An Anisotropic Axisymmetric Model for Wrought Magnesium Alloys

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Abstract: Global environmental concerns have put lightening of structures on the fore front of research in transportation industry. Among other light weight alloys, the transportation industry is considering magnesium intensive light body-in-white structure in automotive applications. Although the research in modeling technique areas is very active, a suitable practical model mimicking the severe asymmetry and anisotropy of magnesium is lacking. Loading-unloading behavior of wrought magnesium alloy over a wide range of strain has been obtained experimentally and subsequently presented here. It is shown that while the material behaves elastically isotropic, it shows a different yield in tension and compression with a high Bauschinger effect. This is attributed to the magnesium multiple deformation mechanisms of slip, extension/contraction twinning, and de-twinning resulting in an asymmetric yield and a directional dependent performance. Up-to-date there is no plasticity model commercially available that can capture this behavior. Therefore, it is necessary to develop a simple and efficient model that can serve as a benchmark tool for plasticity model evaluation. Such model is presented in this paper. The axisymmetric elastic-plastic model of Jahed and Dubey (1997) has been extended to wrought magnesium alloys. An asymmetric yield function is adopted and the obtained behavior in loading and unloading is directly incorporated in the solution process. It is shown that results are significantly different from isotropic assumptions.

Keywords: Anisotropy, AZ31B, Magnesium Alloys, Model, Plasticity, Yield Asymmetry Wrought

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Biographical notes: H. Jahed is currently a professor in mechanical engineering department in University of Waterloo, Canada. His current research interest encompasses fatigue of magnesium alloys, cyclic plasticity, fatigue failure and large deformation FEA. **M. A. Khayamian** received his BSc and MSc in mechanical engineering from Shahid Bahonar University of Kerman, Iran, and Iran University of Science & Technology, respectively.

1 INTRODUCTION

Today, energy has become one of the most important concerns of human beings and by every day approach towards the end of fossil fuels this issue is being more highlighted. This has led to extensive efforts to reduce energy or fuel consumption. One of the reducing factors is lowering the weights of transportation vehicles. As a result, using materials with high strength and also low weight for vehicles is highly regarded. On this basis, magnesium and its alloys, the lightest metal on earth, offers a wide range of applications for weight loss. Application of magnesium alloys as a load bearing component, is needed to predict the behavior of this metal.

Due to unusual behavior of this material which includes anisotropy and yield asymmetry there are no existing suitable elastic-plastic solutions. So an axisymmetric solution for magnesium can provide a benchmark solution for more complicated loading cases. On the other hand the Variable Material Properties (VMP) method is a well-established simple method that can consider this type of asymmetric and anisotropic behavior in the solution procedure[1]. Elastic-plastic behavior of magnesium and its alloys have recently been widely studied. In this case, there are two general approaches. The first approach is crystal plasticity which is micro structural based[2-5]. This approach considering magnesium structure (HCP), deformation and slip mechanisms, investigates magnesium behavior under various loadings. But this method, due to computational complexity can be utilized for simple problems such as tension or compression only and it is not applicable to real-life problems yet.

Phenomenological methods, on the other hand, are another approach based on macro-mechanical behavior of materials, are also basis of this research. In this case, the general behavior of material under different loadings is considered and a model is presented which considers material features and also plasticity rules[6-9]. The main objective of this paper is to provide an anisotropic plasticity model using VMP in axisymmetric problems and extending it to magnesium alloy AZ31B.

2 MATERIAL AND TESTING

AZ31B alloy is one of the magnesium alloys which nowadays are used in many different industries. In the past, cast magnesium alloys were more common but in the last decade, wrought magnesium alloys such as AZ31B due to its high strength and formability are in the spotlight of aerospace and automotive industries.

2.1. Loading

To acquire the AZ31B behavior under loading, a sample of this alloy was tested to obtain both tension and compression stress-strain curves (Figure 1) [8]. As it can be seen in Fig. 1, AZ31B has two important behavioral properties: anisotropy and yield asymmetry. The specific feature of wrought magnesium alloys behavior is the yield asymmetry. So that if extruded AZ31B alloy is loaded in tension and compression in the longitudinal direction (LD), a significant difference in yield stress and stress-strain curve is observed which indicates anisotropic behavior in the plastic part, while the behavior in elastic parts are the same.

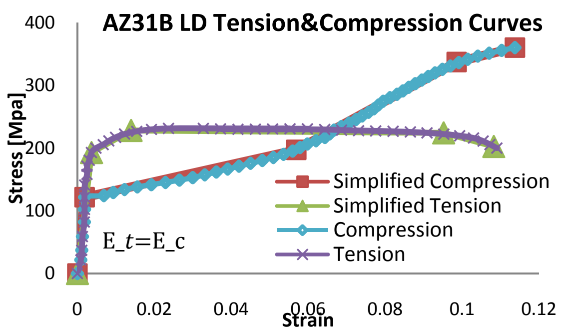

Fig. 1 AZ31B behavior in tension and compression [8]

2.2. Unloading

The unloading behavior ofAZ31B alloy from tensile and compressive loading was also obtained [8] and is shown in Fig. 2.

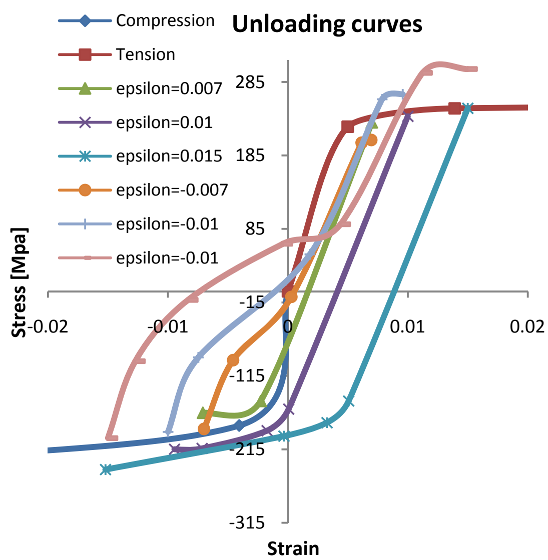

Fig. 2 Unloading behavior of AZ31B in some strains

3 PROPOSED AXISYMMETRIC MODEL

In a cylinder, radial, tangential and longitudinal stresses are principal stresses and the tangential stress has the maximum value. Therefore, the dominant stress is the hoop stress and the principal direction is tangential. Accordingly, the problem of an internally loaded cylinder made of AZ31B is solved in two steps with the following assumptions; only the yield asymmetry of magnesium is taken into consideration in one direction. A solution based on purely isotropic behavior is also obtained for comparison purposes.

3.1. Solution with isotropic behavior

The isotropic solution assumes thatAZ31B behavior in tension is the same as in compression. Noting that in a cylinder under internal pressure the tangential stress is tensile. The tensile behavior of AZ31B is taken as the material behavior. The VMP method solves an elastic-plastic problem in three steps: elastic solution, calculation of the equivalent stress based on yield criterion and updating of $E_{eff}$and υ_{eff} based on stress-strain curves.Since AZ31B is assumed to have isotropic behavior, it obeys the Lame solution. For yield criterion, von Mises yield criterion and its equivalent stress has been used.

3.2. Incorporating Yield Asymmetry in the Solution

In this section the only assumption that should be added to the previous assumptions is yield asymmetry. For solving the cylinder with this new assumption by VMP, each step requires changes that are presented in the following. In thick-walled cylinders under internal pressure for an element on the interior surface, tangential stress,σ_θ is tensile and radial stress, 'σ_r' is compressive. Accordingly, the elastic solution is based on the tensile behavior of AZ31B in tangential direction which is taken to be the same as the extrusion direction and the problem is solved using the Lame solution.

The purpose of selecting appropriate yield criterion and its equivalent stress is to determine the state of stress in each element and whether that element is in plastic part or not. In the case of isotropic materials, tensile and compressive behaviors are similar. Accordingly, von Mises or Tresca yield criterion and its equivalent stress should be used. But for magnesium, due to anisotropic behavior, isotropic criterion such as Mises cannot be used and we need an anisotropic yield criterion such as ductile Coulomb-Mohr criterion which allows different yield strength in tension and compression. For isotropic materials, for example, selecting the Mises criterion, the equivalent stress is:

$$\sigma_{eq} = \sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \quad (1)$$

But for magnesium due to yield asymmetry, finding a unique parameter similar to the yield strength 'σ_0' in isotropic materials is not possible. Therefore, a solution to this problem is to normalize the equivalent stress and stress-strain curve. Comparison may be represented by Eq. (2). For example, for Mises criterion we have:

$$\sigma_{eq} = \frac{\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}}{\sigma_0} < 1 \quad (2)$$

The Coulomb-Mohr criterion (Figure 3) can be represented in normalized stress space as:

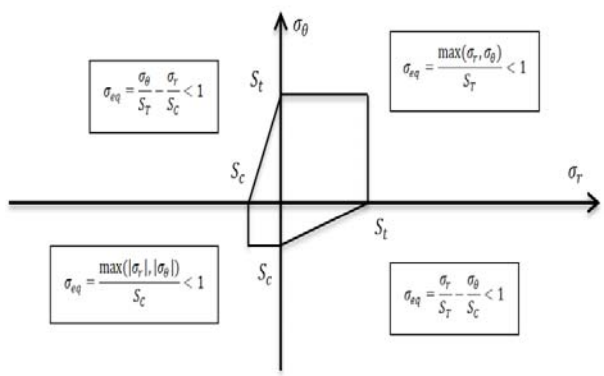

Fig. 3 Coulomb-Mohr yield criterion

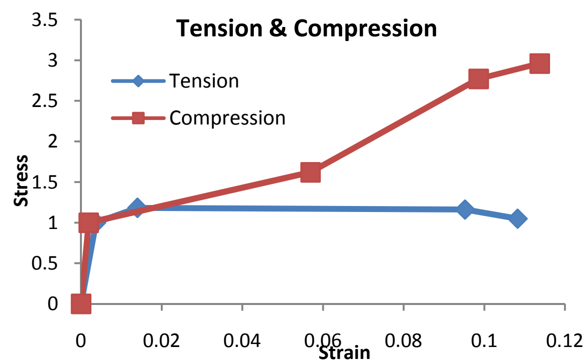

Fig. 4 AZ31Bnormalized stress-strain curves

In Figure 3, 'S_T' and 'S_C' are the yield strengths in compression and tension, respectively. For updating the effective modulus, E_{eff} and effective Poisson's ratio, υ_{eff} with regard to the stress-strain curves, two methods were considered:

1- Using a curve corresponding to the combination of tensile and compressive behavior by means of interpolation between the tension and the compression curves.

2- Updating of E_{eff} and υ_{eff} based on hydrostatic stress. So that if the hydrostatic stress is positive, the updating is done on the tensile curve and if it is negative the updating is done on the compressive curve.

3.3. Unloading

At the end of loading, the unloading stress-strain curve of each element is defined. This is done by the actual stress-strain curves if available; otherwise two different material hardening models isotropic and kinematic hardening is used. These two models represent two different final values of yield stress for predicting reverse yielding. For the case of isotropic, reverse yielding is occurred when:

$$\sigma_0^u = 2\sigma_y \quad (3)$$

σ_0^u is the first yield stress reached during unloading. And for kinematic hardening:

$$\sigma_0^u = (1+\text{BEF})\sigma_0 \quad (4)$$

BEF is the Bauschinger effect factor. The starting points of the unloading curves are the final yield stresses at each element. The unloading curve is then constructed from that point with an elastic limit at a stress difference of σ_0^u and a hardening part with the same hardening curve of the loading. The method of analysis is the same as the one described for loading. However, in this case, each element has to follow its own unloading curve. Results from the second analysis are subtracted from those of loading to obtain the residual field as follows,

$$\sigma_{ij}^R = \sigma_{ij} - \sigma_{ij}^u \quad (5)$$

Superscripts R and u belong to residual and unloading, respectively.

4 NUMERICAL EXAMPLE

To examine the AZ31B behavior in loading and unloading, an open-end cylinder under internal pressure was investigated. Coulomb-Mohr criterion and its corresponding equivalent stress were selected (Figure 3). Also $E_{eff}$and υ_{eff} updating was done based on two assumptions mentioned above. The results were plotted for 'σ_θ' in the following Figures 5, and 6. In unloading process, based on anisotropy and hydrostatic stress assumption, elastic unloading was chosen as the first step. Although the more accurate solution should be based on actual stress-strain unloading curves depicted in Fig. 2.

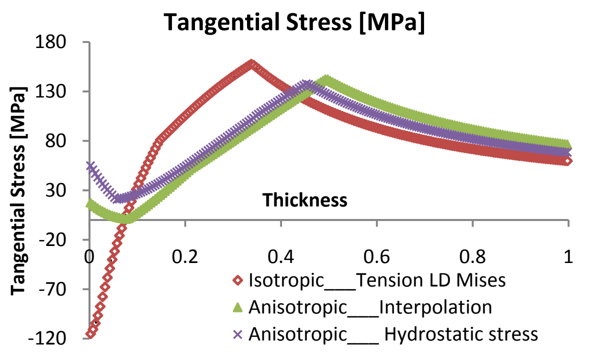

Fig. 5 Comparison of tangential stress after application of the internal pressure based on different assumptions

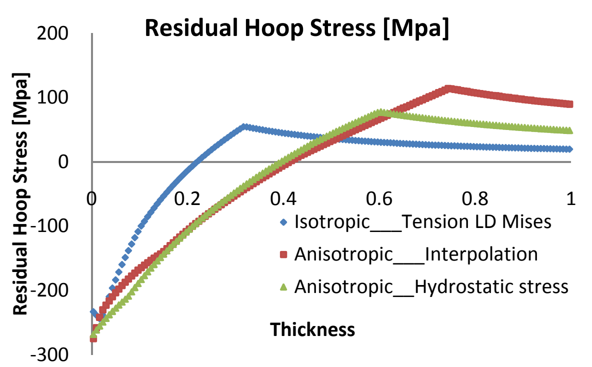

Fig. 6 Comparison of residual tangential stress based on different assumptions

5 RESULTS AND DISCUSSION

In the case of loading, results based on isotropic assumption represent just one discontinuity which shows the elastic and plastic boundary. In this assumption, 34% of the wall thickness experience plasticity. The mentioned percentage for two other assumptions, updated based on interpolation and hydrostatic stress, are 49% and 45%, respectively. Results of the two anisotropic assumptions are very similar. The first part of the corresponding curve is descending because the material is getting stiffer due to an increase in the slope of the compressive stress-strain curve in the third part (Figure 4). In unloading, the residual tangential stress shows about 2% reverse yielding in the cylinder with the isotropic behavior assumption which is not seen in other assumptions. Comparative results for 'σ_θ'represents a significant difference between isotropic and anisotropic assumptions for AZ31B. An extension of the present solution to a more accurate model which includes anisotropy and yield asymmetry in both radial and tangential directions is under consideration.

6 CONCLUSION

As was discussed, wrought magnesium alloys have special and different behavior compared to other metals which include anisotropy and yield asymmetry. Due to different tensile and compressive behavior, obtaining a solution for the problem of an AZ31B cylinder under internal pressure using VMP needs a suitable yield criterion which can include yield asymmetry. This criterion was selected and implemented. Analysis results of the anisotropic AZ31B cylinder compared to an isotropic model showed significant differences. The following conclusions may be drawn from this study:

- This method represents a benchmarking solution in axisymmetric problems for AZ31B and all anisotropic materials which include yield asymmetry.
- In loading, the cylinder experiences less plasticity based on anisotropic assumptions than the isotropic assumption and the form of the curve and stress value for tangential stress near the bore shows large differences between the two assumptions.
- In unloading, the residual tangential stress based on isotropic and anisotropic assumptions shows significant differences especially for reverse yielding.
- Significant differences between results of isotropic and anisotropic assumptions prove that wrought magnesium alloys should be considered anisotropic.
- Anisotropic plasticity models.

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