 48-63 percent. Cheng et al. (2005) used Volume of Fluid Models (VOF) and the multi-phase mixture flow to simulate the flow over the stepped spillways. They concluded that Mixture Model had evident advantages over VOF model.

Chen et al. (2002) numerically simulated stepped spillways by using finite difference method and the application of unstructured mesh. However, despite the limitations and the high costs of experimental models, numerical modeling has been less conducted on these spillways in comparison to experimental investigations.

2. Materials and Methods

2.1. Experimental Model

In the original model, the spillway sill was 20m long, inlet channel was 44m wide, Probable Maximum Flood was 203 m\(^3\)/sand water height over the spillway for the Probable maximum flood was 2.9m. Guide wall in the right hand side of the spillway is an arc with the radius of 25m and its beginning is in the form a half-circle of radius 1.5m and the length of the wall from the beginning to the sill was 8.17m. The distance between the beginning of the wall, after the semi-circular, up to the location of the joint to the sill was 6.16m. Guiding wall of the left hand side of the spillway consisted of four sections including a straight part of 0.85m long, a circular arc of 25m radius and an angle, an arc with the radius of 1.25m and an angle and a length of 5.33m leading to steeped mountain cutting level. Spillway control structure includes a curved and the ogee part.

By considering the center of the coordinates on the spillway crest, spillway ogee profile would be as \( y = 0.228x^{1.85} \) and the upstream curve of the ogee would be identified by:

\[
\frac{x^2}{0.479} + \frac{(0.406 - y)^2}{0.165} = 1
\]

2.2. Numerical model

Numerical modeling of the present study includes the Spillway of upstream Siahbisheh Dam with 1:15 scale for the experimental model which is modeled Water Research Institute. Modeled Spillway width, height and slope are 1.33, 2.853m and 18.43 degree, respectively. Step height is 4.76cm and its width is 14cm. Experiments and numerical modeling were performed by using four flow...
rates of 25, 60, 150 and 203 m$^3$/s that are shown in fig. 1 and 2. Modeling was performed by using CFD Method to create the geometry of the spillway.

2.3. Boundary Conditions

The water inflow velocity can be calculated according to the discharge and the water depth at the inlet. The air boundary was set as pressure-inlet conditions on which atmospheric pressure was assumed. Because the boundary between water and air at the downstream outlet could not be distinguished, it was defined as a pressure boundary, or free-flow condition. Through the time-dependent simulation, the water flowed over the spillway and produced air-water two-phase flow.

Spillway geometry and boundary conditions have been introduced into the software as shown in fig. 3. The position of the different sections of the model is described.

![Fig. 1. Longitudinal section of the spillway control structure.](image1)

![Fig. 2. Schematic view of the spillway and ramp model.](image2)
2.4. Details of the Numerical Model

Experimental model and simulated model geometries are the same and the tri/pave pattern was used for meshing the model. In order to prepare the flow field geometry, its meshing is done by using a software, and the mixture model is applied to model the free surface (fig. 4). Standard scheme was used for pressure secession, Quick pattern for secession of the movement terms in momentum equations, the first-order Upwind scheme for secession of the convective terms of the turbulent equations. PISO (Pressure-Implicit with Splitting of Operators) algorithm is also used for pressure-velocity coupling and 0.10s time step for solving the equations, based on convergence pattern of the simulation. Also, using the discount factors less than one, for the pressure, momentum and Reynolds stress prevents the divergence of the solutions.

2.5. Appropriate Turbulence Model

Results obtained from Reynolds Stresses Model are in a good agreement with real ones and shows the eddy flow inside the steppes pretty well. Therefore Reynolds Stresses Model (RSM) was used as turbulence model to simulate the flow field (fig. 5).

2.6. Continuity Equation for the Mixture Flow

Continuity equation for the mixture flow is defined as:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{U}_m) = 0$$

(2)

Fig. 3. Position of different boundary conditions on the Siahbisheh spillway model

Fig. 4. Geometry of the step spillway
In which the mixture density and the mixture velocity are defined as follows:

\[
\rho_m = \sum \alpha_i \rho_i \tag{3}
\]

\[
u_m = \frac{1}{\rho_m} \sum \alpha_i \rho_i u_i \tag{4}
\]

Where \( \alpha_k \) and \( \rho_k \) are the volume fraction and density of phase \( k \), respectively. The mixture velocity, \( u_m \) represents the velocity of the mass centre of the mixture flow. Note that \( \rho_m \) may vary even though the component densities keep constant.

### 2.6.1. Momentum equation for the mixture flow

Momentum equation in terms of the mixture variables takes the form:

\[
P_m U_M + \nabla \cdot (P_m U_M U_M) = -\nabla p_m + \nabla \cdot (T_M + T_{DM}) + \nabla \cdot (M_{EF} \nabla U_M) + P_M G \tag{5}
\]

### 2.6.2. The two stress tensors

The two stress tensors are defined as:

\[
\tau = \sum \alpha_i \rho_i u_i \nabla u_i \tag{6}
\]

\[
\tau_{DM} = -\sum \alpha_i \rho_i u_i u_{mk} \tag{7}
\]

Where \( u_{mk} \) is the diffusion velocity for the mixture flow. The two stress tensors represent respectively the average viscous stress and diffusion stress due to the phases slip. In “Equation (5)” the pressure of the mixture flow is defined by the relation

\[
\nabla p_m = \sum \alpha_i \nabla p_i \tag{8}
\]

In practice, the phase pressures are often taken to be equal, i.e. \( p_k = p_m \).

### 2.6.3. Continuity equation for phase \( k \)

From the continuity equation for secondary phase \( k \), the volume fraction equation for secondary phase \( k \) can be obtained.

\[
\frac{\partial}{\partial t} (\alpha_k \rho_k) + \nabla \cdot (\alpha_k \rho_k u_k) = -\nabla \cdot (\alpha_k \rho_k u_{mk}) \tag{9}
\]

### 2.6.4. The relative velocity

Before solving the continuity "Equation (8)" for phase \( k \) and the momentum "Equation (4)" for the mixture, the diffusion velocity \( u_{mk} \) has to be determined. The diffusion velocity of a phase is usually caused by the density differences, resulting in forces on the bubbles different from those on the fluid. The additional force is balanced by the drag force.

\[
u_{mk} = u_{qk} - \sum \alpha_i \rho_i \frac{u_{mk}}{\rho_k} \tag{10}
\]

Where \( u_{qk} \) is the slip velocity between air
and water, defined as the velocity of air relative to the velocity of water. Following Manninen et al. (1996), \( u_{qk} \) is defined as:

\[
u_{qk} = \left( \frac{\rho_m - \rho_k}{\mu_{eff,m}} \right) \frac{d_k^2}{18} [ g - (\mu_m \nabla)u_m - \frac{\partial u_m}{\partial t} ]
\]

Where \( \mu_{eff,m} \) is the effective viscosity of mixture and \( d_k \) is the diameter of the particles (or bubbles) of secondary phase \( k \). The bubble diameter used in the simulation is 5mm. Reasonable agreement with the experimental data of Cummings is obtained by using this bubble diameter value. The drag function \( f_{drag} \) is taken from Clift et al. (1996):

\[
f_{drag} = \begin{cases} 
1 + 0.015 \text{Re}^{0.487} & \text{Re} \leq 1000 \\
0.0182 \text{Re} & \text{Re} \leq 1000 
\end{cases}
\]

3. Similarity and Scale Effects.

The exact behavior of two-phase flow of air to replace the free surface profile and high turbulence model, three rules are similar that
Based on the force of gravity is considered similar incident. Common law, the relationship between the Froude number, \( Fr = \frac{q_w}{\sqrt{gh^2/\alpha}} \) should be equal for model and prototype. In this case the kinetic energy correction factor is added due to uniform velocity distribution. In Reynolds analogy, the important role of viscosity is considered by using Reynolds number equality of the non-aerated flow in the original structure and model \( Re = \frac{q_w}{\nu} \) in terms of fluid kinematic viscosity, \( \nu \). In the models that surface tension is important, Weber number \( We = \frac{U_m}{\sigma/\rho_m} \) should be equal in prototype and the model. In this relationship, \( \sigma \) is the surface tension (for water in a flume \( \sigma = 0.072N/m \)) and \( h_m = \frac{h_m}{\sin \theta} \) suppose in \( g h_m \) as the longitudinal distance between the corners of the two consecutive steppes. Since satisfying all the above mentioned conditions simultaneously is not possible, and the gravity force has impacts on the system, all of the constructed models to conduct researches on stepped spillways, including the present model are constructed based on Froude number similitude.

4. Measured Velocity

Many studies have been conducted to measure velocity. Measurements or velocities measured directly by double optical fiber, optical probes and Pitot tubes sometimes in streams with high air concentrations do not enjoy a high degree of precision, or indirectly by measuring the depth of the relationship

\[ V = \frac{q_w}{d}, \]

the net flow depth and \( d \) (the following equation) is calculated:

\[
d = \int_C \left( 1 - c \right) dy = \left( 1 - C_{mean} \right) y_{w90}
\]

In the above equation \( y_{w90} \) is the mixture flow depth at point \( C=0.9 \) and \( C_{mean} \) is the average air concentration in the section that is obtained from \( C_{mean} = \frac{1}{y_{w90}} \int_0^{y_{w90}} c \cdot dy \) relationship.

5. Results of the Numerical Model.

The numerical model results are shown in figs. 6-8. This section shows results of the present study and their analysis.

Flow simulation software, was performed for four different flow that velocity profiles at different sections of the dam spillway shown in figs. 9-12. In all the graphs \( \lambda \) is the scale factor, \( h/l \) is height to length ratio of the step and \( q \) is flow discharge per unit width.

6. Numerical Results Verifying

6.1. Comparison and Verification with Hager’s Studies

For the comparison, profiles obtained from one of the Hager’s studies (2003) (who has done lots of investigations in this field) are
Fig. 6. Q=60 and stepped chute contour

Fig. 7. Q=60 and stepped chute velocity plot

Fig. 8. Q=203 and stepped chute velocity plot

Fig. 9. Velocity profile for Q=25, q=4.6, h/l=0.33, λ=15

Fig. 10. Velocity profile for Q=60, q=4.6, h/l=0.33, λ=15

Fig. 11. Velocity profile for Q=203, q=10, h/l=0.33, λ=15

Fig. 12. Velocity profile for Q=150, q=7.5, h/l=0.33, λ=15
considered. In this section the purpose is to observe and investigate the trend of the diagrams and compare them to Hager results. fig. 13.

Hager results obtained for different scales. Horizontal axis represents the average velocity and the vertical axis represents the depth.

Figs. 9 to 12 plot the velocity variations of the air-water mixture on the basis of the distance perpendicular to the flow direction at different sections of the spillway in the present study. Charts also confirm that one of the numerical modeling results is acceptable.

6.2. Numerical Results Verified by Using Bose and Hager Results.

In this section numerical results are compared and verified by using the results of other investigators. According to the results of the experimental studies of Bose and Hager (2003) on different stepped spillways, two equations for the velocity and concentration distribution in terms of dimension less depth is presented as follow:

\[ U_{90} = Y_{90}^{1/4.3} \quad \text{for} \quad 0.04 < Y_{90} < 0.8 \quad 1.05 \]  

\[ U_{90} = 1 \quad \text{for} \quad Y_{90} > 0.8 \]  

Where \( y_{90} = \frac{y}{y_{90}} \) and \( u_{90} = \frac{u}{u_{90}} \), that \( y_{90} \) is the mixed flow depth at the point that there is 90% air and \( U_{90} \) is the velocity at this point. To compare the results of this study, some of the velocity profiles obtained by these relations are presented in figs. 14 and 15.

We can see that the results of the numerical simulation are in good agreement with Hager equation results, only in zones near the bottom the results of numerical method are less than Hager equation predictions.
7. Energy Dissipation Rate

Energy, $H_t$ is calculated upstream of the spillway crest as follows:

$$ H_t = y_0 + \frac{v_a^2}{2g} \quad (16) $$

In the above equation $y_0$ water depth upstream of the spillway, $v_a$ is the adjacent speed to the spillway and $g$ is the acceleration of gravity. To calculate the energy downstream of the spillway and before the jump the following equation was used:

$$ \frac{\Delta H}{H_t} = \frac{H_r - H_i}{H_i} \quad (17) $$

8. Hydraulic and Geometric changes

After comparing the numerical results to experimental and other researchers’ results, different parameters on the stepped spillway such as hydraulic parameters (flow discharge) and the geometry (number and height of the steppes) are changed. Hydraulic changes are considered as flow discharge changes and the results are as follows:

8.1. Effects of Hydraulic changes

figs. 16 and 17 show velocity variations for different flow rates in f and i sections. In these figs., the dimension less velocity versus dimensionless depth is plotted for different flow rates. As can be seen the general pattern of the profiles are almost identical, but generally value of $U_{y0}$ for higher flow rates is greater for the depth of $Y_{y0}=1$. In both f and i sections, $U_{y0}$ decrease to some extent after reaching the $Y_{y0}=1$ for higher flow rates. For low flow rates this happens in higher $Y_{y0}$ that could be due to faster increase in concentration up to $c=0.99$ at higher flow rates. While for low flow rates.
8.2. Effects of Geometric Changes

In this section the influence of geometry changes in the spillway on energy dissipation over the spillway is discussed. Wide range of parameters predicting the dominant flow regime, as mentioned by researchers shows that flower regime determination, especially in regime conversion zones is almost impossible and errors due to personal judgments could not be neglected.

Accordingly, with respect to the existence of conversion flow regime and continuity of the two other regimes it seems that separate studies of the energy dissipation for each regime is not suitable enough and does not give good result. So the effect of flow regime in studying effective parameters on relative energy dissipation of the stepped spillways is ignored and all the regimes are studied as a whole. The effect of geometry changes on energy dissipation rate was investigated in the following situations.

8.2.1. Changing of the number of steppes at constant slope

In one case, the spillway geometry remained without any changes and the slope was kept constant; i.e. the overall slope and slope of each of the steppes. Then the problem was modeled in two cases; first by increasing the number of the steppes into 80 in which steppe’s height and length reduced into 0.51 and 1.53, respectively, and in the second case, by decreasing the number of the steppes into 30, steppe’s height and length increased into 1.36 and 3.9, respectively. That the results in fig. 18 are shown.

8.2.2. Changing of slope and number of steppes

In another case, geometry of the spillway was changed so that by changing the number of the steppes from 60 to 30 and 80, steppes’ slope and height changed, but the general spillway height and steppes’ length remained unchanged. By the reduction in the number of the steppes, spillway slope increased from 18.4 to 33 degrees while the length of the steppes remained unchanged. By the reduction in the number of the steppes to 80, the slope of the spillway sloped decreased to about 14 degree and the height of the steppes increased to 1.36 m. The output of the software showed that the energy dissipation is reduced. By increasing the number of the steppes to 80, the slope of the spillway sloped decreased to about 14 degree and the height of the steppes decreased to 0.51 m (the length of the step was 2.1m). That the results in fig. 19 and 20 are shown.
It can be seen from fig. 21 that an increase in \( \frac{y_c}{\Delta z} \) ratio (increasing the flow discharge with constant height of the spillway) causes a relative decrease in the energy dissipation of the flow. Also, steeper channel slope and less number of the steppes, in the investigated range of variables caused greater energy dissipation. This result is exactly consistent with the conclusions based on the following equation of Chansun (2002) for the effect of the number of the steppes in falling flow on mild channel slope.

\[
\Delta H \frac{H}{H_c} = \left[ \frac{0.54}{1.5} \left( \frac{y_c}{h} \right)^{0.275} + \frac{3.43 \left( \frac{y_c}{h} \right)^{0.55}}{1.5 + \left( \frac{H_j}{y_c} \right)} \right]
\]

\[
d \leq 0.0916 \left( \frac{h}{l} \right)^{-0.276}, \quad 0.2 \leq \frac{h}{l} \leq 0.6
\]

9. Determination of the effective parameters on stepped spillways

One of the important parameters of the stepped spillways is the starting point of the uniform flow that the flow characteristics remain almost constant after this point and the energy dissipation from this point up to the end of the spillway is equal to the height of this point. The longitudinal velocity profiles plotted by using experimental results show that after a specified point, profiles do not change a lot. Longitudinal velocity profiles for the experimental results were presented in the previous sections. Longitudinal velocity profiles for the numerical model will be first discussed in this section. Then starting point of the uniform flow will be determined by using other researchers’ equations and the results will be compared. Fig. 18 shows the longitudinal profiles obtained for different flow discharges.

As we can be seen the longitudinal velocity profiles remain almost constant after reaching a certain point. For example for the discharge 150m\(^3\)/s, section K presents this point. Several relations are presented by the researchers in determining this point. Here in this paper some of them are compared to our results. Christodoulou (2002) equation for the stepped spillways, for example, with a slope angle of \( 25 < \alpha < 55 \) is presented as follow:

\[
L_u = \frac{8.6q_w^{0.713}}{k_s^{0.085}(\sin \alpha)^{0.277}}
\]

Where \( L_u \) is the distance from sill to the starting point of the uniform flow, \( q_w \) is the discharge per unit width, \( S \) is the height of the step, and \( k_s = s \cdot \cos \alpha \) is the vertical depth of the steppes.

Boes and Minor (2003) presented the following equations for the determination of the uniform flow location based on the critical depth, \( h_c \):

\[
\frac{L_u}{h_c} = \frac{15}{\sin \alpha} \quad \alpha = 30^\circ
\]

\[
\frac{L_u}{h_c} = \frac{35}{\sin \alpha} \quad \alpha = 50^\circ
\]

Boes and Hager (2003) also presented an equation for the starting location of the
uniform flow \((L_u)\) in terms of roughness Froude number:

\[
\frac{L_u}{h_c} = 25.52\sqrt{1-0.055x(sin\alpha)^{\frac{1}{3}}} \times (sin\alpha)^{\frac{1}{3}} \tag{22}
\]

\[
F^* = \frac{q_u}{\sqrt{g \sin\alpha \delta}} \tag{23}
\]

In fig. 22 can be seen that the numerical model calculates higher values of \(L_u\) in comparison to the other results. It should be noted that the slope of the spillway in the present model is 18.43 degrees, which is lower than the assumed value for Christodoulou (2002), and Bose and Minor (2003) equations.

**10. Conclusion**

By the comparison between the experimental data and the numerical results obtained by using CFD, it was shown that the mentioned software suitably models flow on stepped spillways. Results of the flow velocity changes for different flow discharges show that the trends of the dimensionless velocity and dimensionless depth are roughly the same. But generally the value of \(U_{90}\) for higher flow rates is greater in \(Y_{90} = 1\). In various sections and for higher flow rates, \(U_{90}\) decrease to some extent after reaching the \(Y_{90} = 1\). For low flow rates this happens in higher \(Y_{90}\) values that could be due to faster increase in concentration. In other words, for lower flow rates concentration changes from \(c=0.9\) to \(c=0.99\) happens in deeper ranges of depth. Generally, dimension less velocity up to \(Y_{90}=1\) is less for higher flow rates. By an increase in \(y_c/\Delta z\) ratio the relative decrease of the energy dissipation of the models is reduced. It can be argued that by increasing the flow discharge and hence, increasing the speed and Froude number, flow thickness on the steppes in stepped spillways increases. This reduces the effect of step’s roughness on the flow and thus reduces the amount of energy dissipation. Furthermore, by increasing the critical depth, the energy of the flow passing the steppes is increased while the step height is still constant, so relative energy dissipation is decreased.

In this study, changes in geometry were performed so that the overall height of the spillway remained constant. In one case spillway slope was constant and the number of the steppes was changed, and in another case spillway slope was changed. In any case of slope change, spillways with different number of steppes were modeled. Flow discharge variations were applied in
each spillway model and the hydraulic changes were investigated. It was seen that increasing the slope and the number of the steppes up to a specific value increased energy dissipation. Optimized case of the spillway was determined this way. This number was determined as 60 for the present study. In other words, more and less than 60 steppes cause lower energy dissipation. However this number of steppes can be different for other spillways, so that there is no fixed relationship between the energy dissipation and the number of the steppes and the slope of the spillway. Also the trend of the slope increase has a greater effect on energy dissipation and is effective in maximizing the energy dissipation in comparison to the number of the steppes. Also it was observed that the relative energy dissipation decreases by increasing the flow discharge.

References