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Fuzzy $\bigwedge_e$ sets and continuity in fuzzy topological spaces

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Abstract. We introduce a new class of fuzzy open sets called fuzzy $\bigwedge_e$ sets which includes the class of fuzzy e-open sets. We also define a weaker form of fuzzy $\bigwedge_e$ sets termed as fuzzy locally $\bigwedge_e$ sets. By means of these new sets, we present the notions of fuzzy $\bigwedge_e$ continuity and fuzzy locally $\bigwedge_e$ continuity which are weaker than fuzzy e-continuity and furthermore we investigate the relationships between these new types of continuity and some others.

Keywords: Fuzzy $\bigwedge_e$ set, fuzzy $\bigvee_e$ set, fuzzy locally $\bigwedge_e$ set, fuzzy $\bigwedge_e$ continuity, fuzzy locally $\bigwedge_e$ continuity.

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1. Introduction

The concepts of fuzzy sets and fuzzy topology were firstly given by Zadeh in [26] and Chang in [5], and after then there have been many developments on defining uncertain situations and relations in more realistic way. The fuzzy topology theory has rapidly began to play an important role in many different scientific areas such as economics, quantum physics and geographic information system (GIS). For instance, Shi and Liu mentioned that the fuzzy topology theory can potentially provide a more realistic description of uncertain spatial objects and uncertain relations in [25] where they developed the computational fuzzy topology Which is based on the interior and the closure operator.

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Besides, the concepts of fuzzy topology and fuzzy sets have very important applications on particle physic in connection with string theory and ϵ^{∞} theory studied by El-Naschie [18], [19], [20].

Maki [15] introduced the notion of $\bigwedge$ sets in topological spaces. A $\bigwedge$ set is a set λ which is equal to to its kernel (saturated sets), (i.e) to the intersection of all open supersets of λ. Arenas et al.[2] introduced and investigated the notion of λ-closed sets and λ-open sets by involving $\bigwedge$-sets and closed sets.

In 2008, Erdal Ekici [7–11], has introduced and studied the concept of e-open sets in general topology. Seenivasan [23] defined the concept of fuzzy e-open sets and studied fuzzy e-continuous mappings on fuzzy topological spaces. Fuzzy e open sets are weaker than fuzzy δ preopen set, fuzzy δ semi open sets. Using these notion, he studied fuzzy e-continuous mappings in fuzzy topological spaces. In this paper, we extend the notion of e-open sets to fuzzy topological space in the name fuzzy $\bigwedge_e$ sets and fuzzy locally $\bigwedge_e$ sets and study some properties based on this concept. We also introduce the concepts of fuzzy $\bigwedge_e$ continuity and fuzzy locally $\bigwedge_e$ continuity.

2. Preliminaries

Throughout this paper, nonempty sets will be denoted by X, Y etc., $I = [0,\ 1]$ and $I_0 = (0,\ 1]$. For $\alpha \in I$, $\overline{\alpha}(x) = \alpha$ for all $x \in X$. A fuzzy point x_t [14] for $t \in I_0$ is an element of I^X such that $x_t(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{if } y \notin x. \end{cases}$ The set of all fuzzy points in X is denoted by $Pt(X)$. A fuzzy point $x_t \in \lambda$ [14] iff $t < \lambda(x)$. A fuzzy set λ is quasi-coincident with μ [14], denoted by $\lambda q \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. If λ is not quasi-coincident with μ, we denoted $\lambda \bar{q} \mu$. If $A \subset X$, we define the characteristic function χ_A on X by $\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$ All other notations and definitions are standard, for all in the fuzzy set theory.

Here, $(X,\ \tau)$ mean fuzzy topological space (fts, for short) in Chang's sense [5]. For a fuzzy set λ of a fts X, the notion I^X, $\lambda^c = 1_X - \lambda$, $Cl(\lambda)$, $Int(\lambda)$, will respectively stand for the set of all fuzzy subsets of X, the complement, fuzzy closure, fuzzy interior, of λ. By 1_ϕ (or 0_X or ϕ) and 1_X (or X) we will mean the fuzzy null set and fuzzy whole set with constant membership function 0 (zero function) and 1 (unit function) respectively.

Definition 2.1 A fuzzy subset λ in a fuzzy topological space (X, τ) is called

(i) fuzzy regular open (resp. fuzzy regular closed) set [1] if $Int(Cl(\lambda)) = \lambda$ (resp. $Cl(Int(\lambda)) = \lambda$) or if $1_X - \lambda$ is fuzzy regular open set in X.
(ii) fuzzy δ preopen set [3] if $\lambda \leqslant Int(\delta Cl(\lambda))$ (resp. fuzzy δ preclosed set) if $\lambda \geqslant Cl(\delta Int(\lambda))$.
(iii) fuzzy δ semiopen set [17] if $\lambda \leqslant Cl(\delta Int(\lambda))$ (resp. fuzzy δ semiclosed set) if $\lambda \geqslant Int(\delta Cl(\lambda))$.
(iv) a fuzzy e-open set [23] of X if $\lambda \leqslant Cl(\delta Int(\lambda)) \bigvee Int(\delta Cl(\lambda))$.
(v) a fuzzy e-closed set [23] of X if $Cl(\delta Int(\lambda)) \bigvee Int(\delta Cl(\lambda)) \leqslant \lambda$.

The family of all e-open (resp. e-closed) sets of X will be denoted by $eO(X)$ (resp. $eC(X)$.)

Definition 2.2 [23] Let (X, τ) be a fuzzy topological space. Let λ be a fuzzy set of a fuzzy topological space X.

(i) $eInt(\lambda) = \bigvee\{\mu \in I^X : \mu \leqslant \lambda,\ \mu$ is a eO set $\}$ is called the fuzzy e-interior of λ.
(ii) $eCl(\lambda) = \bigwedge\{\mu \in I^X : \mu \geqslant \lambda,\ \mu$ is a eC set $\}$ is called the fuzzy e-closure of λ.

Definition 2.3 Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a mapping from a fts (X, τ_1) to another (Y, τ_2). Then f is called

(1) fuzzy continuous [5] if $f^{-1}(\lambda)$ is fuzzy open set in X for any fuzzy open set λ in Y.
(2) fuzzy δ-semicontinuous [6] if $f^{-1}(\mu)$ is a fuzzy semiopen set of X for each $\mu \in \tau_2$.
(3) fuzzy δ-precontinuous [3] if $f^{-1}(\mu)$ is a fuzzy pre-open set of X for each $\mu \in \tau_2$.
(4) fuzzy e-continuous [23] if $f^{-1}(\mu)$ is a fuzzy e-open set of X for each $\mu \in \tau_2$.

3. Fuzzy $\bigwedge_e$ and fuzzy $\bigvee_e$ sets

In this section, we define the class of fuzzy $\bigwedge_e$ sets which is a weaker form of fuzzy e-open sets and investigate some basic properties of this class. We also present the fuzzy $\bigvee_e$ sets as the dual concept of fuzzy $\bigwedge_e$ sets.

Definition 3.1 Let $(X,\ \tau)$ be a fuzzy topological space and λ be a fuzzy set of X. The fuzzy set $\lambda^{\wedge_e}$ and $\lambda^{\vee_e}$ set of λ are defined as follows:

$$\lambda^{\wedge_e} = \bigwedge \Big\{\alpha : \lambda \leqslant \alpha, \alpha \in eO(X)\Big\},$$
$$\lambda^{\vee_e} = \bigvee \Big\{\gamma : \gamma \leqslant \lambda, \gamma \in eC(X)\Big\}.$$

Proposition 3.2 Let $(X,\ \tau)$ be a fuzzy topological space and λ, μ and μ_i $(i \in \Omega)$ be the fuzzy sets of X. The following statements are valid:

(i) $\mu \leqslant \mu^{\wedge_e}$,
(ii) If $\lambda \leqslant \mu$, then $\lambda^{\wedge_e} \leqslant \mu^{\wedge_e}$,
(iii) $(\mu^{\wedge_e})^{\wedge_e} = \mu^{\wedge_e}$,
(iv) $\bigvee_{i\in\Omega} \mu_i{}^{\wedge_e} \leqslant (\bigvee_{i\in\Omega} \mu_i)^{\wedge_e}$,
(v) $(\bigwedge_{i\in\Omega} \mu_i)^{\wedge_e} \leqslant \bigwedge_{i\in\Omega} \mu_i^{\wedge_e}$,
(vi) $\mu^{\vee_e} \leqslant \mu$,
(vii) If $\lambda \leqslant \mu$, then $\lambda^{\vee_e} \leqslant \mu^{\vee_e}$,
(viii) $(\mu^{\vee_e})^{\vee_e} = \mu^{\vee_e}$,
(ix) $\bigvee_{i\in\Omega} \mu_i{}^{\vee_e} \leqslant (\bigvee_{i\in\Omega} \mu_i)^{\vee_e}$,
(x) $(\bigwedge_{i\in\Omega} \mu_i)^{\vee_e} \leqslant \bigwedge_{i\in\Omega} \mu_i{}^{\vee_e}$,
(xi) $(\mu^c)^{\wedge_e} = (\mu^{\vee_e})^c$.

Proof. We will prove only (v) and (xi). The others can be proved in a similar way.
For all $i \in \Omega$ we have

$$\bigwedge_{i\in\Omega} \mu_i \leqslant \mu_i \ \Rightarrow\ \Big(\bigwedge_{i\in\Omega} \mu_i\Big)^{\wedge_e} \leqslant (\mu_i)^{\wedge_e} \ \Rightarrow\ \Big(\bigwedge_{i\in\Omega} \mu_i\Big)^{\wedge_e} \leqslant \bigwedge_{i\in\Omega} \mu_i^{\wedge_e}.$$

which proves (v). For (xi),

$$\begin{aligned}(\mu^{\bigvee_e})^c &= \left(\bigvee\left\{\gamma : \gamma \leqslant \mu, \gamma \in eC(X)\right\}\right)^c \\ &= \bigwedge\left\{\gamma^c : \mu^c \leqslant \gamma^c, \gamma^c \in eO(X)\right\} \\ &= \bigwedge\left\{\alpha : \mu^c \leqslant \alpha, \alpha \in eO(X)\right\} \\ &= (\mu^c)^{\bigwedge_e}\end{aligned}$$

■

Definition 3.3 Let λ be a fuzzy set of a fuzzy topological space (X, τ). Then λ is called

(1) a fuzzy set $\bigwedge_e$ set if $\lambda = \lambda^{\bigwedge_e}$.
(2) a fuzzy set $\bigvee_e$ set if $\lambda = \lambda^{\bigvee_e}$.

The family of all fuzzy $\bigwedge_e$ sets and $\bigvee_e$ sets will be denoted by $\bigwedge_e(X)$ and $\bigvee_e(X)$, respectively.

Theorem 3.4 μ is a fuzzy $\bigwedge_e$ set iff μ^c is a fuzzy $\bigvee_e$ set.

Proof. It is obvious. ■

Proposition 3.5 Let λ be fuzzy set of a fuzzy topological space (X, τ).

(i) If $\lambda \in eO(X)$, then $\lambda \in \bigwedge_e(X)$.
(ii) If $\lambda \in eC(X)$, then $\lambda \in \bigvee_e(X)$.

Proof. It is obvious. ■

Remark 1 *None of the reverse implications in Proposition 3.5 is valid as shown in the following examples.*

***Example* 3.6** Let λ and μ be fuzzy subsets of $X = \{a, b\}$ and defined as follows:
$\lambda(a) = 0.2$, $\lambda(b) = 0.3$;
$\mu(a) = 0.1$, $\mu(b) = 0.4$..
Then, $\tau = \{0, 1, \lambda\}$ is a fuzzy topology on X. Clearly, it can be shown that $Cl(\delta Int\mu) = 0$ and $Int(\delta Cl\mu) = \lambda$. Since, $\mu \nleq Cl(\delta Int\mu) \bigvee Int(\delta Cl\mu) = (0.2_a, 0.3_b)$, μ is not a fuzzy e-open set. However, $\mu^{\bigwedge_e} = \mu$. Therefore, μ is a fuzzy $\bigwedge_e$ set.

***Example* 3.7** Let λ and μ be fuzzy subsets of $X = \{a, b\}$ and defined as follows:
$\lambda(a) = 0.3$, $\lambda(b) = 0.5$;
$\mu(a) = 0.6$, $\mu(b) = 0.7$..
Then, $\tau = \{0, 1, \lambda\}$ is a fuzzy topology on X. Clearly, it can be shown that $Cl(\delta Int\mu) = \lambda^c$ and $Int(\delta Cl\mu) = 1$. Since, $\mu \ngeq Cl(\delta Int\mu) \bigwedge Int(\delta Cl\mu) = (0.7_a, 0.5_b)$, μ is not a fuzzy e-closed set. However, $\mu^{\bigvee_e} = \mu$. Therefore, μ is a fuzzy $\bigvee_e$ set.

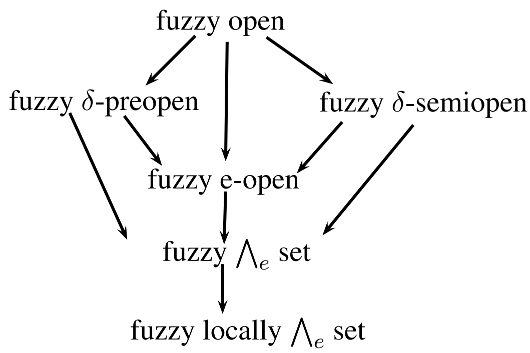
***Example* 3.8** Let λ, μ and ω be fuzzy subsets of $X = \{a, b, c\}$ and defined as follows:
$\lambda(a) = 0.2$, $\lambda(b) = 0.3$, $\lambda(c) = 0.4$;
$\mu(a) = 0.1$, $\mu(b) = 0.1$, $\mu(c) = 0.4$;
$\omega(a) = 0.2$, $\omega(b) = 0.4$, $\omega(c) = 0.4$.
Then, $\tau = \{0, 1, \lambda, \mu\}$ is a fuzzy topology on X. Since, $\omega = (0.2_a, 0.4_b, 0.4_c) \nleq Int(\delta Cl(\omega)) = \lambda = (0.2_a, 0.3_a, 0.4_a)$, ω is not fuzzy δ preopen. On the other hand, $\omega^{\bigwedge_e} = \omega$. That is ω is a fuzzy $\bigwedge_e$ set.

***Example* 3.9** Let λ, μ and ω be fuzzy subsets of $X = \{a, b, c\}$ and defined as follows:
$\lambda(a) = 0.3$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$;
$\mu(a) = 0.6$, $\mu(b) = 0.5$, $\mu(c) = 0.5$;
$\omega(a) = 0.7$, $\omega(b) = 0.7$, $\omega(c) = 0.5$.
Then, $\tau = \{0,\ 1,\ \lambda,\ \mu\}$ is a fuzzy topology on X. Since, $\omega = (0.7_a, 0.7_b, 0.5_c) \nleq Int(\delta Cl(\omega)) = \lambda^c = (0.7_a, 0.6_a, 0.5_a)$, ω is not fuzzy δ semiopen. On the other hand, $\omega^{\bigwedge_e} = \omega$. That is ω is a fuzzy $\bigwedge_e$ set.

***Example* 3.10** Let λ, μ and ω be fuzzy subsets of $X = \{a, b, c\}$ and defined as follows:
$\lambda(a) = 0.3$, $\lambda(b) = 0.5$, $\lambda(c) = 0.2$;
$\mu(a) = 0.4$, $\mu(b) = 0.5$, $\mu(c) = 0.5$;
$\omega(a) = 0.2$, $\omega(b) = 0.2$, $\omega(c) = 0.2$.
Then, $\tau = \{0,\ 1,\ \lambda,\ \mu\}$ is a fuzzy topology on X. Since, $\omega \leqslant Int(\delta Cl(\omega))$ and $\omega \leqslant Cl(\delta Int(\omega))$, ω is fuzzy δ preopen and fuzzy δ semiopen. But ω is not fuzzy open.

***Remark* 2** *Every fuzzy $\bigwedge_e$ set is fuzzy locally $\bigwedge_e$ set but the converse is not true as shown in Example 4.2.*

***Remark* 3** *The following diagram of the implications is true.*



Theorem 3.11 Let λ and λ_i $(i \in \Omega)$ be the fuzzy sets of the fuzzy topological space $(X,\ \tau)$. Then

(1) $\lambda^{\bigwedge_e}$ is a fuzzy $\bigwedge_e$ set.
(2) $\lambda^{\bigvee_e}$ is a fuzzy $\bigvee_e$ set.
(3) If $\{\lambda_i : i \in \Omega\} \subseteq \bigwedge_e(X)$, then $\bigwedge_{i\in\Omega} \lambda_i$ is a fuzzy $\bigwedge_e$ set.
(4) If $\{\lambda_i : i \in \Omega\} \subseteq \bigvee_e(X)$, then $\bigvee_{i\in\Omega} \lambda_i$ is a fuzzy $\bigvee_e$ set.

Proof. (1) By Proposition 3.2(iii), $(\lambda^{\bigwedge_e})^{\bigwedge_e} = \lambda^{\bigwedge_e}$. Hence $\lambda^{\bigwedge_e}$ is a fuzzy $\bigwedge_e$ set.
(2) It is clear from Proposition 3.2 (viii).
(3) For all $i \in \Omega$, we have

$$(\lambda_i)^{\bigwedge_e} = \lambda_i \Rightarrow \bigwedge_{i\in\Omega} \lambda_i^{\bigwedge_e} = \bigwedge_{i\in\Omega} \lambda_i \Rightarrow (\bigwedge_{i\in\Omega} \lambda_i)^{\bigwedge_e} \leqslant \bigwedge_{i\in\Omega} (\lambda_i)^{\bigwedge_e} = \bigwedge_{i\in\Omega} \lambda_i.$$

Since for all $i \in \Omega$, $\bigwedge_{i\in\Omega} \lambda_i \leqslant (\bigwedge_{i\in\Omega} \lambda_i)^{\bigwedge_e}$ holds, thus $\bigwedge_{i\in\Omega} \lambda_i = (\bigwedge_{i\in\Omega} \lambda_i)^{\bigwedge_e}$.
(4) It can be proved in similar manner in (3). ■

4. Fuzzy locally $\bigwedge_e$ sets

In this section, we introduce the class of fuzzy locally $\bigwedge_e$ sets including the class of fuzzy $\bigwedge_e$ sets and give two characterizations of these sets.

Definition 4.1 Let λ be a fuzzy set of a fuzzy toplogical space $(X,\ \tau)$. λ is called fuzzy locally $\bigwedge_e$ set if there exists a fuzzy $\bigwedge_e$ set α and a fuzzy e-closed set β such that $\lambda = \alpha \wedge \beta$.

Remark 4 *Since, $\lambda = \lambda \wedge 1_X$, for every fuzzy set λ, every fuzzy $\bigwedge_e$ set is a fuzzy locally $\bigwedge_e$ set and every fuzzy e-closed set is a fuzzy locally $\bigwedge_e$ set.*

***Example* 4.2** Let λ, μ, γ and δ be fuzzy subsets of $X = \{a,\ b\}$ and defined as follows:
$\lambda(a) = 0.2,\ \lambda(b) = 0;$
$\mu(a) = 0.1,\ \mu(b) = 1;$
$\gamma(a) = 0.2,\ \gamma(b) = 1.$
Then, $\tau = \{0,\ 1,\ \lambda\}$ is a fuzzy topology on X. Clearly, it can be shown that $Cl(\delta Int\mu) = 0$ and $Int(\delta Cl\mu) = \lambda$. Since, $\mu \not\leq Cl(\delta Int\mu) \bigvee Int(\delta Cl\mu) = \lambda = (0.2_a, 0.0_b)$, μ is not a fuzzy e-open set. Hence, $\mu \notin \mu^{\bigwedge_e}(X)$. But μ is a fuzzy locally $\bigwedge_e$ set, since μ can be represented as $\mu = \gamma \wedge \mu$ where γ is a fuzzy $\bigwedge_e$ set and μ is a fuzzy e-closed set. Hence, fuzzy locally $\bigwedge_e$ set need not be fuzzy $\bigwedge_e$ set.

Remark 5 *Every fuzzy e-closed set is fuzzy locally $\bigwedge_e$ set but the converse need not be true as shown in example below.*

***Example* 4.3** Let λ, μ, γ and δ be fuzzy subsets of $X = \{a,\ b\}$ and defined as follows:
$\lambda(a) = 0.3,\ \lambda(b) = 0.5;$
$\mu(a) = 0.6,\ \mu(b) = 0.7;$
$\delta(a) = 0.7,\ \delta(b) = 0.9;$
Then, $\tau = \{0,\ 1,\ \lambda\}$ is a fuzzy topology on X. Clearly, it can be shown that $Cl(\delta Int\mu) = \lambda^c$ and $Int(\delta Cl\mu) = 1$. Since, $\mu \not\geq Cl(\delta Int\mu) \bigvee Int(\delta Cl\mu) = \lambda^c = (0.7_a, 0.5_b)$, μ is not a fuzzy e-closed set. But μ is a fuzzy locally $\bigwedge_e$ set, since μ can be represented as $\mu = \mu \wedge \delta$ where μ is a fuzzy $\bigwedge_e$ set and δ is a fuzzy e-closed set. Hence, fuzzy locally $\bigwedge_e$ set need not be fuzzy e-closed set.

Theorem 4.4 Let λ be a fuzzy set of a fuzzy topological space $(X,\ \tau)$. The following statements are equivalent:

(1) λ is a fuzzy locally $\bigwedge_e$ set.
(2) $\lambda = \alpha \wedge eCl(\lambda)$ for a fuzzy $\bigwedge_e$ set α.
(3) $\lambda = \lambda^{\bigwedge_e} \bigwedge eCl(\lambda)$.

Proof. (1) $\Rightarrow$ (2) : Let $\lambda = \alpha \wedge \beta$ where α is a fuzzy Λ_e set and β is a fuzzy e-closed set. Since $\lambda \leqslant \alpha$ and $\lambda \leqslant eCl(\lambda)$, we have $\lambda \leqslant \alpha \wedge eCl(\lambda)$. On the other hand, $\lambda \leqslant \beta$ and $\lambda \leqslant eCl(\lambda) \leqslant eCl(\beta) = \beta$, $\alpha \wedge eCl(\lambda) \leqslant \lambda$ which completes the proof.

(2) $\Rightarrow$ (3) : If $\lambda = \alpha \wedge eCl(\lambda)$ for a fuzzy $\bigwedge_e$ set α, then $\lambda \leqslant \alpha$. Thus, $\lambda^{\bigwedge_e} \leqslant \alpha^{\bigwedge_e} = \alpha$ which implies $\lambda^{\bigwedge_e} \wedge eCl(\lambda) \leqslant \alpha \wedge eCl(\lambda) = \lambda$. Since $\lambda \leqslant \lambda^{\bigwedge_e}$ and $\lambda \leqslant eCl(\lambda)$, $\lambda \leqslant \lambda^{\bigwedge_e} \wedge eCl(\lambda)$.

(3) $\Rightarrow$ (1) : Since $\lambda^{\bigwedge_e}$ is a fuzzy $\bigwedge_e$ set and $eCl(\lambda)$ is a fuzzy e-closed set, λ is a fuzzy locally $\bigwedge_e$ set. ■

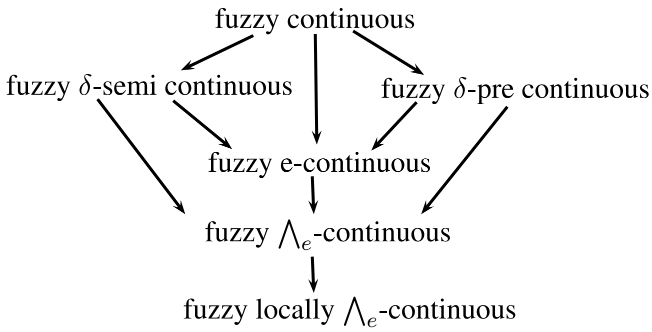
5. Fuzzy $\bigwedge_e$ continuity and Fuzzy locally $\bigwedge_e$ continuity

In this section, we present two weaker forms of fuzzy continuity named fuzzy $\bigwedge_e$ continuity and fuzzy locally $\bigwedge_e$ continuity via the fuzzy $\bigwedge_e$ sets and fuzzy locally $\bigwedge_e$ sets and we obtain some characterizations of these new continuities.

Definition 5.1 Let $f : (X,\ \tau_1) \to (Y,\ \tau_2)$ be a function from a fuzzy topological space $(X,\ \tau_1)$ into a fuzzy topological space $(Y,\ \tau_2)$. The function f is called

(1) fuzzy $\bigwedge_e$ continuous if $f^{-1}(\mu)$ is a fuzzy $\bigwedge_e$ set of X for each $\mu \in \tau_2$.
(2) fuzzy locally $\bigwedge_e$ continuous if $f^{-1}(\mu)$ is a fuzzy locally $\bigwedge_e$ set of X for each $\mu \in \tau_2$.

Remark 6 *It is clear that the implications of the following diagram hold.*



However, none of the implications of this diagram is reversed as shown in the following examples.

***Example* 5.2** Let $X = \{a, b, c\}$ and λ, μ, γ and δ be fuzzy sets of X defined as follows:
$\lambda(a) = 0.4$, $\lambda(b) = 0.6$, $\lambda(b) = 0.5$;
$\mu(a) = 0.6$, $\mu(b) = 0.4$, $\mu(c) = 0.4$;
$\gamma(a) = 0.6$, $\gamma(b) = 0.4$, $\gamma(c) = 0.5$;
$\delta(a) = 0.4$, $\delta(b) = 0.5$, $\delta(c) = 0.5$.
Let $\tau_1 = \{0,\ 1,\ \lambda,\ \mu,\ \lambda\vee\mu,\ \lambda\wedge\mu\}$, $\tau_2 = \{0,\ 1,\ \gamma\}$ and $\tau_3 = \{0,\ 1,\ \delta\}$are fuzzy topologies on X. Consider the identity mapping $f : (X, \tau_1) \to (Y, \tau_2)$ and $g : (X, \tau_1) \to (Y, \tau_3)$defined by $f(x) = x$ and $g(x) = x$, $\forall x \in X$. It is clear that f is fuzzy e continuous, but it is not fuzzy δ-pre continuous. Similarly, g is fuzzy e-continuous, but it is not fuzzy δ-semi continuous.

***Example* 5.3** Let λ and μ be fuzzy subsets of $X = Y = \{a, b\}$ are defined as follows:
$\lambda(a) = 0.2$, $\lambda(b) = 0.3$;
$\mu(a) = 0.1$, $\mu(b) = 0.4$;
Let $\tau_1 = \{0,\ 1,\ \lambda\}$ and $\tau_2 = \{0,\ 1,\ \mu\}$ are fuzzy topologies on X. Consider the identity mapping $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(x) = x$, $\forall x \in X$. Here, μ is fuzzy open set in Y, $f^{-1}(\mu) = \mu$ is a $\bigwedge_e$ set in X. Hence, f is fuzzy $\bigwedge_e$ continuous but f is not fuzzy e continuous as the fuzzy set μ is fuzzy open set in Y, but $f^{-1}(\mu)$ is not fuzzy e-open set in X. Thus, f is fuzzy $\bigwedge_e$ continuous but f is not fuzzy e-continuous.

***Example* 5.4** Let λ and μ be fuzzy subsets of $X = Y = \{a, b\}$ are defined as follows:
$\lambda(a) = 0.2$, $\lambda(b) = 0$;
$\mu(a) = 0.1$, $\mu(b) = 1$;
Let $\tau_1 = \{0,\ 1,\ \lambda\}$ and $\tau_2 = \{0,\ 1,\ \mu\}$ are fuzzy topologies on X. Consider the identity mapping $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(x) = x$, $\forall x \in X$. Here, μ is fuzzy open set

in Y, $f^{-1}(\mu) = \mu$ is a fuzzy locally $\bigwedge_e$ set of X. Hence, f is fuzzy locally $\bigwedge_e$ continuous but f is not fuzzy $\bigwedge_e$ continuous as the fuzzy set μ is fuzzy open set in Y, but $f^{-1}(\mu)$ is not fuzzy $\bigwedge_e$ set in X. Thus, f is fuzzy locally $\bigwedge_e$ continuous but f is not fuzzy $\bigwedge_e$ continuous.

***Example* 5.5** Let λ, μ, γ and δ be fuzzy subsets of $X = \{a, b, c\}$ defined as follows:
$\lambda(a) = 0.3$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$;
$\mu(a) = 0.6$, $\mu(b) = 0.5$, $\mu(c) = 0.5$;
$\gamma(a) = 0.3$, $\gamma(b) = 0.5$, $\gamma(c) = 0.2$;
$\delta(a) = 0.2$, $\delta(b) = 0.2$, $\delta(c) = 0.2$.
Let $\tau = \{0,\ 1,\ \lambda,\ \mu\}$ and $\eta = \{0,\ 1,\ \delta\}$ are fuzzy topologies on X. Consider the identity mapping $f : (X,\ \tau) \to (Y,\ \eta)$ defined by $f(x) = x$, $\forall x \in X$. Here, the identity function $f : (X,\ \tau) \to (Y,\ \eta)$ is fuzzy e-continuous but not fuzzy continuous because for any $\delta \in \eta$, $f^{-1}(\delta) \notin \tau$.

Theorem 5.6 Let $f : (X,\ \tau_1) \to (Y,\ \tau_2)$ be a function from a fuzzy topological space $(X,\ \tau_1)$ into a fuzzy topological space $(Y,\ \tau_2)$. The following statements are equivalent:

(1) f is fuzzy $\bigwedge_e$ continuous.
(2) For all $\mu^c \in \tau_2$, $f^{-1}(\mu) \in \bigvee_e(X)$.
(3) For all fuzzy set λ of Y, $(f^{-1}(Int\lambda))^{\bigwedge_e} \leqslant f^{-1}(\lambda)$.

Proof. (1) $\Rightarrow$ (2) : Let $\mu^c \in \tau_2$. Since f is $\bigwedge_e$ continuous,
$f^{-1}(\mu^c) = (f^{-1}(\mu))^c \in \bigwedge_e(X)$. Thus, $f^{-1}(\mu) \in \bigvee_e(X)$.
(2) $\Rightarrow$ (1) : It can be proved in the above manner.
(1) $\Rightarrow$ (3) : Since $Int\lambda \in \tau_2$ and f is $\bigwedge_e$ continuous.

$$(f^{-1}(Int\lambda))^{\bigwedge_e} = f^{-1}(Int\lambda) \leqslant f^{-1}(\lambda).$$

(3) $\Rightarrow$ (1) : Let $\mu \in \tau_2$. Then $Int\mu = \mu$. By assumption,

$$(f^{-1}(\mu))^{\bigwedge_e} = (f^{-1}(Int\mu))^{\bigwedge_e} \leqslant f^{-1}(\mu).$$

Since, $f^{-1}(\mu) \leqslant (f^{-1}(\mu))^{\bigwedge_e}$ always holds, $f^{-1}(\mu) \in \bigwedge_e(X)$. ■

Theorem 5.7 Let $f : (X,\ \tau_1) \to (Y,\ \tau_2)$ be a function from a fuzzy topological space $(X,\ \tau_1)$ into a fuzzy topological space $(Y,\ \tau_2)$. The following statements are equivalent:

(1) f is fuzzy locally $\bigwedge_e$ continuous.
(2) For all fuzzy set λ of Y, $f^{-1}(Int\lambda) = (f^{-1}(Int\lambda))^{\bigwedge_e} \bigwedge eCl(f^{-1}(Int\lambda))$.
(3) For all fuzzy set λ of Y, $f^{-1}(Cl\lambda) = (f^{-1}(Cl\lambda))^{\bigvee_e} \bigwedge eInt(f^{-1}(Cl\lambda))$.

Proof. (1) $\Leftrightarrow$ (2) : Since $Int\lambda$ is a fuzzy open set, the proof is immediate from Theorem 4.4
(1) $\Rightarrow$ (3) : $(Cl\lambda)^c$ is a fuzzy open set, so by Theorem 4.4

$$\begin{aligned} f^{-1}((Cl\lambda)^c) &= (f^{-1}(Cl\lambda))^c \\ &= ((f^{-1}(Cl\lambda))^c)^{\bigwedge_e} \bigwedge eCl((f^{-1}(Cl\lambda))^c) \\ &= ((f^{-1}(Cl\lambda))^{\bigvee_e})^c \bigwedge (eInt(f^{-1}(Cl\lambda)))^c, \end{aligned}$$

$$f^{-1}(Cl\lambda) = (f^{-1}(Cl\lambda))^{\bigvee_e} \bigvee eInt(f^{-1}(Cl(\lambda)).$$

(3) ⇒ (1) : Let λ be a fuzzy open set. Thus, $Cl(\lambda^c) = \lambda^c$. By assumption,

$$\begin{aligned} f^{-1}(Cl(\lambda^c)) &= f^{-1}(\lambda^c) \\ &= (f^{-1}(Cl(\lambda^c)))^{\bigvee_e} \bigvee eInt(f^{-1}(Cl(\lambda^c))) \\ &= (f^{-1}(\lambda^c))^{\bigvee_e} \bigvee eInt(f^{-1}(\lambda^c)) \\ &= ((f^{-1}(\lambda))^c)^{\bigvee_e} \bigvee eInt((f^{-1}(\lambda))^c) \\ &= ((f^{-1}(\lambda))^{\bigwedge_e})^c \bigvee (eCl((f^{-1}(\lambda))))^c. \\ &= (f^{-1}(\lambda))^{\bigwedge_e} \bigwedge (eCl((f^{-1}(\lambda))))^c. \end{aligned}$$

Hence, $f^{-1}(\lambda) = (f^{-1}(\lambda))^{\bigwedge_e} \bigwedge eCl((f^{-1}(\lambda)))$ which means $f^{-1}(\lambda)$ is a fuzzy locally $\bigwedge_e$ set. ■

Theorem 5.8 Let $f : X \to Y$ be a fuzzy $\bigwedge_e$ continuous function and $g : Y \to Z$ fuzzy continuous function. Then $g \circ f : X \to Z$ is a fuzzy $\bigwedge_e$ continuous function.

Proof. It is clear from the equality $(g \circ f)^{-1} = f^{-1}(g^{-1}(\mu))$. ■

Theorem 5.9 Let $f : X \to Y$ be a fuzzy continuous function. If $g : X \to X \times Y$ the graph map of f is fuzzy $\bigwedge_e$ continuous, then f is a fuzzy $\bigwedge_e$ continuous function.

Proof. Let μ be an open set of Y. Then $1_X \times \mu$ is an open set of $X \times Y$. By Lemma 2.4 in [1],

$$g^{-1}(1_X \times \mu) = 1_X \bigwedge f^{-1}(\mu) = f^{-1}(\mu) \in \bigwedge_e(X).$$

■

Conclusion

This paper deals with the recent concepts in the literature. The concept of fuzzy $\bigwedge_e$ sets is studied via e-open sets. So, this paper is related to [7–11] in the literature.

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