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Ranking Efficient DMUs Using the Variation Coefficient of Weights in DEA

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Abstract

One of the difficulties of Data Envelopment Analysis(DEA) is the problem of de_ciency discrimination among efficient Decision Making Units (DMUs) and hence, yielding large number of DMUs as efficient ones. The main purpose of this paper is to overcome this inability. One of the methods for ranking efficient DMUs is minimizing the Coefficient of Variation (CV) for inputs-outputs weights. In this paper, it is introduced a nonlinear model for ranking efficient DMUs based on the minimizing the mean absolute deviation of weights and then we convert the nonlinear model proposed into a linear programming form.

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INTRODUCTION

Data envelopment analysis (DEA) was initiated by Charnes et al., 1978 as a method to assess relative efficiency of homogeneous decision making units with multiple inputs and multiple outputs. Then, Banker et al., 1984 extended basic DEA models under returns to scale. In DEA we sometimes encounter extreme values or zeroes in input and/or output weights for examined DMUs. In some cases we meet the unfitness of weights, i.e., a solution giving a big weight to variables with less importance or giving a small or zero weight to important variables. (Bal et al., (2008) As regards, the most models of DEA are introduced the more than one efficient DMU in evaluating the relative efficiency DMUs, thus the investigating rank of efficient DMUs is an interesting research topic. A DMU is called extremely efficient if it cannot be represented as a linear combination (with nonnegative coefficients) of the remaining DMUs (Cooer et al., 2007). In data envelopment analysis, there are several methods for ranking of the extreme efficient DMUs, e.g. AP (Andersen et al., 1993) method, MAJ (Mehrabin, Alirezaee and Jahanshahloo [14]) method. (Andersen et al., 1993) proposed a new procedure to rank efficient DMUs. The AP method exhibits the rank of a given DMU by removing it from the reference set and by computing its super efficiency score. However, the AP model may be infeasible in some cases. (Mehrabian et al.,1999) suggested as MAJ model for complete ranking efficient DMUs, but their approach lacks infeasibility in some cases, too. In order to overcome the drawbacks of the AP (Andersen et al., 1993) and MAJ (Mehrabian et al., 1999) models, (Jahanshahloo et al., 2004) presented a method to rank the extremely efficient DMUs in DEA models with constant and variable returns to scale by using L1-norm. According to a complex treatment was applied in (Jahanshahloo et al., 2004) to convert the nonlinear model based on L1-norm into a linear one which provide an approximately optimal solution, hence, (Wu et al., 2010) have also used an effective transformation to convert the nonlinear model in (jahanshahloo et al., 2004) into a linear model. Also (jahanshahloo et al., 2004) have applied gradient line for ranking efficient units. (Bal et al., 2008), suggested DEA model for ranking of DMUs, when the efficient one is more than one;first,they solved multi-plier CCR model of DEA for obtaining efficiency and optimal weights (inputoutput) DMUs; then,they defined $\bar{u}$, and $\bar{v}$, in which $\bar{u}$ is the mean of the optimal weights of output and $\bar{v}$ be the mean of optimal weights of the input in evaluation of DMUk. They defined Coefficient of Variation (CV) based on inputs-outputs of weights,and suggested a method to rank DMUs based on CV. (Rezai Balf et al., 2012) applied Tchebycheff norm for complete ranking efficient units. (Amirteimoori et al., 2005) introduced a method for ranking of extreme efficient DMUs, based on distance. (Hashimoto, 1999) proposed a super efficiency DEA model with assurance region in order to rank the DMUs completely. (Torgesen et al., 1996) suggested a method for ranking efficient units, by their importance as benchmarks for the inefficient units. (Sexton et al., 1986) investigated a ranking method for DMUs based on a cross-efficiency ratio matrix. The cross-efficiency ranking method computes the efficiency score of each DMU that determines a set of optimal weights using linear programs corresponding to each DMU. Then by taking the average of scores of given DMU is obtained the rank of that DMU. (Liu et al., 2008) determined one common set of weights for ranking efficient DMUs, that DMUs are ranked according to the efficiency score weighted by the common set of weights. In (Mwhrabian et al., 1999) is suggested a DEA model for ranking of DMUs based on de_ning the coefficient of variation for input-output weights. (Khodabakhshi et al., 2012) proposed a method to rank the efficient DMUs. According to their method, firstly the minimum and maximum efficiency values of each DMU are computed under the assumption that the sum of efficiency values of all DMUs is equal to unity. Then, the rank of each DMU is determined in proportion to a combination of its minimum and maximum efficiency values. (Shetty et al., 2010) suggested a method for ranking efficient units, which is created the average of the corresponding inputs and outputs of all DMUs. Early, (Jahanshahloo et al., 2013) modified the model which was proposed by (Bal et al., 2008).They introduced two new models for ranking efficient DMUs based on L1-norm and using mean of input-output weights. In this paper, an alternative method suggests for complete ranking DMUs. The rest of the paper is organized as follows. In Section 2 , we review the concept of DEA framework. In Section 3, we explain the ranking method introduced in (Bal et al., 2008), Section 4 proposes the new model for ranking DMUs based on modify the model introduced by (Bal et al., 2008). Section 5 includes Some numerical examples are also given. The last Section concludes the study.

Data Envelopment Analysis

DEA is a methodology for assessing the relative efficiency of decision making units (DMUs) where each DMU has a multiple inputs used to secure a multiple outputs.

It is assumed in DEA that there are n DMUs and for each DMU_j $(j=1,\ldots n)$ is considered a column vector of inputs (X_j) in order to produce a vector of outputs (Y_j), where $(X_j=x_{1j}, x_{2j},\ldots x_{mj})$ and $(Y_j=y_{1j}, y_{2j},\ldots y_{sj})$. It is also assumed that $X_j\geq 0$, $Y_j\geq 0$, $X_j\neq 0$ and $j=1,\ldots n$ for every.

The following nonlinear fractional programming problem measures the level of DEA relative efficiency (h_k) of the kth DMU (X_k, Y_k):

$$
\begin{aligned}
h_k &= Max \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \\
s.t. &\quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad j=1,\ldots,n \\
&\quad u_r \geq 0, \qquad r=1,\ldots,s \\
&\quad v_i \geq 0, \qquad i=1,\ldots,m
\end{aligned} \tag{1}
$$

Here, $\lambda=(\lambda 1,\ldots, \lambda_n)^T$ is a column vector of unknown variables used for components of the input and output vectors by a combination. θ^* represents the efficiency score of DMU_k in (1), where the superscript (*) indicates optimality.

DMU_k is relative efficient if and only if on optimality, the objective of (1) equals to one and all the slacks are zero.

This fractional program can be converted into a linear programming problem where the optimal value of the objective function indicates the relative efficiency of DMU_k. The reformulated linear programming problem, also known as the CCR model, is as follows:

$$
\begin{aligned}
h_k &= max \sum_{r=1}^{s} u_r y_{rk} \\
s.t. &\quad \sum_{i=1}^{m} v_i x_{ik} = 1, \\
&\quad \sum_{r=1}^{s} u_r y_{rk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0, \; j=1,\ldots,n, \\
&\quad u_r, v_i \geq 0, \qquad r=1,\ldots,s, \; i=1,\ldots,m
\end{aligned} \tag{2}
$$

The model (2) can be solved by using any linear programming software, such as GAMS. The solution to model (2) assigns the value 1 to all efficient DMUs. The super efficiency concept is proposed to differentiate completely among all efficient DMUs when there are more than one efficient DMUs. One of the super efficiency models for ranking efficient DMUs in DEA was introduced by Andersen and Petersen (1993). This method enables an extreme efficient unit "k" to achieve an efficiency score greater than one by removing the kth constraint in the envelopment LP formulation, as shown in model (3) (Adler et al., 2002).

$$
\begin{aligned}
h_k &= max \sum_{r=1}^{s} u_r y_{rk} \\
s.t. &\quad \sum_{i=1}^{m} v_i x_{ik} = 1, \\
&\quad \sum_{r=1}^{s} u_r y_{rk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0, \; j=1,\ldots,n, \; j\neq k \\
&\quad u_r, v_i \geq 0, \qquad r=1,\ldots,s, \; i=1,\ldots,m
\end{aligned} \tag{3}
$$

Now, we review the original model of Bal et al. (2008) is presented as follows. The Coefficient of Variation CV, the ratio of sample standard deviation to the sample mean, measures the variability of the weights relative to their mean (or average). It compares the relative dispersion in one type of data with the relative dispersion in another type of data. u_r for $r=1, 2,\ldots, s$ denote the weight on output r and let $\bar{u}$ denote the mean of the u_r for $r=1, 2,\ldots, s$. Then the CV for the weights u_r is defined as follows:

$$
CV = \frac{\sqrt{\sum_{r=1}^{s} (u_r - \bar{u})^2 / (s-1)}}{\bar{u}}.
$$

Similarly , it can be calculated the CV for the weights v_i for $i=1, 2,\ldots, m$ in the following way:

$$CV=\frac{\sqrt{\sum_{i=1}^{m}(v_i-\bar{v})^2/(m-1)}}{\bar{v}}.$$

They suggested following model by combining the coefficient of the variation for input–output weights to the model (2) which is called Coefficient of Variation data envelopment analysis model (CVDEA model):

$$\begin{aligned} h_k\quad &=\max\sum_{r=1}^{s}u_r y_{rk}-\frac{\sqrt{\sum_{r=1}^{s}(u_r-\bar{u})^2/(s-1)}}{\bar{u}}-\frac{\sqrt{\sum_{i=1}^{m}(v_i-\bar{v})^2/(m-1)}}{\bar{v}}\\ s.t.\quad &\sum_{i=1}^{m}v_i x_{ik}=1,\\ &\sum_{r=1}^{s}u_r y_{rk}-\sum_{i=1}^{m}v_i x_{ik}\le 0,\ j=1,\dots,n,\\ &u_r\ ,v_i\ge 0,\qquad r=1,\dots,s,\ i=1,\dots,m \end{aligned} \tag{4}$$

This nonlinear optimization model,based on the CCR model, can be easily solved with Krash-Kuhn-Tuker algorithm. When there are more than one efficient DMUs, the CV is incorporated into the model 3 instead of model 2 in the minimization process and then all efficient DMUs are ranked over again.

The proposed model

For solving of The model was suggested by Bal et al. (2008) by any software of optimization as GAMS, we encounter the error of division by zero and so, using the Krash-Kuhn-Tuker algorithm for large scale problem is not economic. In this regard, an alternative model is proposed, the following model which minimize the mean absolute deviation of weights namely, the average of the absolute deviations of weights u_r and v_i from their mean.

$$\begin{aligned} h_k\quad &=\max\sum_{r=1}^{s}u_r y_{rk}-\frac{\sum_{r=1}^{s}|u_r-\bar{u}|}{s}-\frac{\sum_{i=1}^{m}|v_i-\bar{v}|}{m}\\ s.t.\quad &\sum_{i=1}^{m}v_i x_{ik}=1,\\ &\sum_{r=1}^{s}u_r y_{rk}-\sum_{i=1}^{m}v_i x_{ik}\le 0,\ j=1,\dots,n,\\ &u_r\ ,v_i\ge 0,\qquad r=1,\dots,s,\ i=1,\dots,m \end{aligned} \tag{5}$$

In order to linearize above nonlinear model we let $a'_r=\frac{1}{2}(|u_r-\bar{u}|+u_r-\bar{u})$ and $b'_r=\frac{1}{2}(|u_r-\bar{u}|-(u_r-\bar{u}))$ and also we let $a''_i=\frac{1}{2}(|v_i-\bar{v}|+u_i-\bar{u})$ and $b''_i=\frac{1}{2}(|v_i-\bar{v}|-(v_i-\bar{v}))$. Then, model (5) can be transformed into the following linear programming problem:

$$\begin{aligned} h_k\quad &=\max\sum_{r=1}^{s}u_r y_{rk}-\frac{1}{s}\sum_{r=1}^{s}(a'_r+b'_r)-\frac{1}{m}\sum_{r=1}^{m}(a''_i+b''_i)\\ s.t.\quad &\sum_{i=1}^{m}v_i x_{ik}=1,\\ &\sum_{r=1}^{s}u_r y_{rk}-\sum_{i=1}^{m}v_i x_{ik}\le 0,\ j=1,\dots,n,\\ &a'_r-b'_r=u_r-\bar{u},\qquad r=1,\dots,s\\ &a''_i-b''_i=v_i-\bar{v},\qquad i=1,\dots,m\\ &u_r\ ,v_i\ge 0,\qquad r=1,\dots,s,\ i=1,\dots,m \end{aligned} \tag{6}$$

When there are more than one efficient DMUs, the following super efficiency model is used for ranking all efficient DMUs:

$$\begin{aligned} h_k\quad &=\max\sum_{r=1}^{s}u_r y_{rk}-\frac{\sum_{r=1}^{s}|u_r-\bar{u}|}{s}-\frac{\sum_{i=1}^{m}|v_i-\bar{v}|}{m}\\ s.t.\quad &\sum_{i=1}^{m}v_i x_{ik}=1,\\ &\sum_{r=1}^{s}u_r y_{rk}-\sum_{i=1}^{m}v_i x_{ik}\le 0,\ j=1,\dots,n,\ j\ne k\\ &u_r\ ,v_i\ge 0,\qquad r=1,\dots,s,\ i=1,\dots,m \end{aligned} \tag{7}$$

According to linearize the (5) the above nonlinear model can be written the following linear programming form:

$$\begin{aligned} h_k\quad &=\max\sum_{r=1}^{s}u_r y_{rk}-\frac{1}{s}\sum_{r=1}^{s}(a'_r+b'_r)-\frac{1}{m}\sum_{r=1}^{m}(a''_i+b''_i)\\ s.t.\quad &\sum_{i=1}^{m}v_i x_{ik}=1,\\ &\sum_{r=1}^{s}u_r y_{rk}-\sum_{i=1}^{m}v_i x_{ik}\le 0,\ j=1,\dots,n,\\ &a'_r-b'_r=u_r-\bar{u},\qquad r=1,\dots,s\\ &a''_i-b''_i=v_i-\bar{v},\qquad i=1,\dots,m\\ &u_r\ ,v_i\ge 0,\qquad r=1,\dots,s,\ i=1,\dots,m \end{aligned} \tag{8}$$

It should be noted that the CVDEA method is not stable by changing the measurement unit but the proposed method over come this drawback.

Illustrative examples

In this section, we employ the above DEA model (5) on the data sets of examples used in (Bal et al., 2008).

Example 1 (Efficiency evaluation of six nursing homes). Two inputs and two output variables for six nursing homes are staff hours per day, including nurses, physicians etc. *(x_1)*; supplies per day, measured in thousands of dollars *(x_2)*; total medicare-plus medicaid-reimbursed patient days *(y_1)*; and total privately paid patient days *(y_2)*, respectively, and the related data are given in Table 1. For detailed descriptions of the data see Sexton (1986).

When our proposed model and CVDEA, DEA, AP models are applied to the data, the results demon-strated in Tables 2-5 are obtained. As shown in Table 2, DMU A, DMU B, DMU C, DMU D are efficient by applying DEA model (models 2 and 3). Also, in Table 2, we compute efficiency and optimal weights of inputs and outputs by using AP model. Using CVDEA model, the results are depicted in table 3. Table 4 presents the efficiency and super efficiency according to proposed model. In Table 5, the ranks of DMUs are obtained by the DEA under the super efficiency, the CVDEA model, and the proposed

Table 1: Data of six nursing homes

DMU	y_1	y_2	x_1	x_2
A	1.40	0.35	1.50	0.2
B	1.40	2.10	4.00	0.7
C	4.20	1.05	3.20	1.2
D	2.80	4.20	5.20	2.0
E	1.90	2.50	3.50	1.2
F	1.40	1.50	3.20	0.7

Table 2: Results of the DEA model

DMU	Efficiency	Super efficiency	u_1	u_2	v_2	v_2
A	1	2	0.714	0	0	5.000
B	1	1.395	0	0.476	0	1.429
C	1	1.412	0.238	0	0.172	0.374
D	1	1.131	0	0.238	0.069	0.321
E	0.977	0.977	0.115	0.304	0.110	0.513
F	0.867	0.867	0.162	0.427	0.155	0.722

Table 3: Results of the CVDEA model

DMU	Efficiency	u_1	u_2	v_2	v_2
A	1	0.571	0.57	0.517	1.120
B	0.863	0.176	0.293	0.181	0.392
C	0.991	0.189	0.189	0.227	0.227
D	0.983	0.103	0.165	0.138	0.138
E	0.948	0.158	0.259	0.212	0.212
F	0.735	0.190	0.312	0.256	0.256

Table 4: Results of the proposed model

DMU	Efficiency	u_1	u_2	v_2	v_2
A	1.000	2.400	2.400	2.574	2.574
B	0.694	0.833	0.833	0.893	0.893
C	0.959	0.767	0.767	0.823	0.823
D	0.833	0.399	0.734	0.500	0.500
E	0.767	0.732	0.732	0.785	0.785
F	0.667	0.966	0.966	1.036	1.036

Table 5: Ranks of the DMUs for the models

DMU	DEA	CVDEA	Proposed model
A	1	1	1
B	3	5	5
C	2	2	2
D	4	3	3
E	5	4	4
F	6	6	6

Table 6: Data of seven departments in a university

DMU	y1	y2	y3	x1	x2	x3
1	60	35	17	12	400	20
2	139	41	40	19	750	70
3	225	68	75	42	1500	70
4	90	12	17	15	600	100
5	253	145	130	45	2000	250
6	132	45	45	19	730	50
7	305	159	97	41	2350	600

Table 7: Results of the DEA model

DMU	Efficiency	Super efficiency	u_1	u_2	u_3	v_2	v_2
1	1	1.829	0.983	1.172	0	0	0.250
2	1	1.048	0.719	0	0	0	0.133
3	1	1.198	0	0	1.333	0	0.033
4	0.820	0.819	0.911	0	0	6.415	0.006
5	1	1.220	0	0.432	0.288	0	0.05
6	1	1.190	0.639	0	0.347	0	0.137
7	1	1.266	0.121	0.334	0.105	0.732	0.030

Table 8: Results of the CVDEA model

DMU	Efficiency	Super efficiency	u_1	u_2	u_3	v_2	v_2	v_3
1	1	1.368	0.847	0.971	0.893	0.870	0.193	0.618
2	0.983	0.983	0.462	0.403	0.438	0.106	0.124	0.063
3	0.990	0.990	0.293	0.162	0.293	0.756	0.021	0.514
4	0.820	0.820	0.812	0.420	0.350	1.107	0.109	0.179
5	1	1.311	0.038	0.293	0.366	0.101	0.032	0.125
6	0.980	0.980	0.440	0.440	0.440	0.133	0.133	0
7	1	1.253	0.178	0.178	0.178	0.513	0.025	0.033

method. In this example,by comparing of results we will see ranking of DMUs based on AP,CVDEA methods have exactly similar results (see Tables 5). Also with comparing results proposed methods with CVDEA and AP methods we will see the rank of DMU B to DMU E are different (Table 5). For more details about the rank of DMUs, see the results in Tables 2-5.

Example 2 (Efficiency evaluation of seven departments in a university). The input–output variables for seven departments in a university are defined as follows and the related data are given in Table 6: y1 number of undergraduate students y2 number of postgraduate students y3 number of research papers x1 number of academic staff x2 academic staff salaries in thousands of pounds x3 support staff salaries in thousands of pounds.

Table 7 reports the results of ranking for 6 extremely efficient DMUs (DMU1, DMU2, DMU3, DMU5, DMU6, DMU7) in DEA model. The results of ranking DMUs based on super efficiency DEA and CVDEA model are in-cluded in table 8. Again, when our proposed model is applied to this data, the results in Table 9 is ob-

Table 9: Results of the proposed model

DMU	Efficiency	u_1	u_2	u_3	v_2	v_2	v_3
1	0.895	1.635	1.635	1.635	2.127	2.127	2.127
2	0.915	0.896	0.896	0.896	1.165	1.165	1.165
3	0.793	0.455	0.455	0.455	0.592	0.592	0.592
4	0.510	1.018	1.018	1.018	1.324	1.324	1.324
5	0.929	0.339	0.339	0.339	0.441	0.441	0.441
6	1.000	0.942	0.942	0.942	1.225	1.225	1.225
7	0.778	0.330	0.330	0.330	0.542	0.349	0.157

Table 10: Ranks of the DMUs for models

DMU	DEA	CVDEA	Proposed model
1	1	1	4
2	6	5	3
3	4	4	5
4	7	7	7
5	3	2	2
6	5	6	1
7	2	3	6

tained. In Table 10, the ranks of DMUs obtained by the DEA under super efficiency, the CVDEA model, and the proposed model due to comparison of ranking scores. By using AP and CVDEA methods for ranking DMUs, we will see the rank of DMU1 and DMU7 are the same (Table 10). The result of ranking DMUs based on

proposed method is very different the results of AP and CVDEA methods.

CONCLUSION

In this paper, we provides a simpler nonlinear model for ranking efficient DMUs based on minimizing the mean absolute deviation of weights and then we convert the nonlinear model proposed into a linear program-ming form. Considering the computational complexity of nonlinear CVDEA model, the proposed treatment in this article is easier to be utilized. The results show that the proposed method well performs.

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