The Effects of Local Variation in Thermal Conductivity on Heat Transfer of a Micropolar Fluid Flow over a Porous Sheet

Reza Keimanesh*
Department of Mechanical Engineering,
K. N. Toosi University of Technology, Iran
E-mail: reza_keimanesh@yahoo.com
*Corresponding author

Cyrus Aghanajafi
Department of Mechanical Engineering,
K. N. Toosi University of Technology, Iran
E-mail: aghanajafi@kntu.ac.ir

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Abstract: This study is considering a micropolar fluid flow over a porous stretching sheet in the presence of thermal radiation and uniform magnetic field. The effects of local variation in thermal conductivity of micropolar fluid on heat transfer rate from the sheet are investigated; besides, the impacts of radiation, magnetic field and porous sheet on variations of thermal boundary layer thickness are considered. The results show that the increase of thermal conductivity thickens thermal boundary layer, so heat transfer rate decreases. In addition, intensification of magnetic field and the presence of radiation lower the absolute values of temperature gradient on the wall, and reduce the cooling rate of the sheet. On the contrary, the increase of suction and material parameter has positive influence on cooling rate of the sheet.

Keywords: Magnetic field, Micropolar fluid, Radiation, Thermal conductivity


Biographical notes: R. Keimanesh received his MSc in Mechanical Engineering from K. N. Toosi University of Technology, Tehran, Iran, and BSc in Mechanical Engineering from Bu-Ali Sina University, Hamedan, Iran. Currently, he is working on renewable energy systems and energy efficiency in power plants. C. Aghanajafi is currently Professor at the Department of Mechanical Engineering, K. N. Toosi University of Technology, Tehran, Iran. His research interests include heat transfer and thermodynamics.
1 INTRODUCTION

Fluid flows on an elastic sheet play important roles in industrial applications such as extrusion process in metals industry, continuous glass casting, etc. Since many fluids, such as micropolar fluids cannot be modeled as Newtonian ones, the importance of studying the behavior of such fluids is undeniable. On the other hand, electromagnetic reactions are one of the new events in modern technology, and many researchers have focused on it. Also, the significance of radiation heat transfer in final quality of products, produced in high temperature processes, is crucial. Therefore, the investigation of micropolar fluid flow in different governing conditions is the favorite topic of many researchers.

Wang [1] explained unsteady developing flow around a stretching sheet. He transformed Navier-Stokes equations to nonlinear ODEs with similarity solution, and represented an exact solution for them. Mahapatra and Gupta [2] considered stagnation-point flow of a steady, two-dimensional and incompressible viscoelastic fluid flow on a stretching surface at constant temperature. The existence of steady, viscous flow on a shrinking sheet with considering the suction effects was explained by Miklavcic and Wang [3]. They concluded that there is a dual solution for some suction values, and there is no solution for boundary layer in a special span of suction. Wang [4] described the stagnation flow towards a shrinking sheet, and achieved dual solution for some values of shrinking ratio and for some stagnation flow rates. Fang [5] studied the boundary layer flow over a continual shrinking sheet with power-law velocity of surface. Ishak et al. [6] described non-Newtonian power-law fluid flow over a shrinking sheet with suction. They solved boundary-layer equations and considered the effects of power-law index and suction parameter. Bachok et al. [7] analyzed two-dimensional stagnation-point of nanofluid over an exponentially stretching/shrinking sheet with similarity transformation for three types of nanoparticles in different solid volume fractions and elastic parameters. Also, they found that the solutions for a shrinking sheet are not unique. Xu et al. [8] described the unsteady flow and heat transfer of nanofluids over a stretching elastic surface, and reported a linear relationship between the film thickness and the unsteadiness parameter. To consider micropolar fluid flow, the mathematical model, presented by Eringen [9] is often used. A good work by on incompressible micropolar fluid flow over a stretching sheet was done by Sankara and Watson [10]. Hassanien and Gorla [11] studied the heat transfer to a micropolar fluid passing through the stretching sheet with suction and blowing effects. They considered surface mass transfer rate and power-law constant for the wall temperature as the remarkable parameters in improvement of heat transfer rate. Alomari et al. [12] presented analytical solution for micropolar fluid flow over a continuous moving surface. In their study the microinertia density is variable, and the effects of viscous dissipation were considered. Yacob and Ishak [13] investigated the flow and heat transfer characteristics for a steady two-dimensional micropolar fluid flow over a shrinking sheet. They found that the solution exists only with adequate suction, and there is dual solution for certain suction and material parameters. Hassanien et al. [14] presented an analysis to consider heat transfer features of combined forces and free convection flow of a micropolar fluid. They observed that heat transfer rate is reduced by increase of micropolar material parameter. Mehraban Rad and Aghanajafi [15] analyzed laminar flow in a microchannel under uniform wall temperature in the presence of radiation. They reported that the existence of thermal radiation and using nanofluids improve heat transfer. Ali et al. [16] considered unsteady and axisymmetric boundary layer flow over a shrinking sheet with radiation. They solved the boundary layer equations with Keller-box method, and understood that the separation of boundary layer is delayed by suction parameter. Hussain et al. [17] considered radiation heat transfer in an unsteady thermal boundary layer of a micropolar fluid over a permeable stretching sheet. They used homotopy analysis method, and reported that dimensionless temperature increases by unsteadiness and radiation parameters, but it decreases by stagnation parameter. Ouaf [18] established an exact solution for radiation heat transfer with magnetohydrodynamics (MHD). Fang and Zhang [19], in regard to suction effects, presented a closed-form exact solution of MHD viscous flow on shrinking porous sheet. They observed that the boundary layer thickens by reduction of suction parameter. Taklifi et al. [20] investigated the effect of MHD on the total heat transfer from a porous fin, attached to a vertical isothermal surface by using Rosseland approximation and Darcy model. They reported that heat transfer rate is declined by intensification of magnetic field strength for negative Eckert numbers and it would be enhanced for positive Eckert numbers. Noor et al. [21] studied the hydromagnetic flow with radiation on an inclined surface. They found that flow with injection has more influences on distributions of velocity and temperature than suction has, but their effects on concentration distribution are exactly opposite. Taklifi and Aghanajafi [22] analyzed the effect of MHD on steady two-dimensional laminar mixed flow over a vertical porous surface. They solved the transformed boundary layer equations with Keller-box method, and reported that raising the magnetic field parameter results in increase of dimensionless velocity and reduction of dimensionless temperature. Mukhopadhyay [23] studied heat transfer and boundary layer flow with MHD on an exponentially stretching sheet by using shooting method. He understood that fluid velocity is reduced by increase of magnetic parameter while heat transfer rate at the surface in presence of thermal stratification is enhanced.

In this research, the micropolar fluid flow is supposed to be two-dimensional, incompressible and steady; besides, the
magnetic field is uniform and the body forces are neglected, and the fluid passes over a stretching, permeable sheet. The simultaneous effects of thermal radiation, magnetic field, porosity on heat transfer rate from the stretching sheet, and considering the impacts of thermal conductivity are the most significant aspects of this study which distinguishes it from other researches.

2 PROBLEM FORMULATION

Fig. 1 shows the physical schematic of the problem. It is micropolar fluid flow over a porous, stretching sheet. By using boundary layer approximation, the governing equations for micropolar fluid with heat transfer are written by:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  
(1)

Momentum equation:

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \nu + \frac{\lambda}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\rho \partial N}{\partial y} - \frac{\sigma B_0^2 u}{\rho} \]  
(2)

Angular momentum equation:

\[ u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \left( \frac{\gamma}{\rho j} \right) \frac{\partial^2 N}{\partial y^2} - \frac{\lambda}{\rho j} \left( 2N + \frac{\partial u}{\partial y} \right) \]  
(3)

Energy equation:

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2 u^2}{\rho C_p} \]  
(4)

In regard to large optical thickness for desired medium, the medium is called diffusion, so the Rosseland approximation for radiation can be applied:

\[ q_r = -\frac{4 \xi T^4}{3k} \frac{\partial T}{\partial y} \]  
(5)

So:

\[ \frac{\partial q_r}{\partial y} = -\frac{16\xi T^3}{3k} \frac{\partial^2 T}{\partial y^2} \]  
(6)

Term \( T^4 \) is expanded by Taylor series:

\[ T^4 = T^4_c + 4T^3_c (T - T_c) + 6T^2_c (T - T_c)^2 + \ldots \approx -3T^4_c + 4T^3_c T \]  
(7)

Finally, the energy equation is transformed into:

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\xi T^3_c}{3\kappa \rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2 u^2}{\rho C_p} \]  
(8)

According to the problem, the boundary conditions are written as follows:

Due to the linear stretch of the sheet:

\[ u = cx \ , \ c > 0 \]  
(9-a)

Because of the porous wall, it has suction/blowing effect, which is supposed as a velocity in perpendicular direction, this value is assumed constant. So:

\[ v = v_w \ , \ \text{suction} \ v_w < 0 \ \text{and blowing} \ v_w > 0 \]  
(9-b)

Angular velocity is defined as:

\[ N = -m \frac{\partial u}{\partial y} \]  
(9-c)

And:

\[ T = T_w \]  
(9-d)

And far from the wall (\( y \to \infty \))

\[ u \to U_c = 0 \ , \ N \to 0 \ , \ T \to T_{\infty} \]  
(10)

By using similarity solution, the PDEs are transformed into ODEs. Stream function can be used because the flow is assumed two-dimensional and incompressible. Finally, by substituting similarity transformations (Eq. (11)), the governing equations are reduced into non-linear ODEs.

\[ u = \frac{\partial \psi}{\partial \eta} \]  

\[ v = -\frac{\partial \psi}{\partial x} \]  

\[ \psi = \left(c v^2\right)^{1/2} x f(\eta) \]  

\[ N = c x \left(c / v^2\right)^{1/2} h(\eta) \]  

\[ \eta = \left(c / v^2\right)^{1/2} y \]  

\[ T = T_{\infty} + (T_w - T_{\infty}) \theta(\eta) \]  

Momentum equation:
\[ [1 + K] f^\prime(\eta) + f(\eta) f^\prime(\eta) - f^{\prime 2}(\eta) + K h^\prime(\eta) \]
\[ -(Ha)^2 f^\prime(\eta) = 0 \] (12)

Angular momentum equation:
\[ \left[ 1 + \frac{K}{2} \right] h^\prime(\eta) + f(\eta) h^\prime(\eta) - f^\prime(\eta) h(\eta) \]
\[ -K [2h(\eta) + f^\prime(\eta)] = 0 \] (13)

Energy equation:
\[ (1 + R) \theta^\prime(\eta) + Pr f(\eta) \theta^\prime(\eta) \]
\[ + Pr (Ha)^2 E c f^{\prime 2}(\eta) = 0 \] (14)

By applying similarity transformations, the boundary conditions (9) and (10) are converted into form bellows:
\[ f(\eta) = S, f^\prime(\eta) = 1, h(\eta) = -mf^\prime(\eta), \]
\[ \theta(\eta) = 1, at \eta = 0 \] (15)
\[ f^\prime(\eta) \rightarrow 0, h(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, at \eta \rightarrow \infty \] (16)

The applied dimensionless parameters in the above are defined as follows:
\[ Pr = \frac{\mu C_p}{k}, Ec = \frac{c^2 p^2}{C_p(T_w - T_{\infty})}, K = \frac{\lambda}{\mu}, R = \frac{16 \varepsilon T_{\infty}^3}{3k_k}, \]
\[ Ha = B_0 \sqrt{\frac{\sigma}{\rho c}}, S = -\frac{v_w}{(CV)^{1/2}} \]

Here, the boundary value problem is transformed into initial value problem by using shooting method; finally, they are solved by fourth-order Runge-Kutta method in Maple package.

3 RESULTS AND DISCUSSIONS

Constant m, in boundary conditions is a value between 0 and 1. As m=0, then N=0 and it shows that particle flow is concentrated, and micro-elements are close to the wall and cannot rotate [24], this status is known as the strong focus on micro-elements [25]. On the other hand, m=0.5 is the representative of vanishing the asymmetric part of stress tensor, known as the weak focus on micro-elements [26]. Also, to model turbulence boundary layer m=1 is applied [27]. In this study, the governing equations are solved, and the results are extracted for two microrotation parameter, consisting of m=0, 0.5.

According to Eqs. (12-14), variation in thermal conductivity is ineffectual in \( f^\prime(\eta) \) and \( h(\eta) \), but it is an important parameter in temperature profiles because of varying Prandtl (Pr) number and radiation parameter, so this status could be very useful when only the variation of temperature profiles is required. Therefore, in this study, a constant as \( \alpha \) is defined to consider the changes of Pr number and radiation parameter by local variation in thermal conductivity, as follows:
\[ Pr = \frac{\mu C_p}{k'}, R = \frac{16 \varepsilon T_{\infty}^3}{3k'k} \quad \text{while} \quad k' = \alpha k \]

In order to validate the results, the value of \( -\theta^\prime(0) \) in this research is compared with [28], [29], [30], [31], and they are presented in Table 1. As it is shown, the results are close together, and certify the present results.

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Figs. 2-8 show temperature profiles thermal conductivity for various parameters. The negative values of wall temperature gradient show that heat transfer is from the sheet to the fluid.
As it is shown in Fig. 2, for \( m=0.5 \), increase of \( \alpha \) enhances the thickness of thermal boundary layer, and declines the temperature gradient on the sheet (absolute values), so heat transfer rate from the sheet decreases. It is noticeable that when Prandtl number decreases, thermal diffusion could suppress viscous forces, results in increasing temperature in the boundary layer.

Also, it is seen that the thickness of thermal boundary layer is developed, and the absolute temperature gradient is lowered in the presence of radiation, so heat transfer from the sheet is declined. This is because of the radiation effect on increasing the rate of energy transport to the fluid [32]. According to Eq. (5) the effect of thermal radiation intensity on thermal boundary layer might be explained by absorption coefficient, which its decrease causes more radiation heat flux over the fluids.
The existence of magnetic field effect is appeared by Hartmann number \((\text{Ha}=0)\). Figs. 3, 4 show that the presence of magnetic field thickens thermal boundary layer, and decreases the absolute temperature gradient on the sheet; hence, the cooling rate of the sheet is lowered. This behavior occurs because of Lorentz force, induced by magnetic field which creates a drag force and lower the fluid’s velocity while temperature in the thermal boundary layer is enhanced [32]. Also it is clear that increase of \(\alpha\), results in thicker thermal boundary layer and lower heat transfer from the sheet. As it is shown in Figs. 5 and 6, the thickness of thermal boundary layer of a micropolar fluid \((K\neq 0)\) is less than a Newtonian fluid \((K=0)\), and heat transfer from the sheet to the micropolar fluid is more because of more absolute gradient temperature.

Since there are suction/blowing effects in the sheet, mass transfer parameter is emerged \((S\neq 0)\). Figs. 7, 8 show the effect of porosity of the sheet in different \(\alpha\) for two quantities of microrotation parameter. When the fluid moves over the porous sheet, it permeates through the sheet. Therefore, the thermal boundary layer decreases, and heat transfer from the sheet is improved.

### 4 CONCLUSION

In this study, the effects of local variation in thermal conductivity on micropolar fluid flow and heat transfer from the porous, stretching sheet in the presence of radiation and magnetic field effects are studied. The results show that, as the thermal conductivity increases, the thickness of thermal boundary layer is raised; consequently, the heat transfer rate from the sheet is reduced while the velocity profile and angular velocity profile are constant. In addition, it is concluded that the absolute values of temperature gradient are lowered, by intensification of magnetic field; conversely, suction parameter leads in enhancement of the cooling rate of sheet.

### 5 NOMENCLATURE

- \(B_0\): Magnetic field intensity (Tesla)
- \(c\): Stretching coefficient \((s^{-1})\)
- \(C_p\): Specific heat \((\text{J.kg}^{-1}.\text{K}^{-1})\)
- \(E_c\): Eckert number
- \(f\): Dimensionless steam function
- \(h\): Dimensionless angular velocity
- \(\text{Ha}\): Hartmann number
- \(j\): Micro-inertia per unit mass \((\text{m}^2)\)
- \(k\): Thermal conductivity of fluid \((\text{W.m}^{-1}.\text{K}^{-1})\)
- \(K\): Dimensionless material parameter
- \(m\): Microrotation parameter
- \(N\): Angular velocity \((\text{rad.s}^{-1})\)
- \(\text{Pr}\): Prandtl number
- \(q_r\): Radiative heat flux \((\text{W.m}^{-2})\)
- \(R\): Radiation parameter
- \(S\): Mass transfer parameter
- \(T\): Temperature (K)
- \(u\): Velocity component in x-direction \((\text{m.s}^{-1})\)
- \(v\): Velocity component in y-direction \((\text{m.s}^{-1})\)
- \(x\): Horizontal coordinate \((\text{m})\)
- \(y\): Vertical coordinate \((\text{m})\)

### Greek symbols

- \(\alpha\): Dimensionless constant
- \(\psi\): Stream function
- \(\gamma\): Spin-gradient viscosity \((\text{kg.m.s}^{-1})\)
- \(\eta\): Similarity variable
- \(\kappa\): Absorption coefficient \((\text{m}^{-1})\)
- \(\lambda\): Vortex viscosity \((\text{kg.m}^{-1}.\text{s}^{-1})\)
- \(\mu\): Dynamic viscosity \((\text{kg.m}^{-1}.\text{s}^{-1})\)
$\nu$ Kinematic viscosity (m$^2$.s$^{-1}$)  
$\theta$ Dimensionless temperature  
$\rho$ Density of fluid (kg.m$^{-3}$)  
$\sigma$ Electrical conductivity (S.m$^{-1}$)  
$\xi$ Stefan–Boltzmann constant (W.m$^{-2}$.K$^{-4}$)  

Subscripts  
$w$ At the wall  
$\infty$ Condition far from the surface

REFERENCES