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Optimized Joint Trajectory Model with Customized Genetic Algorithm for Biped Robot Walk

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Abstract

Biped robot locomotion is one of the active research areas in robotics. In this area, real-time stable walking with proper speed is one of the main challenges that needs to be overcome. Central Pattern Generators (CPG) as one of the biological gait generation models, can produce complex nonlinear oscillation as a pattern for walking. In this paper, we propose a model for a biped robot joint trajectory in order to be able to walk straight, exploiting polynomial equations for the support leg's joints and Truncated Fourier (TFS) Series equations for the swing leg's joints in the sagittal plane and frontal plane. Four customized genetic algorithms (GA-1 to GA-4) with different implementations for the crossover steps are used as evolutionary algorithms to optimize equation parameters and achieve the best speed and performance in walking motion. These four GAs differ in crossover step and parent selection parts. After a primary evaluation to make sure the next generation is better off than before, we consider a clever comparison feature between the best of two generations (parent and child) in GA-4. The algorithms have been tested on the Darwin humanoid robot in the Webots simulator environment where the results show that the GA-4 model has the best performance and achieves the desired fitness value.

Keywords: *Humanoid Robot Walk, Central Pattern Generator, Genetic Algorithm, Truncated Fourier series.*

1. Introduction

1.1. Gait Generation Models

Professor Ichiro Kato's group was the first group that worked on humanoid robots at Waseda University from 1970 [1]. Two decades after that, researches on humanoid robots expanded and other groups such as MIT, NASA (USA) and the University of Tokyo initiated various studies on this subject [2, 3].Gait generation for biped robots is an important and open problem. Many researchers have proposed several models for a human-like walk. There are several models to generate walking gait for humanoid robots.

1) Trial and error model: In this model, each step is divided into several phases, and equations are designed for each phase. The following sub-models of this model are: a) a joint space model [4], b) a virtual forces model [3]. The joint space model computes suitable temporal trajectories between the present joint limits of the walking motion [3]. In the virtual forces model, which is proposed by Jerry Pratt et al [4],the equations are produced based on the force reaction and joint torque control. The disadvantages of this model are: I) the dynamics of the robot using this model is very simple and therefore, the model cannot be used for a robot with an adult human size. II) This model is very slow due to its equations design.

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2) Mass distributed model: To generate more accurate gaits and also taking into account the dynamic effects of the robots, this model studies the equation of pendular motion on centre of mass for some biped links [5]. The submodels are: a) Two Masses Inverted Pendulum Model (TMIPM) [6] and b) Multiple Masses Inverted Pendulum Model (MMIPM) [7]. The main advantage of this model is that walking motion is stable but it can only be used for offline trajectory generation, but not in real-time due to the high computation time.

3) Mass concentrated model: This model simplifies the dynamic of the whole body of the robot in one mass, namely "center of gravity", and the kinematic equations of this mass are computed. There are two sub-models for this model: a) the 2D Inverted Pendulum Model (IPM), b) the 3D Inverted Pendulum Model [8]. In the 2D IPM, the motion of the robot is studied in sagittal plane (x-z plane) and all the mass of the body is concentrated in the centre of gravity. In 3D IPM, to get more stability and have a walking motion similar to a human, the robot's motion is also studied in frontal plane and this model introduces ZMP as a stability criteria when the robot stands on one of its legs during walking (single support phase). Due to the simplicity and low computation time, this model can be used for real-time applications.

4) Central Pattern Generator (CPG): Is inspired from biology, which produces multidimensional rhythmic signals and these signals are applied to the robot's joints [9]. Shik [10], Cruse [11] and Brewer [12] used this model as a gait generation model for a biped robot. In this model accurate knowledge of the dynamic of the robot is not needed.

Taga's work on biped locomotion [9] proves that CPG can be used to make humanoid robot's walk. In [13] a genetic algorithm is used for optimizing trajectory generation of the walking biped robots. In [14] Kim uses a method based on PSO algorithm for optimizing CPG parameters. In [15] authors introduce a biologically inspired method for a biped gait generation using particle swarm optimization. Human biomechanics are used to mimic the stability condition and the walking cycle composition. All joints are set to simple initial positions at the beginning of each iteration in order to prepare the robot for a stable walking motion. A forward position should ensure a forward step and a stable walking motion. Truncated Fourier series was used as a nonlinear oscillator to model gait trajectory of each joint of the robot in 2006 [16].

1.2. Evolutionary Algorithms

Each gait generation model has various parameters that must be tuned. Evolutionary algorithms are one of the most popular algorithms to optimize these parameters in minimum time in order to achieve continuous dynamic walking [17] [18]. Each evolutionary algorithm has individuals that search the solution space of the given problem. A fitness value is assigned to individuals to evaluate the performance of each one. The probability of an individual to be selected as a parent depends on the fitness value. The individual with the better fitness value has a better chance of being selected.

The evolutionary algorithms such as generic, simulated annealing, particle swarm optimization (PSO) algorithm are inspired from nature. These algorithms are based on iteration and the probability and are used in optimization problems. Genetic Algorithms are used as an evolutionary algorithm to minimize walking trajectory energy consumption in [17]. A comprehensive study on intelligent control techniques is performed in [18] and GA is used to optimize neural network parameters to control robot walking motion. The investigated GA models of [19] use the crossover and mutation operations of [18]. In [20] different implementations of GA operators are used to optimize gait parameters, such as two-point crossover and Gaussian mutation. To avoid being stuck in the local best answer in the solution space, the authors of [21] designed explicit fitness sharing. In [22] Adaptive PSO is used which tunes the inertia weight dynamically to search local space more or speed up the convergence to the global best position. The adaptive GA is also used in [23] to optimize the gait parameters. This algorithm can adaptively change the possibilities of the GA crossover operator and mutation operator during the process of evolution.

The rest of this paper is organized as follows: Section 2 explains our proposed model. Section 3 focuses on experimental setup, and section 4 presents the experimental results, and finally Section 5 concludes the paper.

2. Proposed Model

2.1. Motion Pattern

The proposed model is a sub model of CPG, which produces rhythmic signals as a trajectory gait generation. The goal of this experiment is to have a straight, stable walking with proper speed .The Darwin kid size robot is used for the proposed algorithm that has 20 degrees of freedom (Fig. 1). For the walking trajectory, only joints of the legs are considered. As can be seen in Fig. 1, each leg has six degrees of freedom and only these joint are derived. The walking trajectory is divided into several types. Angular trajectory is defined as the trajectory in which the angle of each joint is plotted at a certain time slice.

A straight walking cycle consists of two steps which are completely symmetric, so only one step is needed to produce because after each step, only the role of the support and swing legs are changed, respectively. In our studies on human forward walking, the angle of each leg's joints in one period of walking signal (one step) with 0.04 seconds interval is recorded (Fig. 2). In the sagittal plane, the joints of the hip, knee and ankle for the support leg have smooth trajectories.

This trajectory helps humans to achieve faster and stable walking motion. Polynomial Equation (Eq. (1)) is used as a novel method for the support leg to produce the gait Equation to model the trajectory (Table1). a_1, a_2, …, a_n are the constant coefficients for the support leg and t is the time. Due to this equation's low computation time, it's possible to use it in real-time. Truncated Fourier Series (TFS) (Eq. (2)) is used to generate the trajectory of the swing leg in the sagittal plane (Table1) A_i, B_i and C_i are the constant coefficients and t is the time. As pattern generators, the advantages of these equations are their simplicity and the possibility to be used for real-time calculations. TFS is also used on the frontal plane to maintain robot stability during walking motion.

$$\theta_{h,k,a} = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_2 t^2 + a_1 t + a_0 \quad (1)$$

$$\theta_{h,k,a} = \sum_{i=1}^{n} A_i . \sin(B_i t + C_i) \quad (2)$$

Table. 1. Joints equation types

Joint name (plane name)	Support Leg (Left Leg)	Swing leg (Right leg)
Hip (Yaw)	Sinusoid	Sinusoid
Hip (Roll)	Sinusoid	Sinusoid
Hip (Pitch)	Polynomial	Sinusoid
Knee (Pitch)	Polynomial	Sinusoid
Ankle (Pitch)	Polynomial	Sinusoid
Ankle (Roll)	Sinusoid	Sinusoid

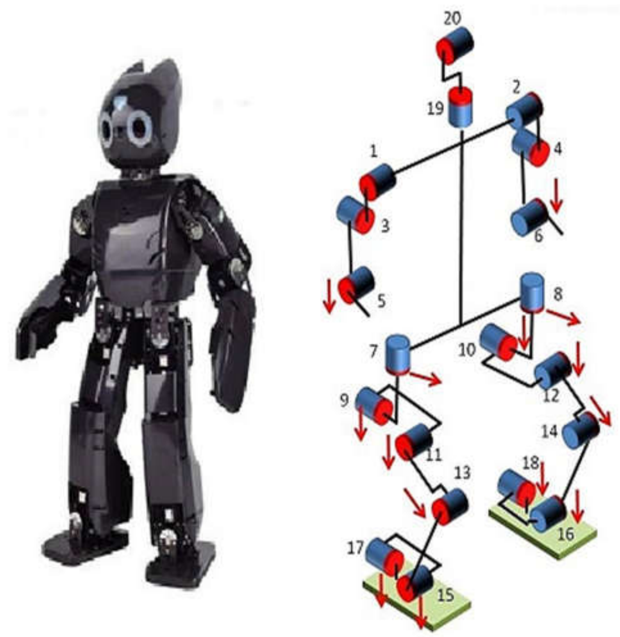
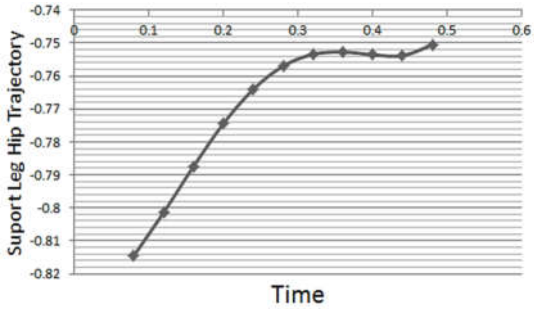
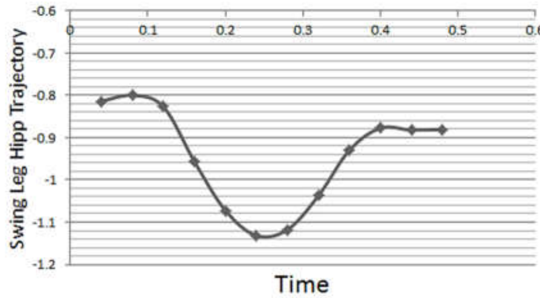


Fig. 1. The Darwin robot model with 20 degrees of freedom



(a)



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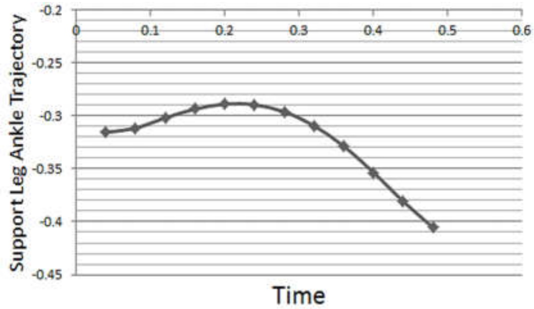

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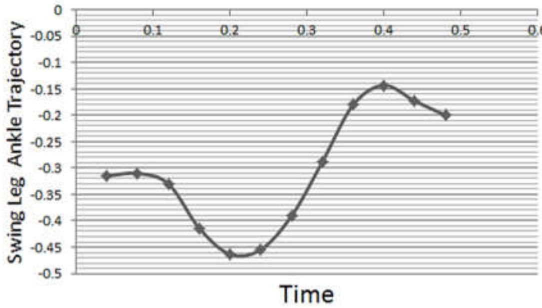

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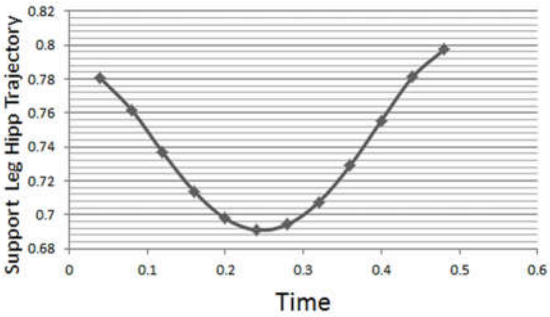

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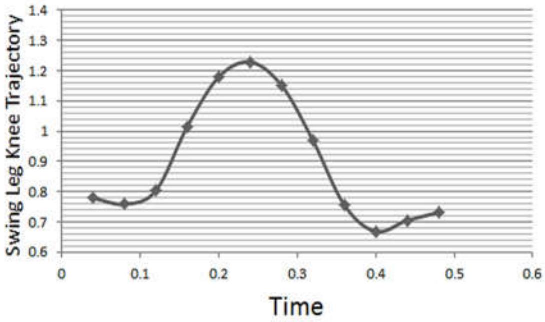
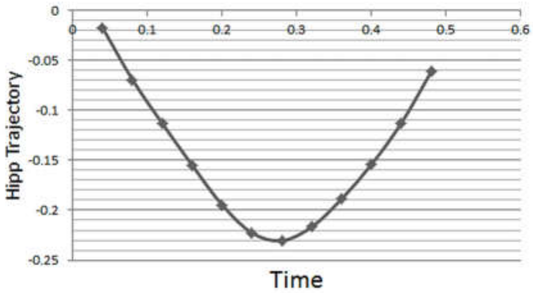
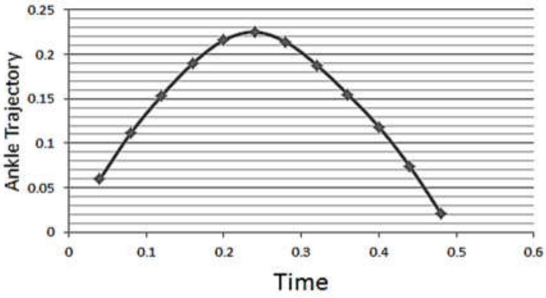


(f)



(g)



(h)

Fig. 2. (a) Hip joint angular trajectory for the support leg in the sagittal plane, (b) Hip joint angular trajectory for the swing leg in the sagittal plane, (c) Knee joint angular trajectory for the support leg in the sagittal plane, (d) Knee joint angular trajectory for the swing leg in the sagittal plane, (e) Ankle joint angular trajectory for the support leg in the sagittal plane,(f) Ankle joint angular trajectory for the swing leg in the sagittal plane, (g) Hip joint angular trajectory for both legs in the frontal plane, (h) Ankle joint angular trajectory for both legs in the frontal plane.

Fig. 3. The Webots environment (simulating software)

2.2. GA Models

In this paper, four GAs are used to optimize the coefficients of the proposed motion trajectory equations (Eq. (1) and Eq. (2)). These GAs consist of four steps, in the first step the initial population is created, in the second step, the fitness of each individual is evaluated, in the third step some individuals are selected based on their fitness value and sent to the crossover operator to produce two new individuals. This operation is then until the size of the population of new individuals reaches the main population size. In the last step, the mutation operator changes some gens of the new individuals (the coefficient values in our model).

GA-1 to GA-4 have different implementations in the selection step and the crossover part. In all of the four models the initial population is created randomly, the mutation is executed on 10% of the new population and only changes a single gen (coefficient) by random. Fig. 4 shows the structure of one chromosome. Each coefficient of the Eq. 1 and Eq. 2 is considered as one gen. Each chromosome consists of 87 gens. There are 15 gens for the coefficients of the polynomial equation and 72 gens for the coefficients of the TFS equation (totally, 87coefficients are considered for all TFS and Polynomial equations (Eq. 1 and Eq. 2)) and cost is introduced as the fitness value of the chromosome after their evaluation.

GA-1 Model: the crossover operator selects two individuals by random and swaps random parts of each one to produce new individuals and repeat this operation until the maximum population size is reached.

GA-2 Model: this model uses the Roulette wheel method to select the best parents. In the crossover step, parts of these parents are swaped randomly in order to create new population in crossover step.

GA-3 Model: similar to the GA-2 Model, the Roulette wheel method is used to select best parents but in the crossover step, the swaping method is different. In this model all gens of one individual (all coefficients of the Equations) which are responsible for producing motion in the x-z plane and all gens that produce motion in the y-z plane are selected to be swaped in order to create two new individuals.

GA-4 Model: in this model Roulette wheel method is used to select parents for the next generation, random parts of each parents are swapped in the crossover operation. When creation of the new generation is done, evluation for all new chromosomes is started, the best chromosomes of two generations (parent and child generations) are selected, taking into acount the size of the population, to produce the next generation. This operation is then repeated. The advantage of this model is that when the Roulette wheel method is used as a selection method there is always a chance for chromosomes with low fitness to be choosen in order to create new generations that produce chromosomes with new abilities. Finally only the best chromosomes from two generations (parents and child generations) are kept.

The main implementation choices for each of these models are shown in Table 2. Eq. (3) shows the fitness function that is used in this paper.

$$\left(\sum_{i=1}^{n} \frac{\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} - |(y_i - y_{i-1})|}{x_{destination} - x_{offset}}\right) \qquad (3)$$

The goal of this experiment is to have a straight, stable walking with proper speed, so evaluating fitness with this criteria has several interesting properties, such as ensuring that the robot is moving forward (in X direction) and favouring higher velocities. At first, for each gen the fitness function has the initial value equal to zero. After each two steps, the fitness function is calculated and added to the fitness function. This calculation is repeated after each two steps until the termination conditions is met.x_{offset} is the initial position of robot, $x_{destination}$ is the final position of the robot when moving straight ahead in the main simulation time.x_{i-1} and x_i are the current positions of robot before and after each two steps i and i-1. the X direction and also y_{n-1} and y_i in Y direction so that the GA can determine good individuals from their fitness, $|(y_i - y_{i-1})|$ is used to penalize fitness if the robot walks in the Y direction. Three terminating conditions are considered for evaluating the fitness function: 1- Robot reaches the center of the field from the Penalty mark (170 cm). 2-The main simulation time finishes (15 seconds). 3- The Robot falls down.

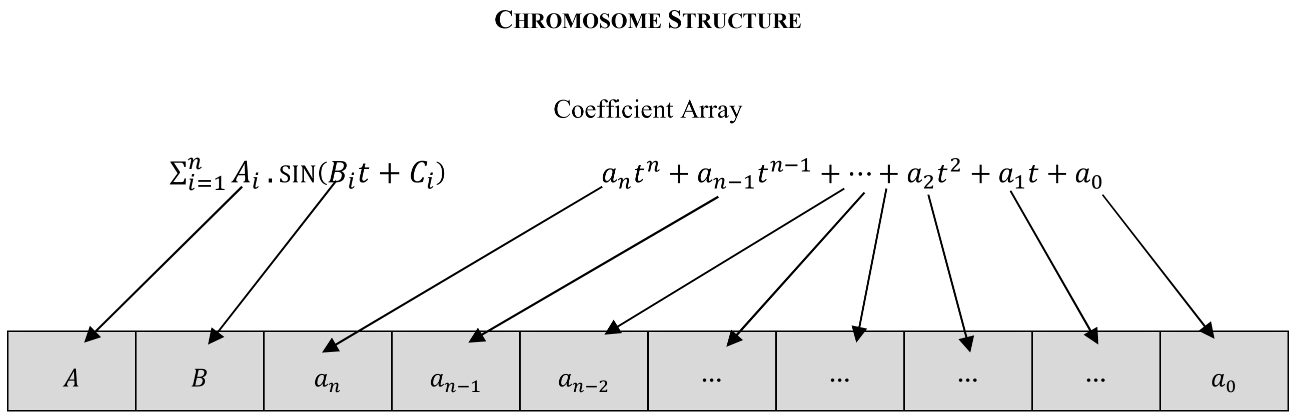


Fig. 4. The chromosome structure (totally each chromosome has 87 GENS).

Table. 2. Four different implementations of GA

Model of implementation	Selection	Crossover	Mutation
GA-1	Random	Random	10%
GA-2	Roulette wheel	Random	10%
GA-3	Roulette wheel	All pitches and all rolls	10%
GA-4	Hybrid	Random	10%

3. Experimental Setup

3.1. Framework

Our proposed method is evaluated on the Darwin humanoid robot and experiments are performed using the Webots simulator as a generic three-dimensional simulator (Fig. 3). More than 400 universities and research institution work with the Webots. The Four basic stages that Webots provides are: 1) the model stage of the robot for designing physical body of the robot. 2) the program stage, in this stage the behaviour of the robot is programmed 3) simulating two previous steps, so the results can be seen in the simulation environment 4) transfering the program to the real robot and run in the real world so the results can be compared with simulation results. The robot model has 20 DOF (Degrees Of Freedom) with a height of about 45.5cm, and a mass of 2.8kg (Fig. 1). Webots has the ability to define supervisor controller to check positions of the robot. The environment function calls of the Webots includes stop running process, revert the process and etc. At the beginning of the simulation the robot is set to a special position (penalty mark point) and robot's joints are set to special offsets to get ready for walking. The robot starts walking until terminating conditions occur. In this situation the real time supervisor calls the revert method of the Webots and the robot will be set to its initial position and this process will be repeated.

3.2. Algorithm Steps

Only 12 joints of the robot's legs (each leg has 6 DOFs) moved according to Table 1. Although other DOFs are effective in the walking behavior, their main role is in smoothing the robot's walking motion. At first, custom stable walk steps are generated in minimum time for the robot. Each step lasts 0.48 second and the angle positions of the joints of robot's leg are recorded every 0.04 second. There are three DOFs in each leg movement in sagittal plane: one in the hip, one in the ankle and one at the knee. Fig. 2 (a), (c) and (e) show hip, knee and ankle trajectories for support leg on the sagittal plane, which are modelled in the MATLAB by polynomial equations. For the swing leg on sagittal plane hip, knee and ankle trajectories are modelled in the

MATLAB by a Sinusoid Fourier series Equation (Fig. 2(b), (d) and (f)). For balancing, there are two DOFs in each leg moving in the frontal plane: one in the hip, one in the ankle. The hip and the ankle trajectories can be seen for the support leg in Fig. 2(g) and (h), so the same equation can be calculated for the hip and the ankle trajectories for both legs (swing leg or support leg). The trajectories for hip and ankle joints can be modelled by a Sinusoid Fourier series equation.

4. Experimental Results

The results of the four proposed GAs can be found in Fig. 5. According to equation (4), to reduce the width of the confidence interval and to obtain more conclusive results, these results are calculated by averaging over five repetitions for each of the GA-1, GA-2, GA-3 and GA-4 models.

$$\begin{aligned} &\text{if } n \geq 30 \text{ then Confidence Interval} = X \pm Z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) \\ &\text{if } n < 30 \text{ then Confidence Interval} = X \pm t_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) \end{aligned} \quad (4)$$

In equation (4) X = sample mean, σ= standard deviation, n = number of terms, α= the desired significance level, $Z_{\frac{\alpha}{2}}$ and $t_{\frac{\alpha}{2}}$ are values corresponding to z table and t table for confidence interval formula [24] [25].

The convergence of all individuals for GA-1 to GA-4 models can be found in Fig.5 (a), (b), (c) and (d). GA-1 has the worst results because all the steps in this model are done randomly. GA-4 has the best convergence because its selection model is based on the best parents and guarantees selecting good individuals for the crossover operations. Due to random selections for swapping parts and generating new individuals, there is always a chance to have individuals with new abilities and unpredictable behaviour and find better solutions.

In the crossover operation of GA-3, all gens of one individual that are responsible for producing motion in the x-z plane are swapped with another individual. In other words, all gens that produce motion in the y-z plane are swapped with another individual to create two new individuals. If all initial individuals have bad gens in just one equation (for example knee equation coefficients for the swing leg), which produces one joint trajectory in the x-z plane or in the y-z plane, this model will never reach a good solution because bad gens are swapped with bad gens to create new individuals. The disadvantage of GA-3 is that it is dependent on good initial state of individuals for reaching to good results, as can be seen in Fig.5 (c.1). GA-1 has the worst results because all steps in this model are done randomly (Fig.5 (a.2)). In the GA-4, individuals have the best convergence to the best solution (Fig.5 (b.2)). Table 3 shows the experimental results with 5 repetitions. After 8 generations, GA-4 reaches near 0.6 of the best fitness value, in this algorithm good individuals are always selected for crossover operation and because of random crossover there is always a chance to produce new individuals with new behaviour (new behaviour for all equations). Fig. 6 shows the best fitness values for the four GA algorithms.

Table. 3. Experimental results

Model	No. of chromosomes	No. of Generations	Best Fitness value (average 5 iteration)	Min. iteration for converging to 0.6
GA-1	20	10	0.353547	-
GA-2	20	10	0.501157	10
GA-3	20	10	0.487875	-
GA-4	20	10	0.832135	8

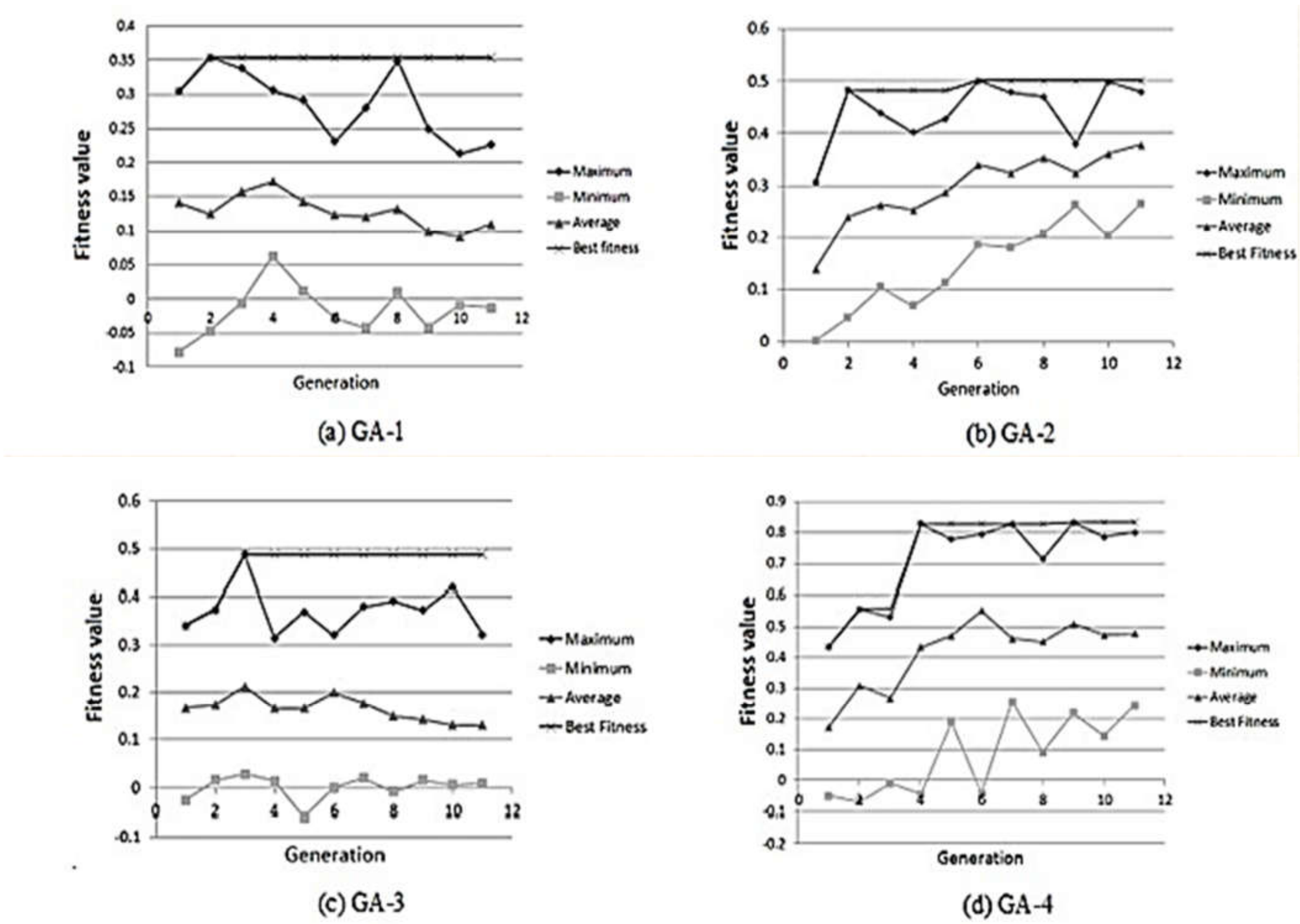


Fig. 5. (a), (b), (c) and (d) show the convergence of chromosomes in GA-1, GA-2, GA-3 and GA-4, respectively and also show the maximum, minimum, average and best fitness values for GA-1, GA-2, GA-3 and GA-4. (The results are achieved by averaging over five repetitions for each of four GA models with 20 Chromosomes and 10 generations).

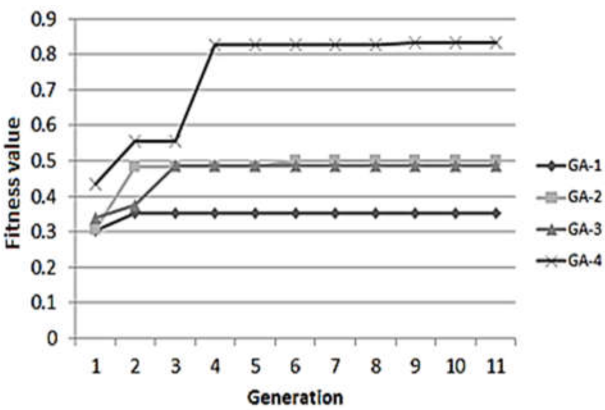


Fig. 6. Best fitness values for four GA models

5. Conclusion

In this paper a new gait generation model were introduced which consists of using optimized polynomial equations as a novel method for the support leg and TFS equation for the swing leg in the sagittal plane. TFS is also used for balancing in the frontal plane. The advantage of this model is its low computation time and possible usage in real-time calculations to achieve walking with proper speed similar to humans. Several implementations of GA have been explored to find the best values for coefficients of the proposed gait generation model in minimum time.

According to the results, GA-4 has the best results and by using clever selection methods like Roulette wheel there's always a chance for normal and bad chromosomes to be chosen, producing new chromosomes with new capabilities. When the evaluation of the new generations is done, the best chromosomes of two generations (parents and children) are selected to produce the next generation this technique leads to the best solution in minimum time.

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