Journal of
Electrical Engineering Science
March 2017, (pp.1-9)
ISSN: 2008-9864

Noiselet Measurement Matrix Usage In CS Framework

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Abstract
Theory of compressive sensing (CS) is an alternative to Shannon/Nyquist sampling theorem which explained the number of samples requirement in order to have the perfect reconstruction. Perfect reconstruction of undersampled data in CS framework is highly dependent to incoherence of measurement and sparsifying basis matrices which the posterior is usually fulfilled by selecting a random matrix. While Noiselets, as a measurement matrix, have very low coherence with wavelets which are the interest of CS, they have never been studied well and compared with other well known Gaussian and Bernoulli measurement matrices, which have been widely used in CS framework, from randomness view point. Therefore, the main contribution of this paper is introducing Noiselets and comparing them with other measurement matrices in two point of view; randomness and quality of recovered images. In case of randomness, the entropy is used as a criterion for computing the randomness. In case of recovered images, the OMP and PDIP algorithms are applied under sampling rates 30, 40, 60%.

Keywords: Compressive sensing (CS), Noiselets, Gaussian measurement, Bernoulli measurement, randomness.

1. INTRODUCTION

Based on Shannon/Nyquist sampling theorem [1] introduced in 1949, for perfect reconstruction of a sampled signal, the sampling rate must be at least twice of the biggest frequency in that signal. Accordingly, for big data, compressing before storage or transmission becomes necessary. Compressive Sensing (CS) [2], is an alternative to Shannon/Nyquist sampling theorem for the acquisition of sparse or compressible signals. In fact, instead of using a periodic impulse for sampling, CS uses random matrices for measurement. Although CS may disregard the Nyquist rate, it was proved that under fulfilled of some circumstances, the signal would be perfectly recovered. CS Signal recovery is usually based on ℓ_1-norm [3], or greedy algorithms such as matching pursuit (MP) [4], orthogonal matching pursuit (OMP) [5], compressive sampling orthogonal matching pursuit (CoSaMP) [6] and primal-dual interior-point (PDIP) [7]. For perfect reconstruction in CS framework, two important factors should be satisfied; 1) sparsity of signal which is usually explored under some sparsity basis like Fourier transform [8], discrete Cosine transform [9], and wavelet transform [10], 2) incoherency between the measurement and the sparse matrices. The incoherency that measures the largest

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correlation between vectors of measurement and sparsifying matrices [11] is a very important factor in CS applications. Precisely, the less incoherency between measurement and sparsifying matrices, the less measured samples are needed whereas the signal is recovered perfectly. Well known measurement matrices, Random Gaussian [2] and Bernoulli [2], have been largely used in CS framework whereas Noiselets [12] are not known yet. Although Noiselets have very low coherence with wavelets as a sparse basis matrix [2], they have not been seriously studied as a good candidate for CS as a measurement matrix. This gave us the motivation to study and compare the randomness of Noiselets with both well known measurement matrices mentioned above; Gaussian and Bernoulli.

As a matter of fact, the randomness parameter of any real matrix is evaluated by obtaining the entropy [13]. According to the fact that low entropy indicates high randomness, experiments on Gaussian, Bernoulli and Noiselet matrices showed that in square sizes, Gaussian has the most randomness of all while in non-square sizes imaginary part of Noiselets have higher randomness. Furthermore, experiments indicated that choosing measurement and sparsifying basis with low coherence will guaranty the perfect reconstruction of images, even though the utilized measurement matrix has low randomness which is Noiselet in our case.

The paper is organized as follows. In Section 2, CS is explained at first. Then Noiselets as a new sampling matrix which has a good incoherence with Haar wavelet basis is introduced. Our proposed method to calculate randomness of measurement matrices and the experimental results for different scenarios are presented in Section 3 and 4 respectively. Finally, we have conclusion in Section 5.

2. BACKGROUND

A. *CS Theory*

The CS framework samples data [1] based on a linear non-adaptive measurement, is written as:

$$y = \varphi x \tag{1}$$

where x denotes signal or data of interest with finite dimension of N written as $x \in R^{N\times 1}$, φ is the sampling or measurement matrix with size $M \times N$ which often considered random Gaussian or Bernoulli and y with size $M \times 1$ is the observed data. As said before, two fundamental requirements need to be fulfilled in CS theory, including: the signal 'sparsity' and the 'incoherency'. A signal x is k-sparse if it has k nonzero or big elements generally in ψ domain. The k-sparse signal x based on basis matrix, ψ, is:

$$x = \psi\ s \tag{2}$$

where $s \in R^{N\times 1}$ with k nonzero elements denotes the sparse representation of signal x and ψ with size $N \times N$ is the sparse basis matrix. Combining Eq. (1) and Eq. (2), the observed signal is,

$$y = \theta s \tag{3}$$

where $\theta = \varphi\psi$ with size $M \times N$ is called dictionary matrix. The second CS requirement called 'incoherency' comes, which means having φ and ψ matrices that are maximally incoherent to each other, then few measurement samples for perfect recovery of signal is needed. Coherency between these two matrices is [2]:

$$\mu(\varphi,\psi) = \sqrt{N} \max_{1\le i,j\le N} \left| < \varphi_k, \psi_j > \right| \quad , \quad \mu(\varphi,\psi) \in [1, \sqrt{N}] \tag{4}$$

where φ_k , ψ_j are k-th row of φ and j-th column of ψ and $\mu(\varphi,\psi) = 1$ means maximum incoherency. Satisfaction of restricted isometry property (RIP) [14] guaranties the maximum incoherency between the sampling matrix, φ, and sparse basis matrix ψ.

Although recovering x or equivalently sparse signal s from y is an ill-posed problem because of M << N, according to CS theory, the original signal can be exactly reconstructed by solving the linear programming problem as long as x is sparse in some domain. Despite of sparseness, ℓ_2 norm gives much attention to the signal energy and fails to recover signal perfectly. Zero-norm is perfect for sparse recovery. However, the corresponding optimization problem is NP-Complete and thus it is intractable but surprisingly when choosing the ℓ_1 norm, it is able to recover signal perfectly as long as it is sufficiently sparse and it is expressed as [1]:

$$\min : \|\hat{s}\|_{\ell_1}, \quad s.t.: y = \theta\,\hat{s} \tag{5}$$

As mentioned in [2], well known random matrices like Gaussian, which are mostly used in CS, are largely incoherence with an arbitary basis matrix ψ with size $N \times N$ and the incoherency is about $\sqrt{2\log N}$. Although Noiselets are not as popular as random sampling matrices, they have good incoherency with fixed basis ψ matrices like Fourier and wavelets [2]. It was proved [2] that the coherency between Noiselets and Haar wavelets is $\sqrt{2}$ and that between Noiselets and Daubechies D4 and D8 wavelets is about 2.2 and 2.9 in order. Hence, this motivated us to analyze the randomness of different CS measurement matrices with respect to noiselets as a measuring matrix.

B. Noiselets

Before explaining Noiselets matrix in detail, three different types of matrix multiplications are expressed. Suppose two arbitrary matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ with the same size. The two mentioned matrices can be multiplied in three different ways as:

$$A * B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} * \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} \tag{6}$$

$$A .* B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} * \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix} \tag{7}$$

$$A \otimes B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{bmatrix} \tag{8}$$

The products in Eqs. (6)-(8) are named matrix multiplication, matrix element-wise multiplication and Kronecker [15]. Noiselets are generated by using the last one product; the Kronecker.

The Noiselet basis, originally presented in [12], has received interest recently due to the following facts: 1) being maximally incoherent to the Haar basis, 2) having a fast implementation algorithm. Thus, they have been employed in CS to sample signals that are sparse in wavelet domain where Haar is the sparse matrix [16], [17]. The procedure of generating Noiselets matrices is explained in following.

It is started with a 1×1 matrix $N_1 = [1]$, then a sequence of noiselet matrices $N_2, N_4, \ldots, N_{2^m}$ with sizes 2×2, 4×4, …, $2^m \times 2^m$, are generated. So, the Noiselet matrix with size $n \times n$ is built up recursively according to:

$$N_n(k,*) = \frac{1}{2}(1-i \quad 1+i) \otimes N_{n/2}(\frac{k}{2},*) \qquad , k = 0,2,4,\ldots,n-2. \tag{9}$$

$$N_n(k,*) = \frac{1}{2}(1+i \quad 1-i) \otimes N_{n/2}(\frac{k-1}{2},*) \qquad , k = 1,3,5,\ldots,n-1. \tag{10}$$

where $N_n(k,*)$ denotes the row vector of N_n. It should be noted that the Noiselet matrices are not real. As an example N_2 and N_4 by using Eqs. (9)-(10) are:

$$N_2 = \frac{1}{2}\begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix} \quad , \quad N_4 = \frac{1}{2}\begin{bmatrix} -i & 1 & 1 & i \\ 1 & i & -i & 1 \\ 1 & -i & i & 1 \\ i & 1 & 1 & -i \end{bmatrix}. \qquad (11)$$

In addition, the elements summation of imaginary parts in every Noiselet matrix is always equal zero, or in other words, the average value of imaginary part of Noiselet matrix, irrespective of its size is zero.

3. MATRIX RANDOMNESS EVALUATION

Choosing a random measurement matrix φ in CS framework that complies RIP and incoherency guaranties the signal perfect reconstruction [18]. So the randomness of a measurement matrix is important for CS implementation. Although both random Gaussian, which is optimal for sparse recovery, and Bernoulli matrices satisfy RIP property [19], [20], they have limited use in practice due to the fact that the structure is imposed on the measurement matrix by many measurement technologies [21], [22]. That is to say, Bernoulli matrices are more feasible than the Gaussian, because the posterior contain highly storage and computation complexity without taking account of the signal vector precision [19], [20]. In this paper we are going to determine and compare the randomness of three mentioned measurement matrices by means of entropy.

The entropy of matrix p with N elements is [13]:

$$h(p) = -\sum_{i=1}^{N} p_i \ln p_i \qquad (12)$$

entropy value is always positive [13] with range of $[0, \ln N]$. The minimum value, $h(p) = 0$, is achieved when only one p_i equals 1 and others equal 0 whereas the maximum value is achieved when all p_i s are equal $1/N$.

Generally, it is concluded [13] that the entropy could be used as a measure of matrix randomness. In this way, the lower entropy will eventuate higher randomness. Before using the entropy as a measure of randomness, elements of measurement matrix should be normalized. As an example, if $\varphi = [\varphi_{11}, \dots, \varphi_{1n}; \dots; \varphi_{n1}, \dots, \varphi_{nn}]$ with size $n \times n$ is an arbitrary matrix, the corresponding normalized matrix is $\varphi_{norm} = [\varphi'_{11}, \dots, \varphi'_{1n}; \dots; \varphi'_{n1}, \dots, \varphi'_{nn}]$ where

$$\varphi'_{ij} = \varphi_{ij} \Big/ \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \varphi_{ij}^2}\,.$$

Obviously, the randomness interval value of any arbitary matrix depends on the two parameters, i.e. elements probability and the matrix size. So randomness of different matrices are incomparable. For example, zero is expected for the randomness of matrix with repetitive elements but the calculated randomness by means of entropy doesn't support this idea. So, in order to have an ideal number for comparison of randomness between different matrices, we have proposed a method which is explained as follows. At first, the input matrix φ' is normalized by the mean value; i.e. $\varphi'' = \varphi'/G$ where G is the average value of the matrix. Then the entropy of new matrix is obtained and considered as the measure of randomness which is as following:

$$Randomness = -\sum_{i=1}^{n}\sum_{j=1}^{n} \varphi''_{ij} \ln \varphi''_{ij} = -\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\varphi'_{ij}}{G} \ln \frac{\varphi'_{ij}}{G}$$

$$= -\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\varphi'_{ij}}{G} (\ln \varphi'_{ij} - \ln G) = -\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\varphi'_{ij}}{G} \ln \varphi'_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\varphi'_{ij}}{G} \ln G \qquad (13)$$

so when matrix φ' has repetitive elements, the average G will be equal to the value of every φ' elements, then according to the definition of randomness in Eq. (13), the randomness of matrix φ' is zero.

4. EXPERIMENTAL RESULTS

In this Section, the randomness of three measurement matrices in CS framework, including: Gaussian, Bernoulli and Noiselet are computed. In this case, the randomness values of different matrix sizes are written in Table 1.

As it can be seen in Table 1, the randomness of Gaussian matrix irrespective of size is always greater than both Bernoulli and Noiselet. In order to clarify this property, the three mentioned matrix with sizes 128×128 and 512×512 are shown in Fig.1. As it is seen, Gaussian has fully random shape whereas Noiselet has repetitive pattern.

Table 1. ***COMPARING THE RANDOMNESS OF GAUSSIAN, BERNOULLI, AND NOISELET (REAL PART).***

	Randomness		
Matrix Size	**Gaussian**	**Bernoulli**	**Noiselet** (Real Part)
4×4	32	11.0904	9.4291
16×16	4×10^3	156.3401	536.3315
64×64	4.7621×10^5	2.8491×10^3	2.1092×10^4
128×128	2.1445×10^6	1.1385×10^4	1.8334×10^5
256×256	4.1836×10^7	4.5354×10^4	7.4316×10^5
512×512	1.8408×10^8	1.8142×10^5	6.2192×10^6

Despite the real part of Noiselet matrix that contains positive elements, imaginary part has both negative and positive elements that are same valued and equal in numbers. This feature of imaginary part makes the matrix average value 0; hence, in our proposed method for calculating the randomness, the division of elements to the average number would be infinity. Owing to the mentioned fact, they are not reported in Table 1.

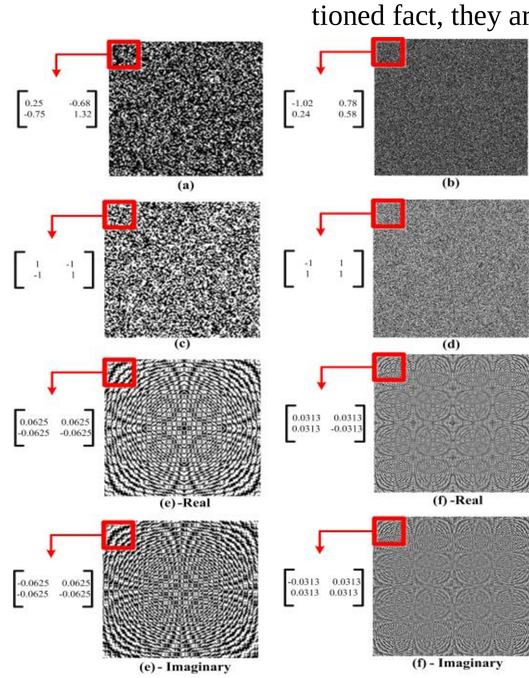

Fig. 1. The pattern of measurement matrices : Gaussian (a)-(b), Bernoulli (c)-(d), and Noiselet (e)-(f) with size 128x128 (first column) and 512x512 (second column).

Whereas it seems that Noiselet cannot be used in CS framework according to randomness value and repetitive pattern shape, one should bear in mind that measurement matrix in CS framework is not square but with size of $M \times N$ where $M << N$. In second part of our experiments, we have calculated the Randomness of Gaussian, Bernoulli and Noiselet matrices in three sampling rates, i.e. $M/N = 30, 40, 60\%$ and written in Table 2. As mentioned before, Noiselets are generated in square sizes. To have a $M \times N$ measurement matrix, a square Noiselet matrix of desired size should be generated at first and then be cut into 30, 40, 60% of its rows size. As an example, for an original matrix with size of 128×128, when $M/N = 40\%$, the measurement matrix size is 6553×16384.

The achieved results show that for every matrix size, the imaginary part of Noiselet has the most randomness and the Bernoulli has the least randomness of all. Randomness of Gaussian is always higher than the real part of Noiselet except in one case which is with sampling rate of 40% and size of 16×16.

For visual comparison among using the three mentioned measurement matrices, OMP [5] and PDIP [7] are used as the recovery algorithms when the sampling rates are 30, 40, 60% and Haar wavelet is used as the sparse matrix. The three original images are shown in Fig.2.

Table 2. ***comparing the randomness of gaussian, bernoulli and noiselet for sampling rate 30,40,60%.***

Sampling Rate (%)	**Size**	**Randomness**			
		Gaussian	**Bernoulli**	**Noiselet** (Real parts)	**Noiselet** (Imaginary parts)
30	**16×16**	1.4631×10^{6}	1.3656×10^{4}	2.2352×10^{5}	3.6917×10^{22}
	32×32	1.3208×10^{9}	2.1877×10^{5}	7.7868×10^{6}	1.4107×10^{25}
	64×64	3.3669×10^{10}	3.4857×10^{6}	2.4602×10^{8}	9.0397×10^{26}
40	**16×16**	7.4315×10^{6}	1.8012×10^{4}	2.4602×10^{8}	5.6283×10^{22}
	32×32	4.7362×10^{9}	2.9056×10^{5}	9.9987×10^{6}	2.1773×10^{25}
	64×64	1.2652×10^{11}	4.6511×10^{6}	3.2789×10^{8}	1.2095×10^{26}
60	**16×16**	7.4940×10^{7}	2.7407×10^{4}	4.4306×10^{5}	1.0442×10^{23}
	32×32	4.9995×10^{10}	4.3730×10^{5}	4.4306×10^{5}	4.2001×10^{24}
	64×64	7.9506×10^{11}	6.9804×10^{6}	4.9207×10^{8}	1.2626×10^{26}

Table 3. ***PSNR values of recovered images of Fig.2 using OMP algorithm.***

		Measurement Matrix								
		Image(a)			**Image(b)**			**Image(c)**		
		Gaussian	Bernoulli	Noiselet	Gaussian	Bernoulli	Noiselet	Gaussian	Bernoulli	Noiselet
Sampling Rates (%)	30	29.22	29.26	34.01	28.61	28.89	32.65	28.24	28.31	29.90
	40	29.92	29.87	35.36	28.98	28.99	32.83	28.34	28.35	30.21
	60	32.33	31.78	36.22	29.68	29.59	33.15	28.51	28.47	30.70

Table 5. ***PSNR values for recovered images of Fig. 2, Using PDIP algorithm.***

		Measurement Matrix								
		Image(a)			**Image(b)**			**Image(c)**		
		Gaussian	**Bernoulli**	**Noiselet**	**Gaussian**	**Bernoulli**	**Noiselet**	**Gaussian**	**Bernoulli**	**Noiselet**
Sampling	**30**	30.27	30.23	**33.75**	29.09	28.89	**32.54**	28.32	27.61	**28.65**
Rates	**40**	32.14	31.65	**34.92**	29.46	29.12	**34.30**	28.79	28.60	**28.89**
(%)	**60**	35.23	34.31	**40.42**	31.53	31.47	**32.17**	29.00	29.06	**29.59**

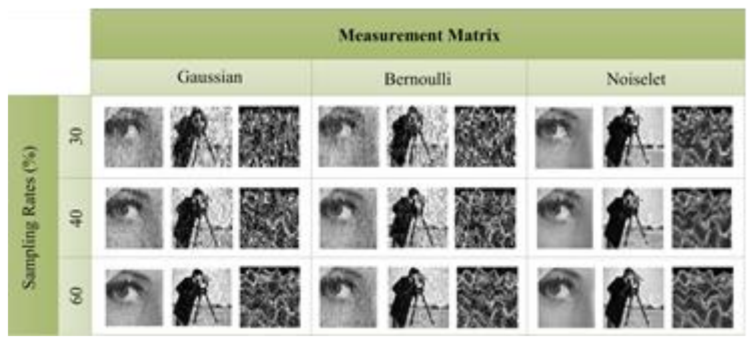

Fig. 2. The recovered images shown in Fig. 2 using OMP algorithm.

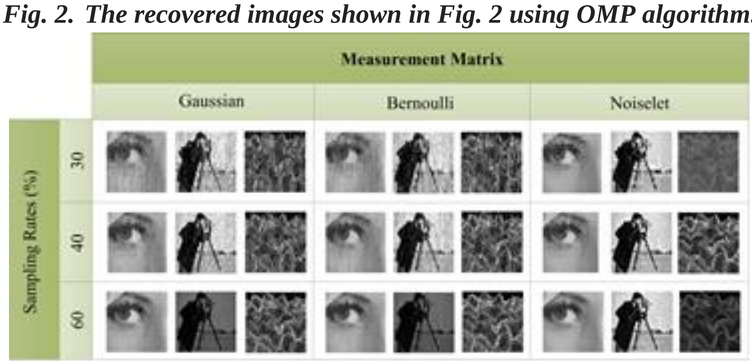

Fig. 3. The recovered images shown in Fig. 2 using PDIP algorithm.

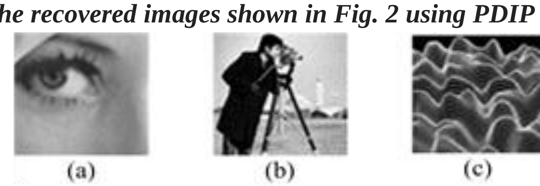

Fig. 4. Test images: (a) Lena eye (b) Cameraman (c) Shape.

We should notify that the real and imaginary parts of Noiselets cannot be used separately as a measurement matrix in CS so we have used complex Noiselets in our simulations. The recovered images are shown in Fig. 3 and Fig. 4.

As far as Noiselets are complex valued, the recovered images using this measurement will also be complex valued; hence, we have used their absolute value to represent images. In this case, the dynamic range of the recovered images are in interval [0,255]. According to visual results shown in Fig. 3 and Fig. 4, it can be concluded that PDIP recovery algorithm has done better recovery in comparison with OMP algorithm. Besides, the visual quality of recovered images also increases by increasing the measurement rates. Furthermore, it is seen that between the three candidate measurements, Noiselets have performed better than others due to the low coherence which they have with Haar sparsifying basis. To support the visual conclusion, we have also calculated the image assessment named, peak signal to noise ratio (PSNR) [23] for the recovered images. The achieved PSNR values are written in Table 3 and Table 4 for OMP and PDIP recovery algorithms respectively.

The PSNR values show the high performance of using Noiselets in comparison with Gaussian and Bernoulli. To put in a nut shell, it should be mentioned that even though Noiselets have less randomness compared to Gaussian measurement matrix, they perform well in CS framework when the sparsifying matrix is Haar wavelet and this all is related to the low coherence of these two matrices.

5. CONCLUSION

Satisfing the RIP condition and being incoherence with basis matrix, CS needs to design a stable measurement matrix. In this paper, the Noiselets properties are studied pricisely in two point of view; matrix randomness by means of entropy and performance of recovery algorithms. Although the randomness of Noiselets in comparison with Gaussian is poor, the performance of both OMP and PDIP recovery algorithms outperforms both Gaussian and Bernoulli when Noiselets as the measurement matrix and wavelet as sparse matrix are used. However, being complex valued of Noiselets is still the bottleneck of using them in real applications.

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