Workspace Analysis of 2-PR(Pa)U- 2-PR(Pa)R New Parallel Mechanism

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Abstract: Parallel kinematic machines, are closed loop structures which have more accuracy, stiffness and ability to withstand high loads. Kinematic of these mechanisms is complicated due to their closed–loop structure, parallel pods, joint constraints and movement constraints. This paper proposes a new parallel mechanism that has four degrees of freedom. In workspace analysis algorithm, conversion of inverse kinematics after providing the moving platform position (position and orientation) from search algorithm, provides basis position for testing the physical limitations of machine. Workspace of the mechanism is obtained by extracting analytical relations and consequently computational programs are written in MATLAB software. Sweep operations is started by dividing the workspace into x – y planes or horizontal sections with fixed spaces of z, then after sweeping all points of the plane, sweep operations of the next plane begins. Constraints and physical limitations considered in this mechanism includes moving restriction of saddle, collision of basis to rails, joint angles and collision of basis to moving platform. If any of these limits are violated, considered point would not be considered in the workspace. Then, to evaluate the correctness of the obtained results of workspace analysis, a suggested mechanism is simulated in SolidWorks software and obtained workspace is validated in this study. Also position kinematic and workspace analysis results are verified experimentally.

Keywords: Inverse kinematic, Parallel mechanism, Workspace analysis

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1 INTRODUCTION

Parallel mechanisms are significantly used in many fields of engineering science and industries such as machining, metrology, flight simulator, simulated earthquake, medical equipment, and etc. In general, these mechanisms have two main bodies which are coupled with each other which act through multiple links in parallel mode [1]. In comparison to arms of traditional series, the potential benefits of parallel structures is as follows: higher kinematic accuracy, lighter weight and better structural rigidity, stable capacity and suitable position of saddle arrangement, low production cost and better load bearing ability; however, from application point of view limited workspace and technical complexity are two important disadvantages of parallel arms. Therefore, the parallel kinematic machine are so suitable when the accuracy, rigidity, high speed and the ability to carry a heavy load is required in a limited workspace [2]. In terms of configuration and movement structure, parallel mechanisms are divided into two types of pods with fixed – length and variable – length: Mechanisms with fixed – length pods, in their own group and along the path of guides, are distinguishable. In these mechanism, the pods with fixed length are used which are connected to a saddle at their end part and the saddles are on motion through guides and this motion is carried out through linear or rotational operators. In mechanisms with variable – length pods, rotational and spherical joints are fixed to fix and moving platforms by bolt and the only variable parameter is the pods length. The position and orientation of moving platform is determined by changing the pods length.

In recent years, much research has been carried out on parallel robots. One of the evolutions which is already in industrial production, especially in the field of manufacturing, is the use of parallel mechanisms independently or as part of other industrial machinery. Parallel robots with six degrees of freedom, generally suffers from small workspace, complex mechanical design, difficulty in making and controlling the move due to complicated kinematic analysis. To overcome these shortcomings, the new structures for parallel robots with fewer than six degrees of freedom are used. On the other hand, in many industrial cases, there is a need to provide facilities with more than three degrees of freedom with parallel arrangement as well as simpler arrangement compared to six degrees of freedom [3]. Parallel mechanisms workspace is a closed volume which is obtained by available points at end position of the machine or robot. Shape of this area in parallel robots is irregular and complicated and there is no fixed and orthogonal axis. Workspace of parallel robots is determined through parameters or kinematic constraints, limits of pods length change, limits of rotation degrees, preventing constraints from pods intersection, and other necessary constraints to prevent additional parts intersection [4], [5]. One of the most effective ways to get work space, is discrete method. In this method, workspace is obtained by selecting a range of points in Cartesian or polar coordinates and then by checking if they follow the mentioned constraints or not. In this method it is possible to consider all the mechanical constraints. The accuracy depends on the number of points and the higher the points number, the higher the computation time will take [6–8].

In this paper, first inverse kinematics position relations of mechanism with 4 degrees of freedom is extracted then by using the discrete search algorithm in MATLAB software, workspace of the mechanism has been calculated. Finally, using animation option of SolidWorks software, validity of the workspace relation is verified. Also position kinematic and workspace analysis results are verified experimentally.

2 INTRODUCING 2-PR(PA)U- 2-PR(PA)R PARALLEL MECHANISM

In this paper, a parallel mechanism with four degrees of freedom is studied. The robot uses a mechanism with 4 pods connected to a platform and its fixed pods length leads to stiffer mechanism.

Parallel mechanism of this robot provides 4 degrees of freedom which includes the displacement in x, y and z axis and rotation about x axis. The fourth degree of freedom (rotational motion) leads to increased maneuverability as well as its usefulness compared to that of mechanisms with three degrees of freedom.

This mechanism is interconnected by two types of fraternal chain 2-PR (Pa) U-2-PR (Pa) R, two chains of PR (Pa) R and two chains of PR (Pa) U in which P, R, Pa and U represent sliding joint, hinge joint, and parallelogram or universal joint respectively (Fig. 1).

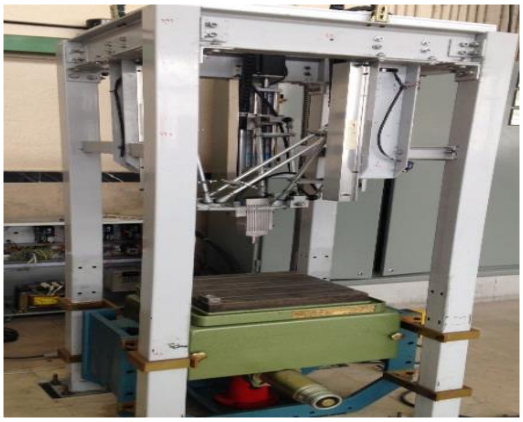

Fig. 1 2-PR(Pa)U-2-PR(Pa)R parallel mechanism with 4 degrees of freedom

To connect moving platform to fixed platform, parallelogram system has been considered. According to the reference [9], the use of a parallelogram in chains leads to increase of rotating ability and robot's stiffness as well. This system provides connection between two joint by two parallel bars which increases the stiffness of the connection. On the other hand, the higher the number of links, the more the stiffness of the whole mechanism will be. The main frame of the mechanism is in a way that provides the best workspace.

3 INVERSE KINEMATICS ANALYSIS OF THE SADDLE POSITION

In inverse kinematics for saddle position, by specifying position of central point of the moving platform, the aim is to obtain the vector position and length of each of the four saddles. The purpose of this analysis is to obtain control commands that should be applied to individual saddles so that suggested mechanisms table with 4 degrees of freedom would follow the path along with orientation correctly.

In Fig. 2, moving platform and fixed platform are defined within the framework of {P} and {O} respectively. Centers of two frameworks are coincident on centers of two platform. {P} framework is connected to the moving platform and moves with it. These two platforms are connected to each other by pods with fixed – length. If the i-th joint point of moving platform to the pods in reference framework of {P} is specified with b_i, the coordinates of q_i in reference framework of {O} is obtained as follows :

$$q_i = p + R_p b_i \tag{1}$$

In this relation, p is central point of moving platform overlapped on origin {P} related to framework of {O} and R_p is the rotation matrix.

$$p = [p_x \quad p_y \quad p_z]^T \tag{2}$$

Since here the only rotation is about X axis, rotation matrix will be as follows:

$$R_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

According to Fig. 2, we can write:

$$L_i = d_i d_{io} + c_i c_{io} + l_i l_{io} \tag{4}$$

By normalizing the Eq. (4) and regarding the law of $A^T A = |A|^2$, we can write:

$$L^2 + d_i^{\,2} + c_i^{\,2} - 2d_i(L_i \cdot d_{io}) - 2c_i(L_i \cdot c_{io}) = l^2 \tag{5}$$

By solving the Eq. (5) for d_i we can write:

$$d_i = L_i \cdot d_{io} \pm \sqrt{(L_i \cdot d_{io})^2 - (L_i \cdot L_i) + l^2 - c_i^{\,2} + 2c_i(L_i \cdot c_{io})} \tag{6}$$

Regarding the geometry of the problem by choosing positive sign for the radical, mechanism constraints are eliminated. Thus negative sign is chosen for the radical:

$$d_i = L_i \cdot d_{io} - \sqrt{(L_i \cdot d_{io})^2 - (L_i \cdot L_i) + l^2 - c_i^{\,2} + 2c_i(L_i \cdot c_{io})} \tag{7}$$

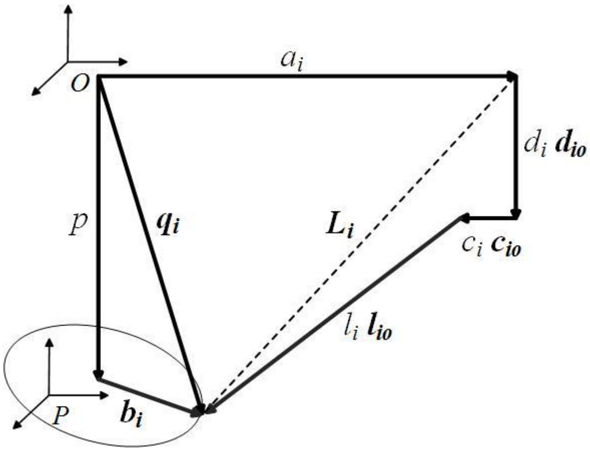

Fig. 2 Schematic of a mechanism pod

4 WORKSPACE ANALYSIS

For general machine tools with three vertical axis (X, Y, Z) connected in series, determining the workspace is very easy and regarding the axis being normal to each other, their number and their motion limitation (± X, ± Y, ± Z), workspace would form a cube in which its three axes form its orthogonal edge. By increasing the axes to 4 or 5, workspace is similar to previous form but, regarding fixed angular limitations, has more degrees of freedom. Due to simple determining of general machine tools workspace, obtaining the working range of machine regarding to size, shape and component relations with each other is relatively easy.

In parallel machine tools, determining the workspace range is very difficult, because there are no fixed axis orthogonal as general machine tools. Workspace of parallel machine tools is obtained by kinematic parameters or constraints of structure such as pods displacement, range of rotational angle, preventing constraints from basis intersection and other necessary constraints to prevent additional parts intersection. Finally, the obtained volumetric range is not geometrically simple.

Generally, regarding the size parameter, size of parallel machine tools workspace is smaller than the size of the workspace in a general machine tools. By adding workspace complexity to parallel machine tools specifications, one of the main disadvantages of machine tools become evident which is clearly obvious in a production environment. Therefore, to reduce the mentioned error effects, it is necessary to consider whole range of workspace regarding machine tool capabilities so that best working conditions within the small and complex workspace would be determined. Presented method in this paper is discrete algorithms. In this method, a search algorithm swept the entire workspace of the machine based on the location of the point within the workspace and then by checking physical constraints or limit of parallel machine tools, verifies every points of the workspace. In general, workspace analysis algorithm can be divided into three main stages:

1- Search algorithm to find the workspace range
2. Converting inverse kinematics
3. Testing physical constraints or limit of machine

In workspace analysis algorithm, conversion of inverse kinematics after providing the moving platform position (position and orientation) from search algorithm, provides basis position for testing the physical limitations of machine. As a result, combination of three abovementioned key stages can provide the required information of the parallel machine tools. However, depending on the configuration, meaning the type, order and the size and different parts of the machine tool, the source data is different, but the process remains the same.

Sweep operations is started by dividing the workspace into x – y planes or horizontal sections with fixed spaces of z, then after sweeping all points of the plane, sweep operations of the next plane begins. This algorithm runs based on the polar – cylinder coordinate system. Constraints and physical limitations considered in this mechanism includes moving restriction of saddle, collision of basis to rails, joint angles and collision of basis to moving platform. Mentioned order will increase the search speed. If any of these limits are violated, considered point would not be considered in the workspace.

4.1. limitation of saddle collision with rail

If the motion range of saddle be in the allowed range, considered points would be within the workspace. This limit can be stated as follows:

$$d_{min} \leq d_i \leq d_{max} \tag{8}$$

4.2. limitation of basis collision with rail

In the considered mechanism, there is a probability of basis collision with rail via specific angles which must be analyzed and avoided. This angle is obtained for different conditions of saddle along the slide joint in a way that there would be no collision between basis and rail (Fig. 3). Necessary condition for preventing collision of basis and rail is obtained by following relations:

$$\theta_{critical} \leq \gamma \tag{9}$$

In the relation above, γ is the angle between the basis and rail.

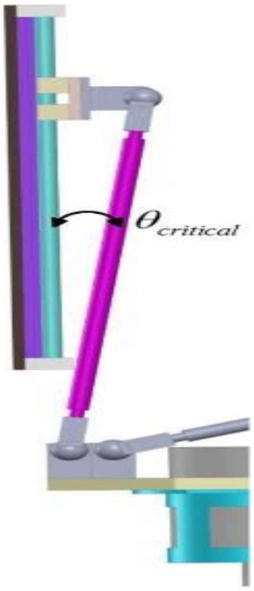

Fig. 3 Angle between basis and rail

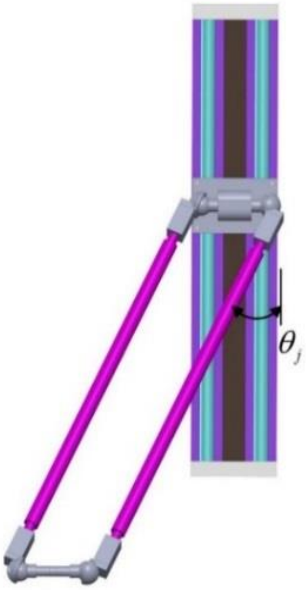

Fig. 4 Allowed value of basis movement to sideways

4.3. limitation of joint angles

The next angle that must be considered is θ_j. This angle is the allowed value of basis movement to sideways (Fig. 4). The maximum allowed value for this angle is 30 degrees. In other words:

$$\theta_j \leq \pi/6 \tag{10}$$

4. 4. limitation of collision of basis to moving platform

In the considered mechanism, as shown in Fig. 5, there is a probability of basis collision to end part of the spindle. Therefore, angle of basis with horizon should be more than θ_p of shown in the Fig. 5.

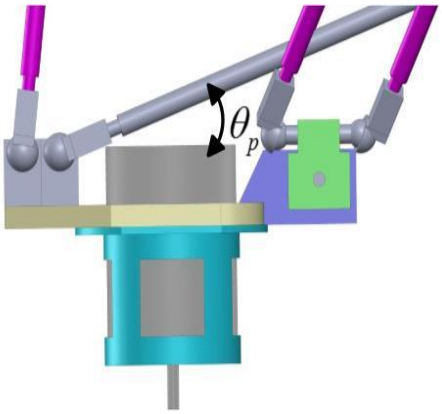

Fig. 5 Collision of basis to moving platform

Figs. 6 and 7 shows workspace range of studied robot for angles 0 degrees and 60 degrees of moving platform to the horizontal position.

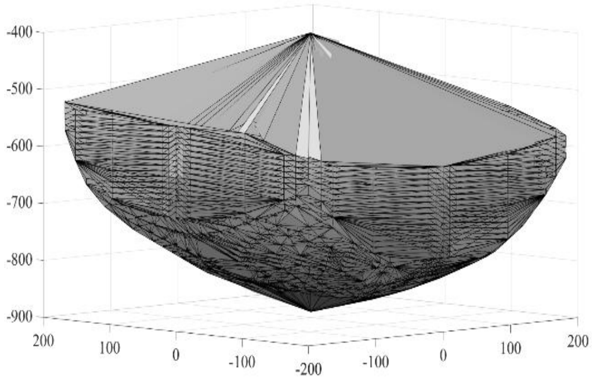

Fig. 6 Workspace of mechanism at an angle of 0 degrees

4. 5 Workspace Volume

For workspace calculating algorithm, cylindrical coordinates are used. The workspace volume relation is as follows:

$$V = \sum_{z_{min}}^{z_{max}} \sum_{\theta=0}^{2\pi} \frac{1}{2} r^2 \Delta\theta \Delta z \tag{11}$$

In the relation above, r is radius of workspace central points. For known values of z and θ, Δz and $\Delta\theta$ are the height and angle changes in cylindrical coordinates respectively, which their values are constant and are determined by user.

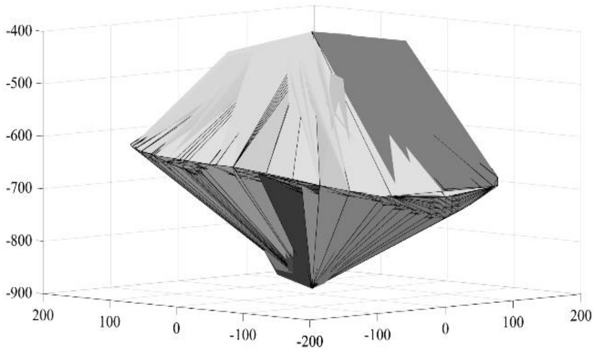

Fig. 7 Workspace of mechanism at an angle of 60 degrees

Choosing a border point on the workspace is based on the last points which violate none of physical limitations. In Fig. 8, the impact of moving platform angle on the size of the workspace is shown. As it is observed, by changing the moving platform angle, mechanism workspace is more limited.

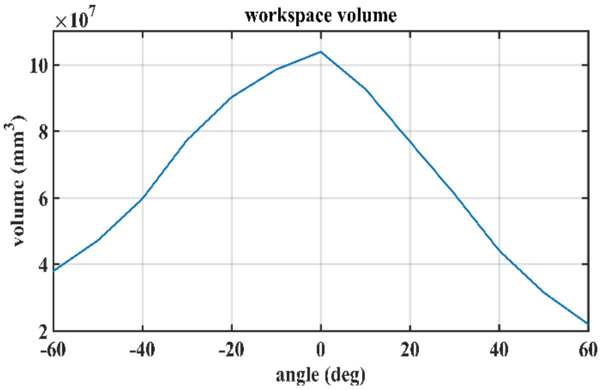

Fig. 8 Workspace volume for different angles of moving platform

5 MECHANISM SIMULATION

In this paper the studied mechanism is simulated in SolidWorks software and kinematic analysis as well as workspace is considered by this modelling. In Fig. 9 a sample of border points of workspace in which mechanical constraints are controlled, is shown. Also position kinematic and workspace analysis results are verified experimentally (Fig. 10).

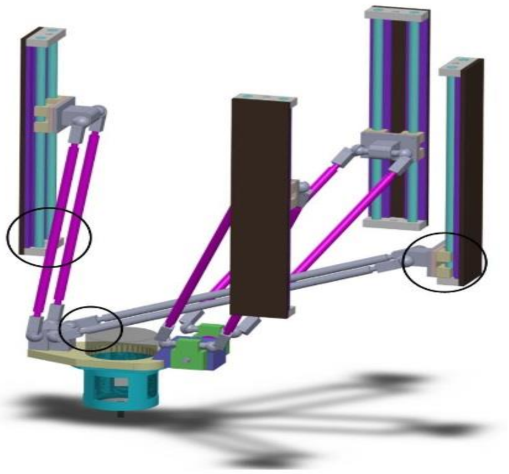

Fig. 9 Controlling points of workspace in SolidWorks software

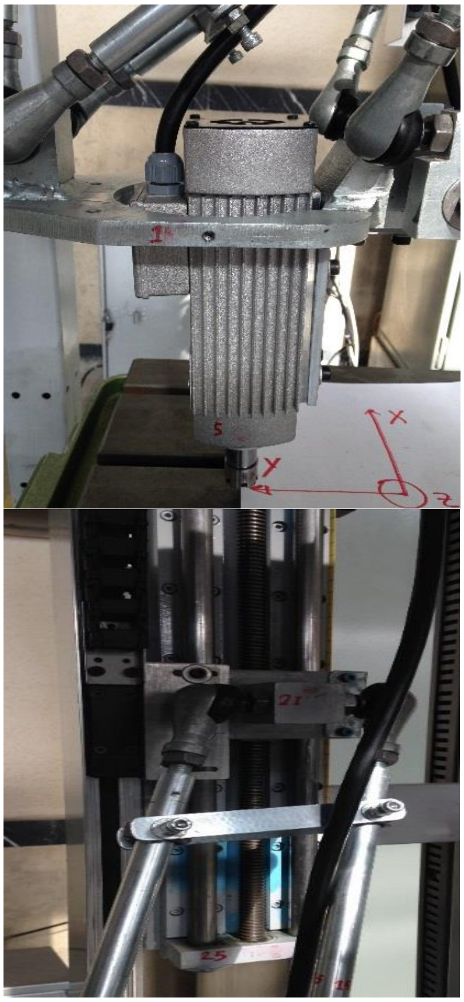

Fig. 20 Controlling position kinematic and workspace analysis experimentally

6 CONCLUSION

In this paper, for the first time, workspace analysis of a parallel robot with 4 degrees of freedom including three degrees of freedom for linear motion and one degree of freedom for rotation has been considered. First inverse kinematics relations of position are written using MATLAB software, then extracted relations are coded.
In workspace analysis algorithm, a search algorithm swept the entire workspace of the machine based on the location of the point within the workspace and then by checking physical constraints or limit of parallel machine tools, verifies every points of the workspace.
Constraints and physical limitations considered in this mechanism includes moving restriction of saddle, collision of pods to rails, joint angles and collision of pods to moving platform.
Regarding the workspace, consideration of mechanism workspace suggests that as the platform angle increase, the workspace volume decrease and when this angle versus to horizon level becomes zero, the highest volume is provided.

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