B2B electronic market analysis using game theory

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Abstract: In the explosive growth of Business to Business (B2B) Electronic Trades, electronic markets have received a great deal of attention recently. The obtained profit of trading in E-B2B market encourage market participants to remain in the market. Market participants consist of: sellers, buyers, and market owner. In this paper the expected profit function for each market participant has been defined in a neutral market based on double auction. Also, the model is simulated and results are shown. Linear programming in game matrix is used to exhibit the capability of the proposed model to support the decision making process. Then the model is exemplified.

Keywords: B2B electronic market; Double auction; Game theory; Linear programming; Neutral market

1. Introduction

B2B electronic market is the online market that facilitates the transactions between businesses to exchange their fabric and products. There are extensive studies analyzing the design and implementation of electronic market mechanisms (Anandalingam et al., 2005; Geoffrion and Krishnan, 2003). The success of B2B e-commerce was accurately predicted by Wang and Lin (2009) for small and medium enterprises. They suggested that the implementation of B2B e-commerce is time consuming and the long-term impact on an organization may be unclear for some time.

In the research by Wang and Benaroch (2004), a mathematical model is introduced that compared the condition of buyers and sellers in both online (electronic) market and offline (traditional) market. This model determines the situation that buyers and sellers will be satisfied in the electronic market. Also, in their analysis process, they considered the integrated and decentralized supply chain.

Berg et al. (2004) used a probabilistic fuzzy modeling for financial markets analysis. Hill and Watkins presented some classifications for seller type, three types (Wang and Hill, 2005) four types (Hill and Watkins, 2009) and their motivations and reinforcement. Then they simulated the trades between seller agents and buyer agents. Based on the Iterative Prisoner’s Dilemma and its associated payoffs, computer-generated sales agents are defined (Hill and Watkins, 2007) to examine five individual moral philosophies (true altruists, true egoists, realistic altruists, tit-for-tats, and realistic egoists). They also considered the loyalty of B2B market participants (Wang and Hill, 2009). The analysis of combinatorial auction is proposed by Choi et al. (2009). For determining the decision strategies, bidding strategies and bid selection strategies have been considered.

In spite of these mathematical models, some game theoretic models are proposed for market analysis. Economists have used the game theoretical approach to model network formation. This line of research suggests that network structures are important in determining the outcomes of economic and social interactions, such as bilateral product exchanges and technology adoption (Jackson, 2005; Sundararajan, 2005). Gan et al. (2005) showed an auction game model for pool-based electricity market. They claimed that the introduced concept of quasi-equilibrium offers an alternative for market studies. Kang et al. (2007) presented a non-cooperative game theory concepts in single auction power pool to win the auction. They considered a game between two suppliers, rather than a supplier and a buyer. They used marginal cost - a set of costs like maintenance, operational and upgrade costs - and the price per unit of power for each power generators to determine the result of auction. Also they performed demand forecasting. Their methodology gives an optimal bidding strategies for competitive power suppliers.

A game theory simulator for assessing the performance of competitive electricity markets...
proposed by Bompard (2008). This simulator is run for one year. According to their research, game theory can be applied to simulate realistic market framework. The behavioral procedures simulated by Menniti et al. (2008) for electricity market with Genetic Algorithm (GA) as an evolutionary game. They used GA to forecast the electricity price and how the competition can influence it over a long period. Lise et al. (2006) investigated market power and the environmental affects of the Northwestern European electricity market with a game theoretic model. Rosenthal (2008) used game theory to illustrate the transfer pricing in a vertically integrated supply chain.

A computer-based learning environment is introduced as a microworld for understanding risks in a deregulated industry (Dyner et al., 2009). Ganeshan et al. (2009) proposed an Optimal procurement portfolios in B2B market to help a procurement manager. Munksgaard and Morthorst (2008) focused on tariffs and investments determination to grow the Danish liberalised power market. An n-person noncooperative bargaining game suggested by Kim and Jeon (2009) that the game leads to a nonlinear programming function, but unfortunately the model may not be directly applied to some cases.

The rest of the paper is structured as follows: In Section 2 the trading model in a neutral market with double auction is proposed. The model involves expected profit for each market participants. The proposed model simulated to exhibit the impact of considered factors. In Section 3 Game theory is used to show the application of the model and model's capabilities to support decision making process. Finally, the conclusion and a short discussion are given in Section 4.

2. The proposed trading model in double auction

In a double auction, sellers and buyers make offer and demand bids and send them to the auctioneer (market owner) as depicted in Figure 1.

The prices suggested by the buyer is $P_b$ and by the seller is $P_s$. The corresponding value for the goods suggested by the buyer is $V_b$. Consider that $P_b < V_b$ because the buyers do not bid a price more than the attributing value. The market owner receives the offers and bids so makes decision how to assign the offers and bids together. There are two conditions:

$$\begin{align*}
\text{If } P_{si} &\leq P_{bj} \text{ then trade is possible} \\
\text{If } P_{si} &> P_{bj} \text{ then nothing}
\end{align*}$$

2.1. Notations

We defined the following quantities:

- $P_s$ Proposed price by the buyer.
- $P_b$ Proposed price by the seller.
- $V_b$ The corresponding value suggested by buyer.
- $\alpha$ Percentage of trade volume charged by the auctioneer to the seller.
- $\beta$ Percentage of trade volume charged by the auctioneer to the buyer.
- $\mu$ Constant cost for market participation.
- $x$ Nonnegative continuous random variable for demand.
- $f(x)$ Probability density function for random variable $x$.
- $F(x)$ Cumulative distribution function for random variable $x$.
- $C$ Seller’s per unit production cost.
- $CON$ The unit of product that can sell or buy in the market.
- $H$ The maximum quantity of bid or order.
- $Q$ Quantity of traded products.
2.2. Assumptions

We will use the following assumptions in the remainder of the paper:

- \( P_s \leq V_b \). This assumption used to avoid trivial problems.
- \( P_s \) is the trade's price. \( P_b - P_s \) charged by auctioneer.
- Sellers and buyers should pay a constant cost (\( \mu \)) and a percentage of trade volume (\( \alpha \) for sellers and \( \beta \) for buyers).
- \( H \) considered as a maximum quantity that can trade in the market. The accurate number of \( H \) depends on the market policy.

2.3. The proposed model

In other papers, profit in E-B2B markets is considered as individual functions, when the business is buying or selling. Also, the owner didn’t considered as a market participants that should be satisfied. The profit is calculated based on their specific considered supply chain.

Our proposed model consists of three functions to calculate the expected profit for each kind of market participants that helps analyzing the market situation. The seller, buyer and owner (auctioneer)'s expected profit can be expressed as Table 1, when a constant cost charged by auctioneer to either sellers and buyers and \( \alpha \) and \( \beta \) as a percentage of trade's volume (\( \alpha \) for sellers and \( \beta \) for buyers). We assumed different percentage charges for buyers and sellers just for increasing the flexibility of the model, absolutely these can be considered as equal. According to Law and Kelton (1982) the distribution function of the quantity of trade in the proposed model will follow the poisson distribution (see the end part of Table 1). Based on the functions in Table 1, the owner can make decisions about how to assign the proposed prices together.

2.4. Simulation of the model

The simulation begins with initializing the proposed prices by the sellers and buyers. Then the expected profit (payoff) for each market participant generates with considering the costs and maximum number of quantity (\( H \)). Figure 2 shows the payoff for each seller when:

Seller1 price = 34, Seller2 price = 54
Seller3 price = 65, Seller4 price = 87
\( H = 90, \alpha = 0.02, \mu = 15. \)

In Figure 2, the seller's payoff for each \( Q \) has been calculated to show the dependency between \( Q \) and payoff for sellers.

| Table 1: Expected profit function for sellers, buyers and the market owner for \( Q_i \). |
|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Seller's payoff \( \sum_{x=Q_i}^{Q} P_s x f(x) \)  | Buyer's payoff \( \sum_{x=Q_i}^{Q} V_b x f(x) \)  | Owner's payoff \( (P_b - P_s)Q_i + \alpha P_s Q_i + \beta P_b Q_i + 2\mu \) |
|\( P_s \leq P_b \)  | \( -\mu \)  | \( -\mu \) |
|\( P_s > P_b \)  | \( 2\mu \)  | \( 2\mu \) |
| Seller's payoff \( \sum_{x=Q_i}^{Q} P_s x \left( \frac{\lambda x e^{-\lambda}}{x!} \right) \)  | Buyer's payoff \( \sum_{x=Q_i}^{Q} V_b x \left( \frac{\lambda x e^{-\lambda}}{x!} \right) \)  | Owner's payoff \( (P_b - P_s)Q_i + \alpha P_s Q_i + \beta P_b Q_i + 2\mu \) |
|\( P_s \leq P_b \)  | \( -\mu \)  | \( -\mu \)  |
|\( P_s > P_b \)  | \( 2\mu \)  | \( 2\mu \) |
Figure 3 exhibits the payoff for each buyer based on possible quantities when they suggested:

Buyer 1 price = 23, Buyer 1 price = 32
Buyer 1 price = 56, \( H = 90, \beta = 0.01, \mu = 15 \).

According to the Figures 2 and 3, when all the trades are possible (in a condition that always \( P_b > P_s \)), the plot is increasing. But the rate of increasing is low in the end of plot in comparison with the beginning.

3. The trade's game

Game theory is a rich area of mathematics for economics, politics, finance, military sciences, and so on. Suppose that there are \( n \) sellers and \( m \) buyers. The owner will construct a matrix game to show the game situation between each seller and buyer. Every element in the matrix poses three values, seller's payoff \( (S_{ij}) \), buyer's payoff \( (B_{ij}) \), and owner's payoff \( (O_{ij}) \).

\[
A = [ (S_{11}, B_{11}, O_{11}) (S_{12}, B_{12}, O_{12}) \ldots (S_{nm}, B_{nm}, O_{nm}) ]
\]

\[
S_{ij} = \begin{cases} 
P_{si} Q_{bj} - C Q_{bj} - \alpha P_{si} Q_{bj}^* - \mu & \text{If } P_{si} \leq P_{bj} \\
\text{Then} - \mu & \text{If } P_{si} > P_{bj} 
\end{cases}
\]

\[
B_{ij} = \begin{cases} 
V_{bj} Q_{bj}^* - P_{bj} Q - \beta P_{bj} Q_{bj}^* - \mu & \text{If } P_{si} \leq P_{bj} \\
\text{Then} - \mu & \text{If } P_{si} > P_{bj} 
\end{cases}
\]

\[
O_{ij} = \begin{cases} 
(P_{bj} - P_{si}) Q_{bj}^* + \alpha P_{si} Q_{bj}^* + \beta P_{bj} Q_{bj}^* + 2\mu & \text{If } P_{si} \leq P_{bj} \\
2\mu & \text{If } P_{si} > P_{bj} 
\end{cases}
\]

\( a_j \) appears the game's payoff when \( i^{th} \) seller is trading with \( j^{th} \) buyer.

Suppose that each seller has different strategies and same for buyers. So, we will have a 4-dimension game matrix that shows the game's current value. For example when the seller \( i \) is trading with buyer \( j \) 2-dimension matrix \( A \) will shows the combinations of these seller and buyer strategies.

A mixed strategy is vector \( X = (x_1, \ldots, x_n) \) for seller selection and \( Y = (y_1, \ldots, y_m) \) for buyer selection that \( x_i \) is the probability of "seller will trade with his \( i^{th} \) strategy" and \( y_j \) is the probability of "buyer will trade with her \( j^{th} \) strategy". Also, \( \sum_{i=1}^{n} x_i = 1 \) and \( \sum_{j=1}^{m} y_j = 1 \) (Barron, 2008).

3.1. Game and linear programming

In this section, a brief review of linear programming in matrix game is mentioned. This method has been used to convey the application of the model and how the proposed model helps to analyze the market situation. Also, it contains some useful suggestions for buyers, sellers, and market owner.
A mixed strategy is vector \( X = (x_1, \ldots, x_n) \) for seller selection and \( Y = (y_1, \ldots, y_m) \) for buyer selection that \( x_i \) is the probability of "seller will trade with his \( i \)th strategy" and \( y_j \) is the probability of "buyer will trade with her \( j \)th strategy". Also, \( \sum_{i=1}^{n} x_i = 1 \) and \( \sum_{j=1}^{m} y_j = 1 \) (Barron, 2008). Linear programming is used to find the optimal mixed strategies for market participants.

At first, assume that \( a_{ij} > 0 \) by adding a constant to \( A \), so \( V(A) > 0 \). Suppose that \( i \)th seller is trading with \( j \)th buyer on \( Q \) quantity and there is matrix \( A \) based on seller's payoff. Market owner wants to increase the profitability for both sellers and buyers. So, owner try to find a dual solution for the game. For sellers, the owner looks for a mixed strategy \( X \), that (when all trades are possible):

\[
E(X, j) = XA_j = x_1 (P_{si}Q_{bj} - CQ_{bj} - \alpha P_{si}Q_{bj} - \mu) + \ldots + x_n (P_{sn}Q_{bj} - CQ_{bj} - \alpha P_{sn}Q_{bj} - \mu) \geq V, 1 \leq j \leq m
\]

where \( \sum x_i = 1, x_i \geq 0 \), and \( V > 0 \). The variables are changed by setting:

\[
x_i' = \frac{x_i}{V}, 1 \leq i \leq n, \quad x' = (x_1', \ldots, x_n')
\]

So,

\[
\sum_{i=0}^{n} x_i' = \frac{1}{V}
\]

Then,

Sellers program:

Minimize \( z_1 = x'J_{m}^{T} \)

Subject to:

\( x'A \geq J_m, x' \geq 0 \)

When the seller's program is solved and the formulation backed to original variables we will have:

\[
V(A) = \frac{1}{\sum_{i=1}^{n} x_i'} = \frac{1}{z_1'}
\]

And for buyers,

\[
Y = (y_1, y_2, \ldots, y_m), y_j \geq 0, \sum_{j=1}^{m} y_j = 1
\]

so,

\[
y_1 (P_{si}Q_{b1} - CQ_{b1} - \alpha P_{si}Q_{b1} - \mu) + \ldots + y_m (P_{sn}Q_{bm} - CQ_{bm} - \alpha P_{sn}Q_{bm} - \mu) \leq u,
\]

\( i = 1, \ldots, n \)

\( u > 0 \) as small as possible,

\[
y_j' = \frac{y_j}{u}, j = 1, \ldots, m, y' = (y_1', \ldots, y_m')
\]

Buyers program:

Minimize \( z_2 = y'J_{m}^{T}, J_m = (1, \ldots, 1) \)

Subject to:

\( y'A \leq J_n, y' \geq 0 \)

So,

\[
V(A) = \frac{1}{\sum_{j=1}^{m} y_j'} = \frac{1}{z_2'}
\]

based on Duality Theorem: \( z_1^* = z_2^* \).

Dual theorem (Barron, 2008): if one of the pair of linear programs has a solution, then so does the other. If there is at least one feasible solution, then there is an optimal feasible solution for both and their values, i.e. the objectives are equal.

Indeed, we have three values in each element for each quantity of trade. So, it is possible to consider two matrices, one based on seller's payoff and another based on buyer's payoff. These payoff matrices are used for finding the optimal mixed strategy for each situation and adapting these obtained strategies with owner's payoff.

3.2. Illustration of game and linear programming

Suppose that the game matrix is \( A \) (based on seller's payoff).
To make all entries positive, added 5 to everything, so:

\[
A' = \begin{bmatrix}
5 & 9 & 4 \\
3 & 1 & 8 \\
4 & 2 & 7 \\
\end{bmatrix}
\]

Sellers program:

Minimize \( z_1 = x_1' + x_2' + x_3' (= \frac{1}{V}) \)

Subject to:

\begin{align*}
5x_1' + 3x_2' + 4x_3' & \geq 1 \\
9x_1' + x_2' + 2x_3' & \geq 1 \\
4x_1' + 8x_2' + 7x_3' & \geq 1 \\
x_1' & \geq 0 \\
V = \frac{1}{z_1} = \frac{1}{x_1' + x_2' + x_3'}
\end{align*}

So, \( x_i = Vx_i' \), is the optimal strategy. As the same manner for buyers:

Buyers program:

Maximize \( z_2 = y_1' + y_2' + y_3' (= \frac{1}{V}) \)

Subject to:

\begin{align*}
5y_1' + 9y_2' + 4y_3' & \leq 1 \\
3y_1' + y_2' + 8y_3' & \leq 1 \\
4y_1' + 2y_2' + 7y_3' & \leq 1 \\
y_1' & \geq 0 \\
\end{align*}

So, we will have:

\[
x_1' = \frac{3}{19}, x_2' = 0, x_3' = \frac{1}{19},
\]

\[
x_1' + x_2' + x_3' = \frac{4}{19}, \quad \frac{1}{V} = \frac{4}{19} \Rightarrow V(A') = \frac{19}{4}
\]

The value of original game is:

\[
V(A) = \frac{19}{4} - 5 = -\frac{1}{4} \quad \text{and} \quad X^* = \left(\frac{-3}{16}, 0, -\frac{1}{16}\right)
\]

and,

\[
y_1' = \frac{3}{19}, y_2' = 0, y_3' = \frac{1}{19},
\]

\[
y_1' + y_2' + y_3' = \frac{4}{19}, \quad \frac{1}{V} = \frac{4}{19} \Rightarrow V(A') = \frac{19}{4}
\]

\[
V(A) = \frac{19}{4} - 5 = -\frac{1}{4} \quad \text{and} \quad Y^* = \left(\frac{-3}{16}, 0, -\frac{1}{16}\right)
\]

So, \( X^* = \left(\frac{-3}{16}, 0, -\frac{1}{16}\right) \) and \( Y^* = \left(\frac{-3}{16}, 0, -\frac{1}{16}\right) \)

are the best strategies for the sellers and buyers. Also, we can consider the matrix based on sellers, buyers, and owner's expected profit. Then, we will have three best strategies for each market participants. The owner can make decisions and assign trades in an near optimal way that is satisfying.

4. Conclusion

This paper has proposed the expected profit in an electronic market that uses a double auction. The proposed model has a series of separated but related expected profit functions for the sellers, buyers and market owner (auctioneer). The proposed functions have the same variables that help the analyzer to consider each market participant's profit in a separated function but related to other market participant's expected profit. The model has been simulated and showed the impact of considered factors in market participants’ expected profit. Also the linear programming in game matrix is suggested to intimate the usage of the model. A hypothetical example is represented that conveys the concept and application of the model. To exploit the best situation for market participants and satisfy them, the optimal strategy was obtained from the example. This model works in a complete information situation. Hereby, the owner knows everything about prices, functions and variables. It can be extended in a non-complete information
situation, when the owner does not know all variables in the model. Also, negotiation in prices can consider in the model to give the opportunity of a flexible trade in market.

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