An optimization technique for vendor selection with quantity discounts using Genetic Algorithm

N. Arunkumar*
Assistant Professor, Dep. of Mechanical Engineering, St. Joseph's College of Engineering, Chennai 600 119, India

L. Karunamoorthy
Professor, Head of the Central Workshop Division, Dep. of Mechanical Eng., Anna University, Chennai 600 025, India

N. Uma Makeshwaraa
Graduate Research Assistant, Dep. of Mechanical Engineering, University of Texas at Austin

Abstract

Vendor selection decisions are complicated by the fact that various conflicting multi-objective factors must be considered in the decision making process. The problem of vendor selection becomes still more complicated with the inclusion of incremental discount pricing schedule. Such hard combinatorial problems when solved using meta heuristics produce near optimal solutions. This paper proposes a multi-component multiple vendor selection model with vendors offering quantity discounts. This problem is then evaluated using Genetic Algorithm with a case study approach. Combinatorial approach is used to group the vendors for selection and Genetic Algorithm to allocate the optimal order quantities for each vendor.

Keywords: Vendor selection; Multi-objective; Combinatorial; Genetic algorithm; Quantity discounts

1. Introduction

In many industries, the cost of raw materials and component parts constitute a major portion of the product cost. For example, the cost of components and parts purchased from external sources by large automotive/textile machinery manufacturers may total more than 50% - 80% of the revenues. In this process of purchasing, supplier selection has long been recognized as important and has been a central focus for much of the industrial marketing research over the past three decades [23]. The supplier selection decisions determine how many and which suppliers should be selected as supply sources and how order quantities should be allocated among the selected suppliers. Selection is said to be efficient when we obtain not only a desirable solution but also an optimal solution.

Traditional supplier selection decisions are mostly based on procurement cost, product quality, delivery performance and supply capacity criteria. The joint consideration of the above criteria complicates the selection decision even for an experienced purchase manager because competing vendors have different levels of achievement under these criteria. For example, the vendor with the least price in a given industry may not have the best delivery performance or product quality. In addition to the multi-objective nature of supplier selection, emergence of a discount pricing schedule becomes a major obstacle for procurement managers in finding the best purchasing strategy. There are various discount models like discounts based on the quantity of each product ordered from a supplier, and discounts based on the total value of all products ordered from a supplier. Quantity discount models involve distinct price breaks for each product and supplier and among which this paper considers incremental quantity discounts.

* Corresponding author. E-mail: n.arunkumar@rediffmail.com
Mathematical programming techniques have been applied to purchasing issues frequently, mainly in the domain of determining order quantities, specifically in environments where complex discounts are offered by the supplier. In this paper, a mathematical model for supplier selection is presented considering practically useful selection criteria including incremental quantity discounts for multiple component vendor selection. The use of Genetic Algorithm for solving such hard combinatorial problem is proposed in this paper.

The rest of this paper is organized as follows: The next section cites the relevant literature to the vendor selection problem. Section 3 discusses the multi-objective supplier selection model. Motivation of this research to use GA is presented in Section 4. Formulation of the mathematical model is presented in Section 5. Section 6 discusses the solution methodology and Section 7 discusses the results of the computational experiment. Finally, conclusions are provided in Section 8.

2. Literature survey

According to the research by Tracey and Tan [27], higher levels of customer satisfaction and firm performance result from selecting and evaluating suppliers based on their ability to provide quality components and subassemblies, reliable delivery, and product performance. Khan Shahadat [20] has indicated that most important supplier selection criteria that would be practical are price, timely delivery along with quality. The evolution of the industrial environment has modified the relative importance of the criteria and some more additional criteria like quantity discounts offered by the supplier are now considered to be practically useful.

Regarding vendor selection methodology, there exists extensive literature on various vendor selection methods. According to Bhutta [3], the vendor selection methods can be broadly classified as linear weighting, cost, mathematical and statistical models.

In linear weighting models weights are given to the criteria, the biggest weight indicating the highest importance. The supplier with the highest overall rating can then be selected. Humphrey et al. [17] use weighted score method for supplier evaluation. This method does not take qualitative factors into consideration. Moreover, the subjectivity of the decision-maker in the identification of weights could be very high.


Although structural method such as AHP helps consistency when assigning weights, a great deal of subjectivity remains embedded in the method. Total Cost of Ownership (TCO) based models attempt to include all quantifiable costs in the supplier choice that are incurred throughout the purchased item’s life cycle [11, 21 and 25]. Since cost estimation involves subjectivity, the results of estimation may be imprecise and random. The largest sources of error in cost estimation are overlooking elements of cost.

Statistical approaches include methods such as cluster analysis and stochastic economic order quantity model. Although stochastic uncertainty is present in most types of purchasing situations, e.g. by not knowing exactly how the internal demand for the items or services purchased will develop, only a very few supplier choice models really can handle this problem. The existing statistical models only accommodate for uncertainty with regard to one criterion at a time [5].

Mathematical Programming (MP) models include Linear Programming, Mixed Integer Programming, Dynamic Programming and others [9, 16 and 28]. Once the criteria are decided, the MP model allows the decision-maker to formulate the decision problem in terms of a mathematical objective function, which then subsequently needs to be maximized (e.g. maximize profit) or minimized (e.g. minimize cost) by varying the values of the variables in the objective function. MP models are most useful in repetitive, high volume-supply situations. A review of vendor selection criteria and methods identifies ten such MP approaches [29]. Data Envelopment Analysis (DEA) was used for evaluation of vendors already selected [31 and 32]. Talluri and Narasimhan [26] propose a framework for effective supplier sourcing which considers multiple strategic and operational factors in the evaluation process. Weber et al.[30] combine MP and DEA method to provide buyers with a tool for negotiations with vendors that were not selected right away. Karpak et al. [19] use goal programming to minimize costs and maximize quality and delivery reliability when selecting suppliers and allocating orders between them. Current and Weber [8] use facility location modeling constructs for the vendor choice problem. Ghousypour and O’Brien [14] combine Analytic Hierarchy Process (AHP) and MP in order to take into tangible criteria as well as intangible criteria and to optimize order allocation among suppliers.
Under quantity discount scenario, Sadrian and Yoon [24] presented a mathematical formulation of single item procurement decision problem under two different business volume discount schedules. Chaudhry et al. [7] developed a mixed integer programming approach to situations involving the sourcing of a single product from vendors offering price breaks which depend on the magnitude of the order quantity.

Most realistic optimization problems, like supplier selection require the simultaneous optimization of more than one objective function. Frequently, the relevant objectives are in conflict. For example, the vendor with the lowest per unit price may not have the best quality or delivery. Hence the firm must analyze the tradeoffs among the relevant criteria when making decisions. Multi-objective approach allows various criteria to be evaluated in their natural units of measurement instead of using a common unit of measurement [28]. This paper proposes such a multi-objective programming model for vendor selection.

Multi-objective Genetic Algorithm has been used for strategic sourcing [10] but has not been used in the context of supplier selection and order allocation. This paper proposes use of combinatorial approach and Genetic Algorithm (GA) for allocation of order quantities to the vendors.

3. Multi-objective supplier selection model

This paper proposes a multi-objective model for supplier selection considering trade-offs among the relevant criteria. There exist various multi-objective techniques among which multi-level programming model is proposed to be suited for the proposed problem. The first step in multi-level programming involves ordering the objectives in terms of importance. Next, it is needed to find the set of points for which the minimum value of the first objective function is attained. Then find the points in this set that minimize the second most important objective.

The method proceeds recursively until all objectives have been optimized on successively smaller sets. Multi-level programming is a useful approach if the hierarchical order among the objectives is of prime importance. This problem considers and prioritizes quality, late delivery and price as the three main objectives and all three have to be minimized.

This problem is solved by taking the optimal value of one objective function and adding it as another constraint for the next objective function. The optimal value is relaxed to get an upper limit which would then be used as constraint for the next objective function. The upper limit and trade-off values for the constraints can be fixed by the purchase manager based on purchasing requirement and company policy as how far the values can be relaxed to get the final solution.

4. Motivation for using GA for model evaluation

Most of the researches use traditional techniques for solving vendor selection problem. Traditional techniques are not efficient when practical search space is large. Numerous constraints make the vendor selection problem more complicated. Genetic Algorithm is different from traditional techniques in the following ways [18].

- GA deals with chromosomes that encode decisions related to the selection or not such supplier and the corresponding percentage of assigned demand, when more than one supplier is selected.
- GA searches from one population of solutions to another, rather than from individual to individual. This gives GAs the power to search noisy spaces littered with local optimum.
- GA use only objective function information to guide themselves through the solution space and not derivatives. When compared to other techniques where it needs variety of information to guide them, GA needs only the measure of fitness (objective function value) about a configuration in the space of solutions.
- GA uses probabilistic transition rules rather than deterministic rules.

Although GAs has been successfully applied to several classes of optimization problem, their application to the problem of optimal vendor selection and allocation of quantity based on discounts is very new. The possible combinatorial nature of supplier selection process with cost, quality and delivery criteria along with the quantity discounts encourages us to use GAs as the search technique. Further, this adaptive search procedure offers a population of comparable yet varied choices of good solutions.

With the help of combinatorial approach various combinations of vendor sets are generated which are then input to the GA optimizer to generate the final solution.
4.1 Multi-objective Genetic Algorithm

Multi-objective optimization problem (MOP) has plural objective functions and requires to search a set of solutions called pareto-optimal solutions. Since GA searches multi-point simultaneously, GA is suitable for MOP. Multi-objective optimization using GA is called Multi-objective GA (MOGA) [12]. This research uses MOGA for model evaluation.

5. MOP model

Vendor selection problems are application-specific as the appropriate constraints and the relative importance of the objectives vary with the problem setting. To demonstrate the applicability of multi-objective programming to the supplier selection problem we take the case of a leading textile machinery manufacturing company in India. This company manufactures draw texturising machine and spinning machines. Texturising machine consists of various units namely creels, heaters, feed units, yarn defect sensing, and defect yarn cutting units, friction unit, oiling unit and take-up units. It requires wide variety of material like castings, machined components, plastics, sheet metals, ceramic guides, etc. to make the machines. The model is formulated considering the constraints and objectives that are used practically by this company and select suppliers. Each component supplied by the vendor will have its own constraints and attributes such as percentage defective, percentage late deliveries and cost per unit. Each vendor has attributes like capacity, minimum order offered by the manufacturer, maximum business offered by the manufacturer and quantity discounts. The manufacturer has only one attribute demand. Let us consider vendor \( j \) supplying component \( i \) to this manufacturer. The notations used in the model are as follows:

\[
\begin{align*}
N & \quad \text{Total number of vendors.} \\
n & \quad \text{Total number of components.} \\
\lambda_{ij} & \quad \text{Defective percentage of } i^{th} \text{ component produced by vendor } j. \\
\beta_{ij} & \quad \text{Late delivery percentage for } i^{th} \text{ component produced by vendor } j. \\
\eta_{ij} & \quad \text{Cost per unit for } i^{th} \text{ component produced by vendor } j. \\
x_{ij} & \quad \text{Number of components of type } i \text{ purchased from vendor } j. \\
C_{ij} & \quad \text{Capacity of the vendor } j \text{ for the } i^{th} \text{ component.} \\
Q_{ij} & \quad \text{Minimum order quantities the vendor } j \text{ will supply for the } i^{th} \text{ component.} \\
B_{ij} & \quad \text{Minimum business for the vendor } j \text{ for the } i^{th} \text{ component.} \\
M_{ij} & \quad \text{Maximum business for the vendor } j \text{ for the } i^{th} \text{ component.} \\
\eta^*_{ij} & \quad \text{Cost per unit for } i^{th} \text{ component produced by vendor } j \text{ without discount.} \\
\eta''_{ij} & \quad \text{Cost per unit for } i^{th} \text{ component produced by vendor } j \text{ with discount.} \\
m_{ij} & \quad \text{middle order quantity for the } i^{th} \text{ component of vendor } j. \\
\phi_i & \quad \text{Upper limit desired by the purchase manager for number of defective items for } i^{th} \text{ component.} \\
\omega_i & \quad \text{Upper limit desired by the purchase manager for number of late deliveries for } i^{th} \text{ component.} \\
D_i & \quad \text{Total demand for component } i. \\
\xi_{ij} & \quad \text{A binary variable that shows the selection of vendor } j \text{ for supplying } i^{th} \text{ component.}
\end{align*}
\]

5.1. Objectives

In this specific vendor selection problem the objectives are to minimizing the total cost, total number of defective items and total number of late deliveries in purchasing multiple components from multiple vendors. Let us consider \( Z_1, \ Z_2 \) and \( Z_3 \) representing the quality, delivery and cost objectives. The objectives can be represented as follows:

\[
\begin{align*}
\text{[Quality Objective]} & & \text{Minimize } Z_{i1} = \sum_{j=1}^{N} \lambda_{ij} x_{ij} & i=1,2,\ldots,n, \\
\text{[Delivery Objective]} & & \text{Minimize } Z_{i2} = \sum_{j=1}^{N} \beta_{ij} x_{ij} & i=1,2,\ldots,n, \\
\text{[Cost Objective]} & & \text{Minimize } Z_{i3} = \sum_{j=1}^{N} \eta_{ij} x_{ij} & i=1,2,\ldots,n.
\end{align*}
\]
One of the criteria considered in this model is the quantity discount. The vendors provide price breaks based on the number of components ordered. This is determined considering the middle order quantity \( m \) as indicated below. If the quantity exceeds \( m \), then the total cost incurred by the manufacturer for buying \( i^{th} \) component from a particular vendor is:

\[
\eta_{ij} = \eta'_{ij} \forall X_{ij} \leq m_j ,
\]

\[
\eta_{ij} = \eta'_{ij} m_j + \eta''_{ij} (X_{ij} - m_j) \forall X_{ij} > m_j. \tag{5}
\]

This type of discount model is termed as incremental quantity discount models. The cost objective (Equation 3) considers the above said incremental discounts when solving the model.

### 5.2. Constraints and trade off values

Trade off values for the objectives are considered as constraints as given below.

- Conform to the upper limit on the number of defective items: Though the objective is to minimize the number of defective items, an upper limit on the number of defective items helps the purchase manager in a practical situation to define how many defective items can be accepted.

- Conform to the upper limit on the number of late deliveries: Similar to number of defective items, number of late deliveries can also have an upper limit based on which the vendors need to be selected.

These two trade-off values are indicated as given below. This model follows a step-by-step approach, where each and every criterion is evaluated one by one. Hence the first objective is quality and the next one is delivery. The maximum number of defective items allowed is given as a constraint for the next objective (late delivery). Similarly when late delivery objective is found, a trade-off value of this and quality is used to arrive at the cost objective. Hence this type of approach makes the purchase manager to define the limits for each attribute (quality and late delivery).

Minimize \( Z_{ij3} \)

Subject to:

\[
\sum_{j=1}^{N} (\lambda_{ij} X_{ij}) \leq \phi_i \quad i=1,2\ldots,n.
\]

\[
\sum_{j=1}^{N} (\beta_{ij} X_{ij}) \leq \omega_i \quad i=1,2\ldots,n. \tag{7}
\]

In the above model, \( \phi_i \) and \( \omega_i \) are not determined, it is the user input based on the trade-off. For example, though the model is able to get a minimum defective of 58, the user may relax it up to 75 to get a different solution. So \( \phi_i \) is chosen as 75 to arrive at a different solution. \( \phi_i \) and \( \omega_i \) may be fixed by the purchase manager based on the company policy. Similarly for cost calculation, the trade-off value is input considering defective items and late deliveries.

### 5.3. Vendor constraints

- Ensure that vendor’s capacity is not exceeded: It is always not possible for the vendors, to deliver as much as required by the manufacturer, as the vendor would have maximum capacity. This is also added as a constraint, so that during order allocation it is taken in to consideration.

- Satisfy the minimum order quantity for the vendors: This is a vendor constraint where he would like to accept the order only if the number of components purchased conforms to his business policy.

Common constraint 1 (Capacity):

\[
X_{ij} \leq (C_{ij} \times \xi_{ij}) \quad \forall i=1,2\ldots,n \quad j=1,2\ldots,N. \tag{8}
\]

Common constraint 2 (Minimum Quantity):

\[
X_{ij} \geq (Q_{ij} \times \xi_{ij}) \quad \forall i=1,2\ldots,n \quad j=1,2\ldots,N. \tag{9}
\]

### 5.4. Manufacturer constraints

- Ensure that firm’s willing business to the vendor is not exceeded: which means the order allocation for each vendor does not exceed the firm’s maximum willing business for each vendor.
- Satisfy minimum business for selected vendors: The manufacturer should also ensure that it offers business to its vendors in line with its minimum business policy.
- Ensure that the demanded quantity is met: This forms the main constraint. While meeting the objectives, it is essential to make sure the demand is always met. This model has an advantage that it considers the number of defective items during vendor selection and ensures that the demand is met.

Common constraint 3 (minimum business):

\[ X_{ij} \geq (B_{ij} \times \xi_j) \quad \forall \ i=1,2,\ldots,n \quad j=1,2,\ldots,N. \]  

(10)

Common constraint 4 (maximum business):

\[ X_{ij} \leq (M_{ij} \times \xi_j) \quad \forall \ i=1,2,\ldots,n \quad j=1,2,\ldots,N. \]  

(11)

Common constraint 5 (demand):

\[ \sum_{j=1}^{N} X_{ij} \geq D_i \quad i=1,2,\ldots,n. \]  

(12)

This model is evaluated using GA by providing these objectives and constraints as inputs to GA optimizer, which gives the selection of vendors and their order quantities for each component.

6. Solution methodology

This multi-objective supplier selection problem is solved using a step-by-step process which considers objectives one by one. The optimal order quantity for each vendor in a combination is allocated using real coded GA. The main advantage of this model with GA when compared to other techniques is that it provides a family of solutions apart from the best solution. This will be useful for the purchase manager to look at the alternative solutions. Combinatorial optimization problems are concerned with the efficient allocation of limited resources to meet desired objectives. Constraints on basic resources restrict the possible alternatives that are considered feasible. Still, there are many possible alternatives to consider and one overall goal determines which of these alternatives is the best. In this problem the vendor combinations are input to the GA optimizer for optimal order allocation based on objectives, constraints and trade off values.

6.1. Chromosome representation of vendors

The number of chromosomes in a population remains constant in GA. In this problem a chromosome represents a collection of vendors who supply a component to the company. It is a collection of genes, which represents vendor’s order quantities. The minimum number of vendors to be selected for that run will determine the length of the chromosome. The chromosome representation with an example of allocation of order quantities for four vendors is as shown in Figure 1. During a particular selection, if the number of vendors set as 4 then the length of the chromosome is 4+1=5. The size of chromosome (M) is determined by the number of vendors to be used in a particular cycle. An additional gene is added with the chromosome to store the sum of the order quantities assigned to the vendors. During the random generation of population, the chromosome comes out of the random number generator only if the total is greater than the demand to be met. This condition starts the search with a population having better fitness and hence the convergence of the result is quick. It can also be programmed without the additional gene, but it will take a longer iteration to converge.

![Figure 1. Chromosome representation of vendor order allocation.](image-url)
Figure 1 indicates two chromosomes; the chromosome with a total of 1995 distorts the path of the search because the order quantity is lesser than the demand. Having the order quantity lesser than the demand has a probability of zero in getting selected in the pool if allowed to evolve. Therefore, this chromosome is not used in the initial population thereby the initial state space is void of impossible solutions to the problems. The initial population in any GA is generated randomly satisfying the constraints.

6.2 Evolution process of GA

The selection mechanism works like the “guidance system” of the “evolution process” in GA as shown in Figure 2. In this figure, ‘A’ is the ratio of number of dead individuals in iteration to the population size. The value of A determines how GA is proceeding.

For example if A value is very high say greater than or equal to 0.7, then it means that the number of dead individuals is very high. Hence the population has very few satisfactory solutions and so a mutation operator is used to fill the population with random solutions. This will make GA to search the space randomly to get the optimum solution.

If the value of A is very little say less than or equal to 0.3, then it means the population has a good number of feasible solutions and crossover operator can be used to produce variants of solution. The value of A was determined in trial and error method to suit the problem.

6.3 Genetic operators

The number of individuals dying during a generation is actually dependent on the closeness of the average fitness value of the population to the best fitness value. The genetic operators used in this optimization will be discussed below.

i) SBX (Simulated Binary Crossover) Crossover

The crossover operator is the main genetic operator. The crossover operator is used where there is a need for hybrid chromosome in the population. The hybrid chromosomes are needed at instances when most of the chromosomes in the population are closer to the solution. When the ratio of the number of dead to the population size is less than 0.3 then SBX crossover operator is chosen. The chromosomes needed for the crossover are chosen from the remaining 70% of the population. The SBX crossover operator is µ, which is a random value selected between 0 and 1 during each cycle.

\[
\begin{align*}
\text{Child}_1 &= \text{Parent}_1 \times \mu + \text{Parent}_2 \times (1-\mu) \\
\text{Child}_2 &= \text{Parent}_2 \times \mu + \text{Parent}_1 \times (1-\mu)
\end{align*}
\]

The order allocation for Child1 and Child2 for a particular run is shown in Figure 3. Order quantity of vendor 1 for child 2 is calculated as follows:

\[V_{1,\text{child }2} = V_{1,\text{parent }1} \times \mu + V_{1,\text{parent }2} \times (1-\mu)\]

\[\therefore V_{1,\text{child }2} = 430 \times 0.36 + 490 \times (1-0.36)\]

\[V_{1,\text{child }2} = 468.\]

<table>
<thead>
<tr>
<th>Vendor 1</th>
<th>Vendor 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>490</td>
<td>370</td>
<td>445</td>
</tr>
<tr>
<td>V1</td>
<td>V5</td>
<td>V3</td>
</tr>
<tr>
<td>430</td>
<td>495</td>
<td>627</td>
</tr>
</tbody>
</table>

Random value \(\mu = 0.36\)

<table>
<thead>
<tr>
<th>Vendor 1</th>
<th>Vendor 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>451</td>
<td>450</td>
<td>562</td>
</tr>
<tr>
<td>V1</td>
<td>V5</td>
<td>V3</td>
</tr>
<tr>
<td>468</td>
<td>415</td>
<td>510</td>
</tr>
</tbody>
</table>

Figure 2. Evolution process of GA.

Figure 3. SBX crossover.
ii) **Mutation operator**

Mutation operator helps the genetic search to move in a random direction thereby eliminating the problem of being stuck in local minima. A simple way to achieve mutation would be to alter one or more genes. In genetic algorithms, mutation serves the crucial role of either (a) replacing the genes lost from the population during the selection process so that they can be tried in a new context or (b) providing the genes that were not present in the initial population. The mutation rate used for this problem is 0.05. This parameter was selected from the range of mutation rate used in past studies and this value was fixed based on this problem using trial method. Using this value, it was possible to get better solution. Figure 4 shows the mutation of vendor 3.

A random vendor in the selection is chosen and the order quantity of that vendor is mutated within the vendor’s capacity. The mutation takes place such that the total order exceeds the demand. Mutation operator is chosen when the number of dead chromosomes to population size ratio exceeds 0.70. This occurs only when the population’s average fitness rate drops to 70% of the best fitness so far. To improve the average fitness of the population new domains in the state space must be analyzed. Hence the mutation operator is selected for inserting random chromosomes in the population.

iii) **Reproduction operator**

The reproduction operator is applied to emphasize good solutions and eliminate bad solutions in a population, while keeping the population size constant. This is achieved by identifying good (usually above average - 95%) solutions in a population, making multiple copies of good solutions, and eliminating bad solutions from the population so that multiple copies of good solutions can be placed in the population. The commonly used reproduction operator is the proportionate reproduction operator where a chromosome is selected for the mating pool with a probability that is proportional to its fitness.

![V1 V5 V3 V6 Total](parent1)

### Figures

**Figure 4.** Mutation operator.

### Table: Parent 1

<table>
<thead>
<tr>
<th>V1</th>
<th>V5</th>
<th>V3</th>
<th>V6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>490</td>
<td>370</td>
<td>445</td>
<td>695</td>
<td>2000</td>
</tr>
</tbody>
</table>

### 6.4 GA control parameters

The following GA parameter values are arrived based on the satisfactory performance of trials conducted for this problem with different ranges of values.

- Population Size = 150,
- Number of Generations (n) = 250,
- Crossover Probability (Pc) = 0.75,
- Mutation Probability (Pm) = 0.05,
- Crossover mechanism: SBX.

### 6.5 Fitness function

When using evolutionary computation techniques such as GA, the fitness of each individual in the population must be calculated before a generation can be computed and the evolution can continue. Once the fitness is obtained, the algorithms perform the natural selection steps (crossover, mutation and/or replication) to form the next generation of the population.

The fitness scores and the system structure both guide the evolution of the individual chromosomes toward an optimal solution. This continues for a specified number of generations, and once completed, returns the best chromosome encountered over the entire run of the algorithm.

Fitness is one of the most important parts of the evolutionary strategy, because it directly relates to what traits are desirable in the population. This correlation is what guides the evolution towards the optimal traits. The fitness is essentially a “score” for the individual represented by the chromosome. The fitness value for this problem is arrived as follows:

The chromosomes are evaluated using the multi-objective functions considered in this paper. The multi-objective functions need to be combined into a single objective function denoted by ‘f’. For this purpose two random weights wt1 and wt2 are generated, such that wt1 + wt2 = 1. For each chromosome, the combined objective function is the minimization of function ‘f’ given by:

**Minimization of late delivery**

\[
f_{\text{Late delivery}} = \frac{\text{Demand}}{\text{Order}} - \text{Defectives} \times wt_1 + \frac{\text{Demand}}{\text{Order}} \times wt_2.
\]
Minimization of Cost

\[ f_{\text{Cost}} = \left( \frac{\text{Demand}}{\text{Order} - \text{Defectives}} \right) \times w_{t_1} + \left( \frac{\text{Demand}}{\text{Demand} + \text{LateDel}} \right) \times w_{t_2}, \]

Such that: \( w_{t_1} + w_{t_2} = 1 \).

7. Case study: Computational results

The model furnished in Section 3 is solved using the details of the textile machinery manufacturing company in India. This paper considers the vendor set consisting of 7 vendors from their vendor database for evaluation of the model.

7.1. Vendor details

Data for seven vendors collected from the leading textile machinery-manufacturing firm for a demand of 2000 components are as shown in Table 1.

7.2. Manufacturer details

The purchasing department needs a maximum demand of 2000 with minimum and maximum business which is the minimum and maximum order quantities that can be placed for each vendor to be 100 and 1200 respectively. Hence if a vendor’s capacity is beyond these limits, this model will take care of allocating the quantities to the vendors appropriately considering manufacturer’s minimum and maximum business for each vendor. The upper limit and trade-off values for the objectives were fixed for this problem using the multi-objective approach indicated in Section 3 and carrying a sensitivity analysis on the obtained values. These values (defectives-75, late deliveries-55) also matched the manufacturing company’s policy for accepting the maximum number of defectives and late deliveries. For this problem the values are as follows:

7.2.1. Price breaks

The price breaks for quantity ordered are given below according to discount each vendor offers as shown in Table 2.

7.2.2. Vendor selection with quantity discounts

Figure 5 indicates the various vendor set combinations against their number of defectives and costs. The notation 1:2:3:5 represents the group of vendors V1, V2, V3 and V5.

Some of the 4, 5 and 6 vendor combinations are not indicated in the graph, since the combinations sum of defective percentage was very high. From this figure it is evident that the vendor combination 1:2:3:5 produces minimum defective items and 1:5:6:7 produces minimum late delivery. But 1:2:3:5 results in more late delivery.

<table>
<thead>
<tr>
<th>Vendors</th>
<th>Vendor’s min order</th>
<th>Vendor’s max capacity</th>
<th>Percent defective</th>
<th>Cost 1 (Rs.) ( (\eta') ) before discount</th>
<th>Percent late deliveries</th>
<th>Cost 2 (Rs.) ( (\eta'') ) after discount</th>
<th>Middle order ( (m) ) in quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>100</td>
<td>600</td>
<td>2.5</td>
<td>10</td>
<td>3.25</td>
<td>9</td>
<td>299</td>
</tr>
<tr>
<td>V2</td>
<td>200</td>
<td>750</td>
<td>4.5</td>
<td>11.5</td>
<td>5.25</td>
<td>10</td>
<td>499</td>
</tr>
<tr>
<td>V3</td>
<td>250</td>
<td>800</td>
<td>5</td>
<td>12</td>
<td>6.25</td>
<td>11</td>
<td>499</td>
</tr>
<tr>
<td>V4</td>
<td>350</td>
<td>750</td>
<td>3.5</td>
<td>9.5</td>
<td>15</td>
<td>9</td>
<td>549</td>
</tr>
<tr>
<td>V5</td>
<td>100</td>
<td>700</td>
<td>1.5</td>
<td>10.5</td>
<td>0.2</td>
<td>10</td>
<td>399</td>
</tr>
<tr>
<td>V6</td>
<td>300</td>
<td>950</td>
<td>6</td>
<td>12.25</td>
<td>2.5</td>
<td>11.5</td>
<td>599</td>
</tr>
<tr>
<td>V7</td>
<td>250</td>
<td>1000</td>
<td>5.8</td>
<td>15</td>
<td>2.35</td>
<td>14</td>
<td>649</td>
</tr>
</tbody>
</table>
Table 2. Price breaks.

<table>
<thead>
<tr>
<th>Vendors</th>
<th>Ranges</th>
<th>Cost (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>$100 \leq X &lt; 300$</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td>$300 \leq X \leq 600$</td>
<td>09.00</td>
</tr>
<tr>
<td>V2</td>
<td>$200 \leq X &lt; 500$</td>
<td>11.50</td>
</tr>
<tr>
<td></td>
<td>$500 \leq X \leq 750$</td>
<td>10.00</td>
</tr>
<tr>
<td>V3</td>
<td>$250 \leq X &lt; 500$</td>
<td>12.00</td>
</tr>
<tr>
<td></td>
<td>$500 \leq X \leq 800$</td>
<td>11.00</td>
</tr>
<tr>
<td>V4</td>
<td>$350 \leq X &lt; 550$</td>
<td>09.50</td>
</tr>
<tr>
<td></td>
<td>$550 \leq X \leq 750$</td>
<td>09.00</td>
</tr>
<tr>
<td>V5</td>
<td>$100 \leq X &lt; 400$</td>
<td>10.50</td>
</tr>
<tr>
<td></td>
<td>$400 \leq X \leq 700$</td>
<td>10.00</td>
</tr>
<tr>
<td>V6</td>
<td>$300 \leq X &lt; 600$</td>
<td>12.25</td>
</tr>
<tr>
<td></td>
<td>$600 \leq X \leq 950$</td>
<td>11.50</td>
</tr>
<tr>
<td>V7</td>
<td>$250 \leq X &lt; 650$</td>
<td>15.00</td>
</tr>
<tr>
<td></td>
<td>$650 \leq X \leq 1000$</td>
<td>14.00</td>
</tr>
</tbody>
</table>

Similarly, Figure 6 indicates the graph against cost for the vendor combinations. From this figure it is evident that although 1:5:6:7 produces less number of late deliveries, the cost is high. With trade off values of number of defectives and late deliveries, vendor set (1:2:5:6) is found to be the best vendor set. Selection based on quality, late delivery and price with quantity discounts and their order allocation for best group of vendors is shown in Table 3.

Figure 7 indicates the fitness curve for the supplier selection. It is found that as the number of generations increases, the solution converges to an optimal value. The optimal value is found at generation number 101. The program was executed in an Intel Pentium P4, 2.40 GHz PC. The solution converged in 36 seconds using the processor specified.

7.3. Comparison of vendor selection model using GA and Integer Linear Programming

We have compared our results obtained using GA with the results obtained using ILP [1]. Same constraints and objectives used in the vendor selection using ILP is used when evaluating the model using GA. Since ILP method (Arunkumar et al. [1]) uses all unit quantity discounts when compared to this model which uses incremental quantity discounts, the results are compared without considering quantity discounts. Hence all the constraints and objectives are the same in both models.

Table 4 and Figure 8 indicate the final vendors selected and their corresponding order allocation using the same input parameters using GA and ILP.

Table 3. Vendor selection with quantity discounts using GA.

<table>
<thead>
<tr>
<th>Vendors</th>
<th>Order</th>
<th>Number of defectives</th>
<th>Number of late deliveries</th>
<th>Middle Order</th>
<th>Cost in Rs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>504</td>
<td>12</td>
<td>16</td>
<td>299</td>
<td>4835</td>
</tr>
<tr>
<td>V2</td>
<td>359</td>
<td>16</td>
<td>18</td>
<td>499</td>
<td>4128.5</td>
</tr>
<tr>
<td>V5</td>
<td>689</td>
<td>10</td>
<td>1</td>
<td>399</td>
<td>7089.5</td>
</tr>
<tr>
<td>V6</td>
<td>517</td>
<td>31</td>
<td>12</td>
<td>599</td>
<td>6333</td>
</tr>
<tr>
<td>Total</td>
<td>2069</td>
<td>69</td>
<td>47</td>
<td></td>
<td>22386</td>
</tr>
</tbody>
</table>

Order – Defectives = 2069 – 69 = 2000 = Demand
It is evident from the table that both the approaches select V1:V2:V5:V6 as the best vendors. There is less difference in the order allocations and the final cost. There is 2.48% difference in the final cost objective between the two methods. Whereas the defectives, late deliveries and demand being the same in both the methods.

When compared to ILP, GA can be used effectively for incremental quantity discounts. Also by using GA it is possible to obtain alternate solutions apart from the best solution which is useful for some practical scenarios.

8. Conclusion

This paper proposed the use of GA selection of vendors offering quantity discounts which is a multi-objective problem and combinatorial in nature.

Existing approaches do not consider the defectives present in the supply while allocating order quantity to each vendor when meeting the demand. Because of this the final total order allocation would be less than the demand by the number of defectives.

One of the innovative approaches proposed in this model is that it considers defectives during order allocation so that the exact demand will always be met. The total order allocated would always be the same as the given demand.

One another advantage of the proposed model is that when trying to get the best solution, the purchase manager is given visibility to frame constraints for subsequent objectives i.e. the tradeoff between quality and delivery can be set. This enables the purchase manager to control the objectives that can be better applied practically.

Advantage of using GA as a design tool is their ability to find solutions to problems in a way completely free of preconceptions about what is possible and what is not.

The results obtained using this approach were found to produce comparable results to that using conventional techniques.

Also this approach provides a way of getting the next set of vendors and hence this will be very helpful to the purchase manager who also would like to find the alternate set of suppliers.

There are various possible extensions to this research. Methods to improve the computational time are one extension to this research. There are other types of discount models. Applying GA to all unit quantity discounts and volume discounts is another area of research.

<table>
<thead>
<tr>
<th>Vendor Number</th>
<th>Order allocation using ILP</th>
<th>Order allocation using GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>598</td>
<td>539</td>
</tr>
<tr>
<td>V2</td>
<td>348</td>
<td>340</td>
</tr>
<tr>
<td>V5</td>
<td>686</td>
<td>558</td>
</tr>
<tr>
<td>V6</td>
<td>435</td>
<td>637</td>
</tr>
<tr>
<td>Defective items</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>Late deliveries</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Total order</td>
<td>2074</td>
<td>2074</td>
</tr>
<tr>
<td>Cost</td>
<td>22513</td>
<td>22962</td>
</tr>
</tbody>
</table>
References


