Passivity-Based Control of the DC-DC Buck Converters in High-Power Applications

Armin Lotfi Eghlim\textsuperscript{1}, Mahdi Salimi\textsuperscript{2}
\textsuperscript{1}Department of Electrical Engineering, Ahar Branch, Islamic Azad University, Rasht, Iran
Email: Armin.lotfi.eghlim@gmail.com
\textsuperscript{2} Department of Engineering, Ardabil Branch, Islamic Azad University Ardabil, Iran

ABSTRACT
In this paper, a novel approach for control of the DC-DC buck converter in high-power and low-voltage applications is proposed. Designed method is developed according to passivity based controller which is able to stabilize output voltage in a wide range of operation. It is clear that in high-power applications, parasitic elements of the converter may become comparable with load value and hence, in this paper all of the converted parasitic elements are modeled during development of the controller. In order to evaluate the accuracy and effectiveness of the proposed method, designed controller is simulated using MATLAB/Simulink toolbox. Presented simulation result proves that the developed controller has acceptable dynamic and steady-state responses which are compared with a standard PI controller in high-power applications.

KEYWORDS: voltage control, nonlinear control systems, uncertainty, Lyapunov method, DC-DC power converters

1. INTRODUCTION
DC-DC converters are used widely in different industrial applications like DC-motor speed control, maximum power point tracking of the photovoltaic systems and communication equipment. Buck, Boost and Buck-Boost converters are different topologies of the standard DC-DC choppers. From controller design viewpoint, these converters are nonlinear and time-variant [1].

Usually linear controllers are used to regulate output voltage of the converter, considering the linearized model [2]. Although the design and implementation of the linear regulators are completely simple, however its application may result in instability of the system in a wide range of operation. In fact, acceptable response may not be obtained using a linear controller in specific applications such as modern processors power supplies where converter load value is changed in a very wide range [3].

Considering nonlinear characteristic of the DC-DC converters, it is better to use a nonlinear control technique in output voltage regulation. Main nonlinear controllers are feedback linearization [4], sliding mode [5], adaptive back stepping [6] and passivity-based control (PBC).

In spite of simplicity of implementation, dynamic response of the system in feedback linearization approach may have considerable overshoot with respect to large
variations in input and reference voltage [7]. Sliding mode control is robust controller respect to uncertain parameters of the model and it is easy to implement [8]. However, variation of the switching frequency, the steady-state error and chattering are the main problems of sliding mode method in practical applications [9]. On the other hand, adaptive back stepping technique has been developed successfully in DC-DC converters [10]. This method is based on suitable Lyapunov function selection. Although, uncertain parameters of the model can be estimated using adaptive rules; however complexity of design and practical implementation are its main disadvantages [11].

In [12], the PBC method is applied for regulation of the buck/boost converter. Although, it has been shown that direct control of the output voltage of this converter is not possible. However, in [12], dynamic response of the converter for different parameters value is not studied and output voltage ripple is relatively large. Therefore, it is not possible to use this method for implementation of the power supplies.

In [13], a multi-variable controller is designed for DC-DC boost converter with PBC technique. Designed controller in [13] is investigated for some variations in input voltage. However, it is not possible to use it in a wide range of operation, because an idealized model is used for controller development in DC-DC converter. Also in [14], combination of flatness and the PBC methods is studied using computer simulation results. It should be noted that the same problems exist in this case as reviewed in [12] and [13]. Another technique for development of the PBC in DC-DC converters is presented in [15]. But it is clear that the proposed control method has steady-state error. In addition, considerable oscillation is seen in dynamic response of the converter.

In [16], application of the PBC for DC-DC converters with constant power loads is studied. However, in modeling of converter, the effect of parasitic elements is not considered. Obviously, in high-power applications, value of the parasitic elements is important during steady state and dynamic responses of the converter. In [17], it is shown that the conventional PBC has not an acceptable response during input voltage variations. To solve this problem, an adaptive controller is designed in [17] to estimate input voltage. It is well-known that implementation of the adaptive controllers requires complex online calculations [18]. Also in [19], the nonlinear PBC is designed for DC-DC buck/boost converter.

Designed controller uses both output voltage and inductor current as feedback signals in two different feedback loops. However, due to un-modeled parameters of the converter, controller may not respond satisfactorily in a wide range of operation. For example, it can’t be used for control of the power supplies in the modern computers processor.

To improve the response of the controller, parasitic resistance of the inductor is modeled in [20]. In addition, a complimentary PID controller is used to eliminate steady-state error of the system. Using this approach for non-minimum phase converters is completely accepted [1]. However, in [20], the linearized model of the converter is used for controller design; so application of this technique in a wide
range of input voltage variation, load resistance and reference voltage changes may result in instability of the converter.

In [21], a feed-forward loop is used for regulation of the buck/boost converter in indirect realization of the PBC method. However, due to lack of direct feedback from the output voltage, proposed method has considerable steady-state error.

In this paper, a novel method is proposed for control of the buck converter based on PBC approach. This nonlinear controller is designed using an exact model of the converter and its stability is proved considering Lyapunov theorem. The impact of the parasitic element (such as ESR of inductor, ESR of power switch and ...) are considered in model development of the converter. Simulation results illustrate that the proposed control method has fast transient response with zero steady-state error and can be used in a wide range of operation in high-power converters.

2. Averaged state-space model of the DC-DC Buck converter with considering parasitic elements

The importance of parasitic elements in characteristic of the buck converter is shown in Fig.1 for different values of load resistance. In low-voltage and high-power applications, parasitic elements have considerable influence on converter model. These parasitic elements can be modeled by equivalent series resistance (ESR) of the inductor, output capacitor and power switches.

2.1 Simplification of the converter model

The buck DC-DC converter is shown in Fig.2.

According to Fig.2, capacitor voltage can be written as:

\[
v_C = v_o - r_c i_c
\]  

(1)

where, \(v_C\), \(v_o\), \(r_c\) and \(i_c\) are the capacitor voltage, output voltage, ESR of the capacitor and capacitor current respectively. Using (1), capacitor current can be written as follows:

\[
i_c = C \frac{dv_c}{dt} = C \frac{dv_o}{dt} - Cr_c \frac{di_c}{dt}
\]  

(2)

so:

\[
Cr_c \frac{di_c}{dt} = C \frac{dv_o}{dt} - i_c
\]  

(3)

where \(Cr_c\) is the capacitor time-constant (\(\tau\)).

\[
\tau \frac{di_c}{dt} = C \frac{dv_o}{dt} - i_c
\]  

(4)

Equation (4) represents a first order differential equation. Capacitor current can be calculated as the solution of equation (4):

\[
i_c = e^{-\frac{t}{\tau}} i_c(t = 0) + \int_0^t e^{-\frac{(t-\hat{t})}{\tau}} \frac{C}{\tau} \frac{dv_o}{df} \, d\hat{t} , \quad \hat{t} > 0
\]  

(5)

Assuming \(\hat{t} > 5\tau\), capacitor current may be approximate as:

\[
i_c \approx \int_0^t e^{-\frac{(t-\hat{t})}{\tau}} \frac{C}{\tau} \frac{dv_o}{df} \, d\hat{t} = \frac{C}{\tau} e^{-\frac{t}{\tau}} \int_0^\hat{t} \frac{dv_o}{df} \, e^{\frac{\hat{t}}{\tau}} , \quad \hat{t} > 0
\]  

(6)

Using hopital theorem, equation (6) can be simplified as:
\[
\frac{d}{dt}\left(\frac{1}{\tau}\right) \approx \frac{\frac{d}{dt}\left(\frac{1}{\tau}\right)}{\tau} + \frac{d}{dt}\left(\frac{1}{\tau}\right) \approx C \frac{dv_o}{dt}.
\]

Equation (7) clearly shows that during long term behavior (\(\tilde{t} > 5\tau\)), the ESR of the capacitor can be eliminated in the modeling of the converter.

1.1. Averaged state-space model of the converter

Considering inductor current \((x_1)\) and output voltage \((x_2)\) as state variables, the DC-DC buck converter can be modeled in the continuous conduction mode of operation using an averaging technique.

During ON-state of the power switch (\(0 < t < uT_S\)), equivalent circuit of the converter is illustrated in fig.3. In this condition, state-space model can be written as:

\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{L}(r_i + r_d)x_1 - \frac{1}{L} x_2 + \frac{1}{L} E \\
\dot{x}_2 &= \frac{1}{C} x_1 - \frac{1}{RC} x_2
\end{align*}
\]  

Similarly, during \((uT_S < t < T_S)\), the power switch is OFF and converter model can be written as:

\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{L}(r_i + r_d)x_1 - \frac{1}{L} x_2 - \frac{1}{L} v_d \\
\dot{x}_2 &= \frac{1}{C} x_1 - \frac{1}{RC} x_2
\end{align*}
\]

According to the averaging technique in state-space modeling [6], DC-DC buck converter can be modeled in CCM operation as follows:

\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{L}(r_i + u r_d)x_1 + \frac{1}{L} (uE - \bar{u} v_d) \\
\dot{x}_2 &= \frac{1}{C} x_1 - \frac{1}{RC} x_2
\end{align*}
\]

where, \(0 \leq u \leq 1\) is control input (duty cycle of the converter) and \(\bar{u} = u - 1\)

2. DEVELOPMENT OF THE PASSIVITY-BASED CONTROLLER

Using exact model of the converter (equation 12, 13), the passivity-based controller is designed for DC-DC buck converter in CCM operation.

2.1. Introduction of the system’s storage energy function

Total stored energy of the converter can be defined as:

\[
V(x) = \frac{1}{2} L x_1^2 + \frac{1}{2} c x_2^2
\]  

This equation can be rewritten in a compact matrix from as:

![Fig. 1. Importance of parasitic elements in DC-DC converter modeling](image)

(For simplification, converter input voltage assumed to be constant and 20 volts. Internal resistance of inductor 0.2 ohms, inductance of circuit 1mH, output capacitor 40 \(\mu\)F and switching frequency 25 KHz have been assumed. Circuit power switches are implemented by SKM100GB124D)
Fig. 2. The DC-DC Buck converter power circuit

\[ V(x) = \frac{1}{2} X^T DX \]  
(15)

where, \( V(x) \) is the Lyapunov function and:

\[ X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad D = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \]  
(16)

At first, for PBC controller design, the following definitions are considered:

**Definition 1**: suppose a given system as:

\[
\begin{align*}
\dot{x} &= f(x, u) \\
y &= h(x, u)
\end{align*}
\]  
(17)

with \( f(x, u) \) locally Lipschitz and \( f(0,0) = h(0,0) = 0 \)

\( f(x, u) \) is passive, if there exists a continuously differentiable positive definite function \( V(x) \) (called the storage function) such that:

Assuming the inductor current \( (x_1) \) as an output of the system \( (y) \), it is clear that in (21) a positive value is subtracted by \( uE y \).

Hence, it is concluded that \( \dot{V}(x) < uE y \).

So, it can be said that the buck DC-DC converter is strictly passive and it could be controlled by applying an appropriate damping factor.

On the other hand, using (12) and (13), the state-space model of the buck converter can be rewritten in matrix form as follow:

\[ D \dot{x} + (J + R)x = \mu \]  
(22)

where :
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\[ D = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} ; \quad J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} ; \]

\[
R = \begin{bmatrix} (r_1 + ur_s + \bar{u}r_d) & 0 \\ 0 & 1/R \end{bmatrix} ; \quad \mu =
\begin{bmatrix} uv_{in} - \bar{u}v_d \\ 0 \end{bmatrix}
\]

(23)

At first to design the controller, system error variables are defined:

\[
e(t) = x - x_d = [e_1 \quad e_2] = [x_{1d} - x_1 \quad x_{2d} - x_2]
\]

(24)

In (24), \(x_{1d}\) is the reference value of the inductor current and \(x_{2d}\) is output reference voltage.

Using (24), equation (22) can be rewritten as:

\[
D\dot{e}(t) + (J + R)e(t) = \mu - [D\dot{x}_d(t) + (J + R)x_d(t) + Re(t)]
\]

(25)

In the second operation, the system is considered to evolve unforced, thus:

\[
\mu - [D\dot{x}_d(t) + (J + R)x_d(t) + Re(t)] = 0
\]

(26)

In (26), to develop the PBC, a damping factor (\(R_ie(t)\)) is injected. The stability of the system can be judged by Lyapunov function.

In (26), \(R_i\) is called damping factor:

\[
R_i = \begin{bmatrix} R_i & 0 \\ 0 & 0 \end{bmatrix}
\]

(27)

Simply, equation(26) can be written as follows:

\[
\begin{cases}
uE = L\dot{x}_{1d} + (r_1 + ur_s + \bar{u}r_d)x_{1d} + x_{2d} + \bar{u}v_d - R_i(x_{1d} - x_1) \\C\dot{x}_{2d} - x_{1d} + \frac{1}{R}x_{2d} = 0
\end{cases}
\]

(28)

2.2. Stability:

Using (28), \(x_{2d}\) can be obtained as follows:

\[
\begin{aligned}
\dot{V}(x) &= \frac{1}{2}e^TDe > 0 \quad \forall e \\
&\Rightarrow \quad \dot{V}(x) = \frac{1}{2}e^TD\dot{e} + \frac{1}{2}e^TD\dot{e}
\end{aligned}
\]

(30)

(31)

Using (12) and (13), system error variables can be calculated as:

\[
\begin{cases}
e_1 = -\frac{1}{L}(r_1 + ur_s + \bar{u}r_d)(x_{1d} - x_1) \\
e_2 = \frac{1}{C}(x_{1d} - x_1) - \frac{1}{RC}(x_{2d} - x_2)
\end{cases}
\]

(32)

(33)

By using (32) and (33) in (31), time-derivative of the Lyapunov function can be simplified as:

\[
\dot{V}(x) = -\frac{1}{2}\left[(R_1 + r_1 + ur_s + \bar{u}r_d)e_1^2 + \frac{1}{R}e_2^2\right]
\]

(34)

which is negative function and result in stability of the system.

2.3. Controller design

Using (28), \(x_{2d}\) can be obtained as follows:

\[
x_{2d} = uE - \bar{u}v_d - L\dot{x}_{1d} - (r_1 + ur_s + \bar{u}r_d)x_{1d} - R_i(x_{1d} - x_1)
\]

(35)

Also, time-derivative of \(x_{2d}\) is calculated using (34):

\[
\dot{x}_{2d} = \dot{u}E + \dot{\bar{u}}v_d - L\dot{x}_{1d} - (\dot{ur}_s + \dot{\bar{u}}r_d)x_{1d} - (r_1 + \dot{ur}_s + \dot{\bar{u}}r_d)\dot{x}_{1d} - R_i(\dot{x}_{1d} - \dot{x}_1)
\]

(36)

According to two-loop control approach, the inductor current reference \(x_{1d}\) can be considered as:

\[
x_{1d} = k_p(x_{2d} - x_2) + k_i\int(x_{2d} - x_2)\,dt
\]

(37)

Considering (12) and (13), time-derivatives of \(x_{1d}\) can be calculated as:
By inserting equation (35), (37), (38) and (39) in (36), the PBC controller for DC-DC buck converter with consideration of the parasitic elements is designed as follows:

\[
\dot{x}_{1d} = \frac{k_p}{c}(x_{1d} - x_1) + \frac{k_p}{Rc}(x_2 - x_{2d}) + k_i(x_2 - x_{2d}) \\
\dot{x}_{2d} = \frac{k_p}{c}r_e(x_{2d} - x_2) + \frac{k_p}{c}r_e(x_1 - x_{1d}) + \frac{k_p}{c}(r_1 + u s + u d)(x_1 - x_{1d}) + \frac{k_p}{c}(x_2 - x_{2d}) + \frac{k_i}{c}(x_1 - x_{1d} - x_2) \\
\dot{x}_{2d} = \frac{k_p}{c}r_e(x_{2d} - x_2) + \frac{k_p}{c}r_e(x_1 - x_{1d}) + \frac{k_p}{c}(r_1 + u s + u d)(x_1 - x_{1d}) + \frac{k_i}{c}(x_2 - x_{2d}) + \frac{k_i}{c}(x_1 - x_{1d} - x_2) + \frac{k_i}{c}(x_2 - x_{2d}) + \frac{k_i}{c}(x_1 - x_{1d} - x_2) + \frac{k_i}{c}(x_2 - x_{2d}) + \frac{k_i}{c}(x_1 - x_{1d} - x_2) + \frac{k_i}{c}(x_2 - x_{2d}) + \frac{k_i}{c}(x_1 - x_{1d} - x_2)
\]

By inserting equation (35), (37), (38) and (39) in (36), the PBC controller for DC-DC buck converter with consideration of the parasitic elements is designed as follows:

\[
\dot{u} = \frac{1}{v_{in} + v_a + (r_d - r_e)}[k_p(x_{2d} - x_2) + k_i(x_2 - x_{2d}) dt]
\]

\[
\left[ \frac{1}{c}(x_{1d} - \frac{1}{R}x_{2d}) + L \left\{ \frac{k_p}{c^2}r_e(x_2 - x_2) + \frac{k_p}{c^2}(r_1 + u s + u d)(x_1 - x_{1d}) + \frac{k_i}{c}(x_2 - x_{2d}) + \frac{k_i}{c}(x_1 - x_{1d}) \right\} + (r_1 + u s + u d - R_d) \left\{ \frac{k_p}{c}(x_{1d} - x_1) + \frac{k_p}{c}(x_2 - x_{2d}) + k_i(x_2 - x_{2d}) \right\} + \frac{R_i}{L}((r_1 + u s + u d)x_1 - x_2 + (u v_{in} - u v_d)) \right]
\]

(40)

3. SIMULATION RESULT

Considering nominal specifications of the DC-DC buck converter in Table I, proposed controller is simulated in MATLAB/Simulink toolbox.

Test 1: Considering the nominal values of the power circuit given in Table I and R (load resistance) = 0.3Ω, input voltage power supply is stepped up from \( V_{in} = 10V \) to \( V_{in} = 15V \) at \( t = 25mS \), then it is changed to Initial value \( V_{in} = 10V \) at \( t = 44mS \). Simulation results are illustrated in Fig.4 using the developed PBC. In Fig.5 response of the standard PI controller in similar conditions is illustrated. It is clear that proposed PBC shows more stable response compared with the PI compensator.

Test 2: In this condition, the converter input voltage is supplied by a diode bridge rectifier with a small capacitor filter. Reference voltage and load resistance are \( V_{ref} = 3 \) and \( = 0.035 mΩ \). Considering the nominal values of the power circuit given in Table I, simulation results of the developed PBC and PI controllers are illustrated in Fig.7.

Test 3: Assuming the load resistance of \( R = 0.3Ω \) and considering the nominal values of the power circuit given in Table I, the reference voltage is stepped up from \( V_{ref} = 5V \) to \( V_{ref} = 8V \) at \( t = 15mS \); then the reference voltage is stepped down from \( V_{ref} = 8V \) to \( V_{ref} = 3V \) at \( t = 30mS \). Simulation results are shown in Fig.7.

Test 4: Considering the nominal values of the power circuit, the input voltage is stepped up from \( V_{in} = 10V \) to \( V_{in} = 15V \) at \( t = 15mS \) and simultaneously load resistance of the converter is changed from \( R = 0.3Ω \) to \( R = 0.27Ω \); then value of reference voltage and load resistance is returned to initial values. Simulation result related to PBC is illustrated in Fig.8.

4. CONCLUSIONS

In this paper, passivity-based controller is designed for output voltage regulation of the DC-DC buck converter in high-power applications. In this approach, effect of the parasitic elements are modeled during state-space modeling of the converter and considering nonlinear nature of the PBC, the developed controller is able to stabilize converter in high-power and small reference values. In spite of large changes in load value, input voltage and model
parameters, simulation results prove accuracy of the developed controller. Compared with standard PI controller, proposed method has better dynamic and steady-state responses.

**TABLE I.** specifications of the dc–dc buck converter

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>input power supply</td>
<td>24V</td>
</tr>
<tr>
<td>2</td>
<td>Converter inductor</td>
<td>120 μH</td>
</tr>
<tr>
<td>3</td>
<td>Output capacitor</td>
<td>330μF</td>
</tr>
<tr>
<td>4</td>
<td>Switching frequency</td>
<td>100KHz</td>
</tr>
<tr>
<td>5</td>
<td>ESR of inductor</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>6</td>
<td>switch static resistance</td>
<td>0.1Ω</td>
</tr>
<tr>
<td>7</td>
<td>Diode dynamic resistance</td>
<td>0.001Ω</td>
</tr>
<tr>
<td>8</td>
<td>Diode voltage in forward bias</td>
<td>0.8V</td>
</tr>
<tr>
<td>9</td>
<td>kp</td>
<td>0.25</td>
</tr>
<tr>
<td>10</td>
<td>$k_I$</td>
<td>1000</td>
</tr>
<tr>
<td>11</td>
<td>$R_I$</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Fig. 5.** PBC response to step changes in the input voltage - test 1

**Fig. 6.** PI controller response to step changes in the input voltage - test 1

**Fig. 7.** Input and output voltage using PBC - test 2

**Fig. 8.** Output voltage using PBC in more detail - test 2
Fig. 9. Output voltage using PI controller - test 2

Fig. 10. Output voltage using PBC in more detail - test 3

Fig. 11. Output voltage using PI controller - test 3

Fig. 12. PBC response to step changes in the input voltage and load resistor change - test 4

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