Journal of Solid Mechanics Vol. 10, No. 3 (2018) pp. 561-570

New Method for Large Deflection Analysis of an Elliptic Plate Weakened by an Eccentric Circular Hole

Sh. Dastjerdi [1, *], L. Yazdanparast [2]

[1] *Young Researchers and Elite Club, Mashhad Branch, Islamic Azad University, Mashhad, Iran*
[2] *Department of Mechanical Engineering, Damavand Branch, Islamic Azad University, Damavand, Iran*

Received 16 June 2018; accepted 15 August 2018

ABSTRACT

The bending analysis of moderately thick elliptic plates weakened by an eccentric circular hole has been investigated in this article. The nonlinear governing equations have been presented by considering the von-Karman assumptions and the first-order shear deformation theory in cylindrical coordinates system. Semi-analytical polynomial method (SAPM) which had been presented by the author before has been used. By applying SAPM method, the nonlinear partial differential equations have been transformed to the nonlinear algebraic equations system. Then, the nonlinear algebraic equations have been solved by using Newton–Raphson method. The obtained results of this study have been compared with the results of other references and the accuracy of the results has been shown. The effect of some important parameters on the results such as the location of the circular hole, the ratio of major to minor radiuses of elliptical plate, the size of circular hole and boundary conditions have been studied. It is concluded that applying the presented method is very convenient and efficient. So, it can be used for analyzing the mechanical behavior of elliptical plates, instead of relatively complicated formulations in elliptic coordinates system.

Keywords: Elliptical plate; Eccentric circular hole; First-order shear deformation theory; Semi-analytical polynomial method (SAPM).

1 INTRODUCTION

THE plates are the structures with many different types of applications in engineering such as ocean engineering structures and firmly it can be indicated that it is very hard to find a structure which at least one plate structure would not be in it. Different geometrical forms of plates can be used in mechanical structures. However, the most applicable plates in structures are regular geometric plates. For example, circular, or annular/circular plates [1, 2]. Analysis of the mechanical behavior of plates is one of issues that have prompted many researchers to do extensive research in this field, including bending, buckling and vibration analyses of plates. Predicting the mechanical behavior of plates can reduce the production costs and subsequent errors dramatically.

Sato [3] explained the importance of using the various types of plates on elastic foundation as structural elements in the wide fields of engineering. The influence of uniform lateral load and in-plane force on the bending of elliptical plates on elastic foundation in elliptical coordinate system is derived exactly. This is while Datta [4] was

[*] Corresponding author. Tel.:+98 5137642570.
E-mail address: dastjerdi_shahriar@yahoo.com (Sh.Dastjerdi).

extracted the Galerkin method to study large deflection bending of thin orthotropic elliptical plates that resting on a Winkler type foundation with clamped edge boundary condition. Kutlu and Omurtag [5] studied large deflection analysis of an elliptic plate embedded in orthotropic elastic foundation by using finite element method. They concluded that the large deflection of an elliptic plate is heavily depended on orthotropic foundation and geometrical parameters of the plate. They considered first-order shear deformation theory and nonlinear strain terms to derive governing equations. Zhong et al. [6] extracted the triangular differential quadrature method for analyzing the flexure of uniformly loaded elliptical Reissner–Mindlin plates resting on Pasternak type elastic foundation. A higher-order boundary perturbation method (BPM) introduced by Parnes [7] to derive Green's function by means of an eccentric source in an elliptic domain. He analyzed clamped and moderately elliptic thin plates subjected to eccentric loading. He also introduced BPM for an elliptic plate that the boundary condition of the plate is simply-supported and subjected to a central lateral point [8]. The accuracy of BPM solution is checked for moderately elliptic plates. Wang and et al. [9] proposed a numerical solution method for solving the boundary value problems in bending of elliptical plates using Barycentric Lagrange interpolation that the governing equation discretized to a system of algebraic equation. They have expressed with citing and comparing with previous articles that the BLI is a fast, high accurate and numerical stability method.

An analytical solution for bending of an anisotropic plate containing an elliptical hole and subjected to out-of-plane bending moments has been studied by Hsieh and Hwu [10]. They used Stroh-like formalism for bending these plates that the deflections, the moments and the transverse shear forces can all expressed in complex matrix form. The fundamental frequencies of a simply supported elliptical plate using Rayleigh-Ritz method was presented by Leissa [11] and also his other articles are good sources that studied on the vibration of the plates [12-18] until 1987.

The analysis of super elliptical plates that consisted different shapes from ellipse to rectangle are available in the literatures. The vibration and buckling of super elliptical plates is presented by Wang et al. [19] using Rayleigh-Ritz method. In the following Altekin and Altay [20] presented the static analysis of point-supported super-elliptical plates using Ritz method. After that Altekin [21] presented the free linear vibration and buckling of super-elliptical plates resting on symmetrically distributed point-supports. Also Altekin [22] investigated the free vibration of orthotropic super elliptical plates on intermediate supports and Altakin [23] presented the static analysis of super-elliptical plates on intermediate point supports by the Ritz method. Laura and Rossit [24] studied thermal bending of thin anisotropic clamped elliptic plates. Zhang [25] used Ritz method to analyze the non-linear bending of super elliptical thin plates with simply supported edge and clamped edge based on classical plate theory. Mc Nitt [26] analyzed the free vibration of a damped semi elliptical plate and a quarter-elliptical plate and computing the lowest natural frequency of the normal mode that is fully clamped in its boundary condition with the classical bending theory but he avoided of the influence of rotatory inertia. Hasheminejad and Vaezian [27] presented a semi-analytical solution based on the elaborated collocation multipolar method for the free vibration analysis of a fully clamped elliptical plate with an elliptical hole. Irie and Yamada [28] have used Ritz method to calculate the free vibration of an orthotropic elliptical plate with a similar hole at the center of it. Natural frequencies and mode shapes are calculated numerically. Also Ghaheri and et al. [29] have used Ritz method for calculating the buckling and vibration of thin, symmetrically laminated, elliptical composite plates under initial in-plane edge loads and resting on Winkler-type elastic foundation. Mathieu function employed to calculate the large deflection of a heated elliptic plate with clamped edges under stationary temperature with Berger's method [30]. Dastjerdi et al. [31] studied the effect of vacant defect on bending analysis of circular Graphene sheets. They presented a new semi-analytical polynomial method for solving the obtained partial differential equations. They concluded that the applied method (SAPM) is so effective and convenient.

Mechanical analysis of elliptical plates has been considered according to classical methods by using complex equations in elliptical coordinate system. In this paper, we tried to study the nonlinear bending behavior of an elliptical plate in the cylindrical coordinates system by using the more convenient formulations. The governing equations are presented based on the first-order shear deformation plate theory and solved by SAPM method. Also, it is tried to perform a comprehensive analysis, that is why an eccentric circular hole is considered in the elliptic plate and the effect of the hole is studied. For validation, the results are compared with the relevant researches in the past and the advantages of the applied method are proved. Furthermore, the effects of some geometrical properties of an elliptical plate which can affect the results are studied. This paper can be an appropriate reference for the future works related to analysis of moderately thick elliptical plates.

2 FORMULATION

2.1 Plate geometry

In this paper, it is tried to formulate the bending behavior of the moderately thick elliptical plates. For this purpose, the first-order shear deformation theory is used. In this theory, the shear force has been considered linearly along the plate thickness. The results of FSDT for moderately thick plates are more accurate than the classical plate theory (CLPT).

First, an elliptical plate including an eccentric circular hole is considered, that shown in Fig. 1. In Fig. 1 a is the major radius of ellipse and b is the minor radius. Center of the hole with r_i radius, is located at a distance of e from the center of ellipse. h is the thickness of the plate. The cylindrical coordinates system is used in this paper. For this aim, the eccentric circular hole center, S arbitrary line is drawn that have θ angle with the X horizontal axis. By changing θ from 0 to 360° the entire range of plate is covered ($O_1P = S$). By changing the θ angle it can be seen that S size is changed. To use the cylindrical coordinate system, r and θ can be made the S size at any desired angle according to the constant values of problem as follows. The elliptic equation in the general case for the plate in Fig. 1 can be written as follows:

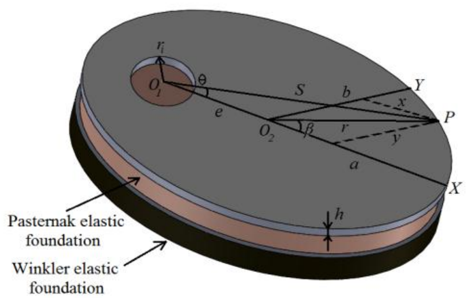

Fig.1
Elliptical plate resting on elastic foundation.

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \tag{1}$$

According to Fig. 1, x and y can be formulated below:

$$\begin{aligned}\sin(\theta) &= \frac{y}{S} \Rightarrow y = S \cdot \sin(\theta) \\ \cos(\theta) &= \frac{x+e}{S} \Rightarrow x = S \cdot \cos(\theta) - e\end{aligned} \tag{2}$$

By substituting Eqs. (2) into (1), S can be derived as follow:

$$\frac{1}{a^2}\left(S \cdot \cos(\theta) - e\right)^2 + \frac{1}{b^2}\left(S \cdot \sin(\theta)\right)^2 = 1 \tag{3}$$

$$\begin{aligned}&\Rightarrow \frac{1}{a^2}\left(S^2 \cdot \cos^2(\theta) + e^2 - 2eS\cos(\theta)\right) + \frac{1}{b^2}\left(S^2 \cdot \sin^2(\theta)\right) = 1 \\ &\Rightarrow \left(\frac{1}{a^2}\cos^2(\theta) + \frac{1}{b^2}\sin^2(\theta)\right)S^2 - \left(\frac{2e}{a^2}\cos(\theta)\right)S + \left(\frac{e^2}{a^2} - 1\right) = 0\end{aligned} \tag{4}$$

According to Eq. (4) it is observed that a quadratic equation is obtained that with its solving, the value of S can be achieved in terms of the a, b and e constants at different θ angles. By solving Eq. (4), S can be derived as follow:

$$S = \frac{\frac{e}{a^2}\cos(\theta) \pm \sqrt{\left(\frac{e^2}{a^4}\cos^2(\theta)\right) - \left(\frac{1}{a^2}\cos^2(\theta) + \frac{1}{b^2}\sin^2(\theta)\right)\left(\frac{e^2}{a^2} - 1\right)}}{\left(\frac{1}{a^2}\cos^2(\theta) + \frac{1}{b^2}\sin^2(\theta)\right)} \tag{5}$$

In Eq. (5), the acceptable result for S is:

$$S = \frac{\frac{e}{a^2}\cos(\theta) + \sqrt{\left(\frac{e^2}{a^4}\cos^2(\theta)\right) - \left(\frac{1}{a^2}\cos^2(\theta) + \frac{1}{b^2}\sin^2(\theta)\right)\left(\frac{e^2}{a^2} - 1\right)}}{\left(\frac{1}{a^2}\cos^2(\theta) + \frac{1}{b^2}\sin^2(\theta)\right)} \tag{6}$$

For the especial case, when the ellipse is considered a circle, $a = b = r_o$. So, S can be written below which was presented before by authors [31].

$$S = e\left(\sqrt{\left(\frac{r_o}{e}\right)^2 - \sin^2(\theta)} + \cos(\theta)\right) \tag{7}$$

2.2 Constitutive equations

The elliptical plate which is considered to be studied is a moderately thick plate. Consequently, to aim for obtaining more accurate results, the first-order shear deformation theory is assumed as follow:

$$U(r,\theta,z) = u(r,\theta) + z\,\phi(r,\theta) \tag{8}$$

$$V(r,\theta,z) = v(r,\theta) + z\,\psi(r,\theta) \tag{9}$$

$$W(r,\theta) = w(r,\theta) \tag{10}$$

As mentioned before, the partial differential constitutive equations are formulated in cylindrical coordinates system. The governing equations for circular Graphene nano-plate including an eccentric defect have been presented by authors [31] before. In this study, the mentioned equations [31], can be applied for elliptical macro plates by neglecting the small scale effects ($\mu = 0$). So, the constitutive equations for moderately thick elliptical plates including an eccentric hole are expressed as follow:

$$\delta u : N_{r,r} + \frac{1}{r}\left(N_{r\theta,\theta} + N_r - N_\theta\right) = 0 \tag{11}$$

$$\delta v : N_{r\theta,r} + \frac{1}{r}\left(N_{\theta,\theta} + 2N_{r\theta}\right) = 0 \tag{12}$$

$$\delta w : Q_{r,r} + \frac{1}{r}\left(Q_{\theta,\theta} + Q_r\right) + \left(q - k_w w + k_p \nabla^2 w + N_r \frac{\partial^2 w}{\partial r^2} + N_\theta\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) + 2N_{r\theta}\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta}\right)\right) = 0 \tag{13}$$

$$\delta \phi : M_{r,r} + \frac{1}{r}\left(M_{r\theta,\theta} + M_r - M_\theta\right) - Q_r = 0 \tag{14}$$

$$\delta \psi : M_{r\theta,r} + \frac{1}{r}\left(M_{\theta,\theta} + 2M_{r\theta}\right) - Q_\theta = 0 \tag{15}$$

The definition for stress resultants ($N_{ij}, M_{ij}, Q_{ij}\left(i, j = r, \theta, r\theta\right)$) and boundary conditions are presented by author [31].

3 SOLVING METHOD

The governing equations (Eqs. (11-15)) are nonlinear partial differential equations. So, finding an analytical solution is likely impossible and numerical solution must be attended. In addition, due to existence of eccentric circular hole, the distance between grid points is not constant in numerical solutions such as differential quadrature method (DQM) and solving by DQM is not possible (Fig. 2). Consequently, an innovative semi-analytical solution called SAPM is applied. The details for SAPM solution have been presented in Ref. [31].

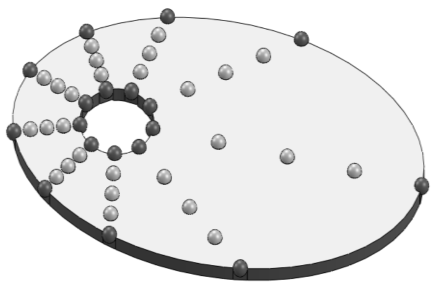

Fig.2
Grid points for elliptical plate including an eccentric circular hole.

4 NUMERICAL RESULTS AND DISCUSSIONS

For validation, the results of this study are compared with the results of the other available works. Tables (1-3) are provided for this purpose as the Ref. [5] was more relevant between available articles. It is observed that the obtained results have appropriate and have adequate accuracy compared to the former study. This indicates that using the semi-analytical method (SAPM) is fairly appropriate. The formulation is simple and at the same, the results will be achieved with the desired precision. Available methods for analysis the elliptic plates are complicated, however, in this paper with using cylindrical coordinate system instead of elliptic coordinates system and simultaneously using SAPM method significant results have been achieved. Therefore, using the governing equations in this study and solving the equations with SAPM method, that enabling the use of the nodal points with non-identical distances, is strongly recommended for subsequent works on elliptic plates. Time and computational complexity are extremely low in this method compared to other methods which it is one of the advantages of this method.

Table 1
Comparison of central dimensionless deflection between [5] and present paper with various geometries and thickness ratios.

$\tilde{q}$	Central deflection ($\tilde{w} = w / h$)					
	[5] (*h*/*b*)			Present paper (*h*/*b*)		
	0.01	0.1	0.25	0.01	0.1	0.25
6	0.4763	0.4837	0.5097	0.4725	0.4769	0.4969
12	0.8820	0.8960	0.9390	0.8741	0.8826	0.9127
18	1.2101	1.2301	1.2831	1.1980	1.2104	1.2446
24	1.4793	1.5047	1.5639	1.4630	1.4761	1.5138
30	1.7064	1.7370	1.8001	1.6859	1.7005	1.7371
36	1.9030	1.9386	2.0043	1.8763	1.8901	1.9281
42	2.0767	2.1170	2.1845	2.0414	2.0577	2.0949
48	2.2327	2.2776	2.3462	2.1880	2.2047	2.2476
54	2.3747	2.4239	2.4932	2.3177	2.3342	2.3810
60	2.5052	2.5586	2.6283	2.4300	2.4486	2.4995

Table 2
Comparison of $\tilde{w}$ between [5] and present paper with various foundation parameters and thickness to width ratios

$\tilde{q}$	Central deflection ($\tilde{w}=w/h$)					
	[5] $\bar{K},\bar{G}$ =20,1 (*h*/*b*)			Present paper $\bar{K},\bar{G}$ =20,1 (*h*/*b*)		
	0.01	0.1	0.25	0.01	0.1	0.25
8	0.3711	0.3716	0.3729	0.3678	0.3651	0.3625
16	0.7168	0.7188	0.7230	0.7138	0.7052	0.7001
24	1.0252	1.0301	1.0387	1.0146	1.0077	1.0030
32	1.2968	1.3053	1.3190	1.2811	1.2717	1.2703
40	1.5364	1.5492	1.5678	1.5145	1.5098	1.5078
48	1.7499	1.7671	1.7902	1.7110	1.7070	1.7158
56	1.9420	1.9639	1.9908	1.9028	1.8899	1.9005
64	2.1166	2.1432	2.1733	2.0662	2.0513	2.0684
72	2.2768	2.3081	2.3407	2.2156	2.1998	2.2144
80	2.4248	2.4607	2.4955	2.34810	2.3372	2.3503

Table 3
Comparison of [32] and present paper central deflection for various types of boundary conditions

Central deflection ($\tilde{w}=wD/pa^4$)			
[32]		Present paper	
Clamped	Simply-supported	Clamped	Simply-supported
9.8480×10^{-4}	1.0137×10^{-3}	9.8387×10^{-4}	0.9978×10^{-4}

The important parameters that can affect the deflection and the maximum stress of an elliptic plate are examined in this section. Some of the mentioned parameters could be considered as the ratio of major diameter to the minor diameter of the ellipse, the boundary conditions, circular hole location in the plate and the hole size. In addition, the effect of each of these factors will be examined. First, the effect of the ratio of major radius a to the minor radius b will be studied on the maximum deflection and stress of ellipse plate. An elliptic plate with the following characteristics is assumed:

$$b=1m, h=0.01m, r_i=0, q=10^3\,\text{N/m}^2, E=1.9\times10^{11}\,\text{N/m}^2, \nu=0.29 \tag{16}$$

Maximum deflection and Von-Misses stress variations due to the variation of the (*a*/*b*) ratio are shown in Figs. 3 and 4. If the plate is considered a full circle, *a* must be equal to b or (*a*/*b* =1). It is observed that increasing the (*a*/*b*) ratio, causes increasing in maximum deflection. Initially, the slope is steeply. In the following, for more than a certain ratio, here (*a*/*b* =2), the slope is dropped and gradually it is decreasing. Ultimately, by increasing the (*a*/*b*), there were no significant change in the deflection. Exactly the same conclusions can be obtained for the maximum stress and interesting thing is that the turning point of change reduction for the maximum deflection and the stress is the ratio (*a*/*b* =2). So, maximum deflection and stress can be predicted with acceptable accuracy for the high ratio of (*a*/*b*). Practically, when the (*a*/*b*) ratio increases, the symmetry of the problem will be dropped strongly and the errors in the analysis will be increased. Therefore, the use of Figs. 3 and 4 are very significant.

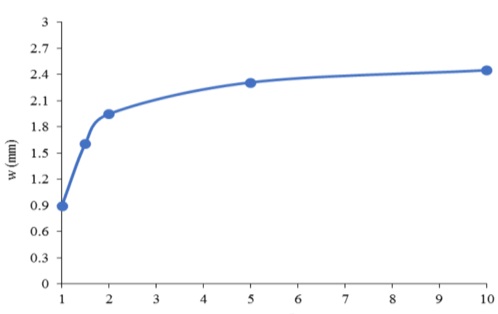

Fig.3
Maximum deflection for a solid elliptic plate due to the variation of *a*/*b* ratio.

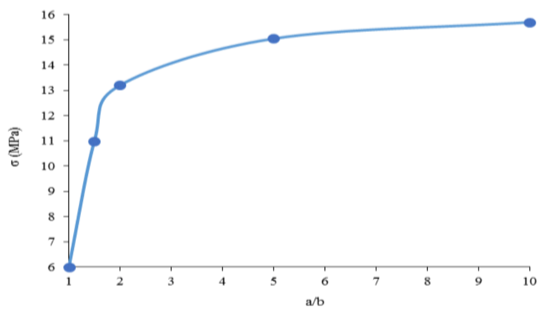

Fig.4
Maximum Von-Misses stress for a solid elliptic plate due to the variation of *a*/*b* ratio.

In present paper, with the aim to cover a wide range of issues, the effect of a circular hole that is eccentric with ellipse center is also analyzed. Nonlinear analysis of the mentioned issue will be discussed in this article for the first time and there is not any research on this subject. In continue, Figs. 5 and 6 show the maximum deflection changes and the stress to the effect of the eccentricity of the circular hole with a constant radius for different types of boundary conditions. Two different boundary conditions are considered. The CC Boundary condition indicates that both plate edges inside the circular hole and the outside of the ellipse, are clamped and FC boundary condition, indicates the free edge inside the circular hole and the clamped edge at the outer of the ellipse. $e = 0$ demonstrates that the center of the circular hole and the ellipse are in conformity with each other and by increasing the value of e, the eccentricity of the circular hole is increased. The plate properties are expressed below:

$$a = 1.5m, b = 1m, h = 0.01m, r_i = 0.3, q = 10^3\,\text{N/m}^2, E = 1.9\times10^{11}\,\text{N/m}^2, \nu = 0.29 \tag{17}$$

Due to Fig. 5, with increasing e for CC boundary condition, the maximum deflection increases, and this process is almost linear. However, for the FC boundary condition, the deflection increased at first and then reduced. So, it can be assumed that there is not any considerable change is occurred. In other words, the eccentricity of the circular hole in the elliptical plate at FC boundary condition does not have significant effect on the maximum deflection. But for the CC boundary condition, the changes are dramatically that it cannot be ignored. Therefore, the boundary conditions can be remarkably effective in the analysis and according to Fig. 5, whatever the plate has more inflexible boundary conditions, the effects of eccentricity on the results of maximum deflection will be more. It can also be seen in Fig. 6 that by increase of e, the maximum stress grows, but this rising is particularly remarkable for CC boundary conditions than the FC. Here there is not any change for FC boundary condition such as deflection and gentle upward trend seen for the maximum stress. But these changes are very insensible and can be ignored.

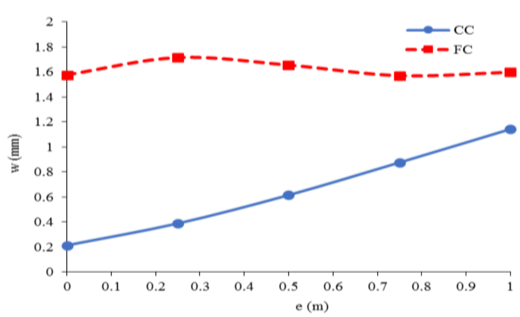

Fig.5
The effect of eccentricity of circular hole on maximum deflection.

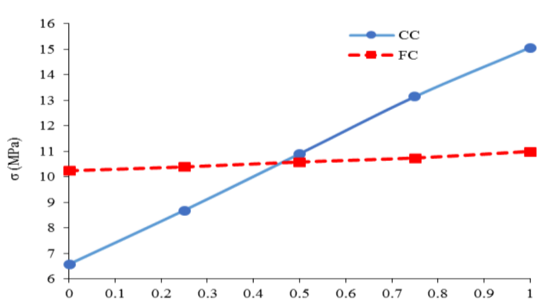

Fig.6
The effect of eccentricity of circular hole on maximum stress.

Another factor that affects the results is the size of the circular hole. Figs. 7 and 8 show the variations on the maximum deflection and the stress. According to Fig. 7, it can be seen that by increasing the radius of the circular hole, the plate area has been reduced and consequently, the maximum deflection is decreased. But this reduction for FC boundary condition is more than CC. By increasing the radius of the hole, the deflection in both CC and FC boundary conditions will be tend to each other. Fig. 8 shows the maximum stress. For the maximum stress, unlike the deflection (Fig. 7), by increasing the radius of hole, the maximum stress of two CC and FC boundary conditions will not approach and the changing process of stress reduction for both CC and FC boundary conditions is almost the same. However, here the maximum stress in the CC boundary condition is lower than FC boundary condition too. So, when there is a free edge in the plate, the stresses on the plate are higher and this issue should be considered in the design.

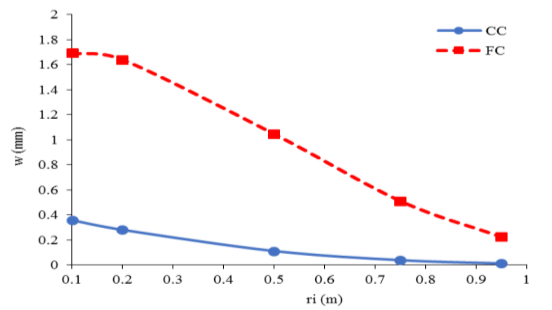

Fig.7
Variations of the maximum deflection due to the increase of the size of circular hole.

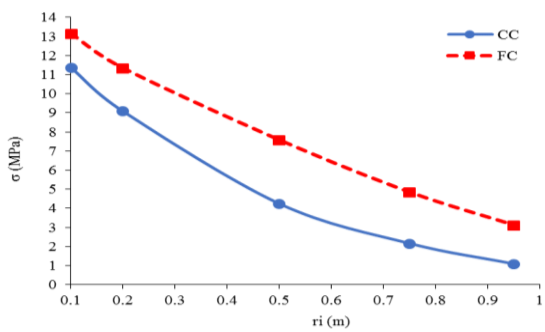

Fig.8
Variations of the maximum stress due to the increase of the size of circular hole.

The final factor which is considered and its impact is examined, is the influence of the eccentricity of the inner hole due to increase of the size of the hole. According to Fig. 9, it can be seen that the maximum deflection rises with growth of the e (as shown in Fig. 5 this conclusion was obtained before). Figs. 9 and 10 have been considered for CC boundary conditions. Gradient of the deflection heightened by increasing the radius of the circular hole and also with more eccentricity of the hole from the ellipse center. Therefore, whatever the hole is more eccentric; the impact of the size of the hole on the deflection is more. In Fig. 10 it can be observed that by increasing the size of circular hole, maximum stress reduced for both e locations and unlike Fig. 9 this decreasing is the same for both $e = 0$ and $e = 0.5a$ and there are insensible difference. As seen in Figs. 9 and 10, the size of the hole has an important role in the obtained results. Another factor that can increase the effect of the size of the hole is its location. For that reason, in this study, the eccentric hole is considered in order to be possible for obtaining a wide range of results by changing the position of the circular hole in the elliptic plate.

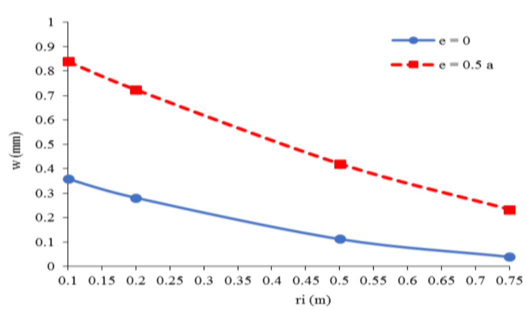

Fig.9
The maximum deflection due to the increase of the size of circular hole for different values of *e*.

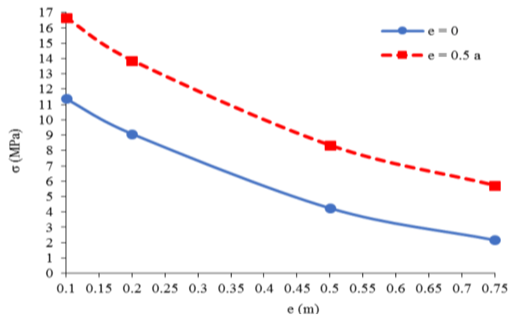

Fig.10
The maximum stress due to the increase of the size of circular hole for different values of *e*.

5 CONCLUSIONS

There is not any specific analytical method for nonlinear bending analysis of the elliptic plates with eccentric circular hole yet except finite element methods. In this study, a very simple formulation is presented for an elliptic plate with an eccentric circular hole in cylindrical coordinates system based on first-order shear deformation theory and the constitutive equations have been solved by using SAPM method. The obtained results have good agreement with the results of available studies. The factors that affecting the results, including the size of circular hole, hole location, and boundary conditions have been investigated in this paper. The significant conclusion is that although the proposed method is considerably simpler in formulations in comparison with the other available methods, however, has remarkable accuracy in the results. Basically, the analysis of elliptic plates with eccentric holes is an axisymmetric problem that cannot be studied with numerical or analytical methods and only the finite element methods should be used. Nevertheless, this study demonstrated that it is possible with presentation a semi-analytical method. Also, it is recommended for applying the presented method for future investigations such as vibration and buckling analyses. In addition, it could be possible to consider the nano-plate by exerting the small scale effects. So, many other researches can be done based on the formulations which has been proposed in this study. The summary of the results can be categorized as follows:

1. The results of the present study at the same simplicity in the formulation by using SAPM have a remarkable accuracy.
2. The ratio of major to minor diameters of the elliptic plates is very important and after a certain ratio, the changes in the results will not be dramatically.
3. Boundary conditions have a significant impact on the results. By increasing the radius of the circular hole, the deflection of the elliptic plate is reduced and this reduction is further for the FC boundary condition rather than the CC boundary condition.
4. By rising the eccentricity of the circular hole (*e*), the maximum deflection of the elliptic plate increases and this growth for CC boundary condition is more than FC boundary condition.

REFERENCES

[1] Dastjerdi Sh., Jabbarzadeh M., 2016, Non-local thermo-elastic buckling analysis of multi-layer annular/circular nano-plates based on first and third order shear deformation theories using DQ method, *Journal of Solid Mechanics* **8**(4): 859-874.

[2] Dastjerdi Sh., Jabbarzadeh M., 2016, Nonlocal bending analysis of bilayer annular/circular nano plates based on first order shear deformation theory, *Journal of Solid Mechanics* **8**(3): 645-661.

[3] Sato K., 2006, Bending of an elliptical plate on elastic foundation and under the combined action of lateral load and in-plane force, *III European Conference on Computational Mechanics*.

[4] Datta S., 1976, Large deflections of elliptic plates exhibiting rectilinear orthotropy and placed on elastic-foundation, *Journal of Applied Mechanics* **43**(4): 690-692.

[5] Kutlu A., Hakkı Omurtag M., 2012, Large deflection bending analysis of elliptic plates on orthotropic elastic foundation with mixed finite element method, *International Journal of Mechanical Sciences* **65**(1): 64-74.

[6] Zhong H., Li X., He Y., 2005, Static flexural analysis of elliptic Reissner–Mindlin plates on a Pasternak foundation by the triangular differential quadrature method, *Archive of Applied Mechanics* **74**(10): 679-691.

[7] Parnes R., 1989, Bending of simply-supported elliptic plates: B.P.M. solutions with second-order derivative boundary conditions, *Journal of Applied Mechanics* **56**(2): 356-363.

[8] Parnes R., 1988, A BPM solution for elliptical plates subjected to eccentric loads, *International Journal of Solids and Structures* **24**(8): 761-776.

[9] Wang Z.Q., Jiang J., Tang B.T., Zheng W., 2014, Numerical solution of bending problem for elliptical plate using differentiation matrix method based on Barycentric Lagrange interpolation, *Applied Mechanics and Materials* **638**: 1720-1724.

[10] Hsieh M.C., Hwu Ch., 2002, Bending of an anisotropic plate weakened by an elliptical hole, *The 3rd Asian-Australasian Conference on Composite Materials*, New Zealand.

[11] Leissa A.W., 1967, Vibration of a simply-supported elliptical plate, *Journal of Sound and Vibration* **6**(1): 145-148.

[12] Leissa A.W., 1969, *Vibration of Plates,* Washington, Office of Technology Utilization, SP-160, NASA.

[13] Leissa A.W., 1978, Recent research in plate vibrations: Classical theory, *The Shock and Vibration Digest* **9**: 13-24.

[14] Leissa A.W., 1978, Recent research in plate vibrations: Complicating effects, *The Shock and Vibration Digest* **9**: 21-35.

[15] Leissa A.W., 1981, Plate vibration research, 1976-1980: Classical theory, *The Shock and Vibration Digest* **13**: 11-22.

[16] Leissa A.W., 1981, Plate vibration research: Classical theory, *The Shock and Vibration Digest* **13**: 19-36.

[17] Leissa A.W., 1987, Recent studies in plate vibrations: part I, Classical theory, *The Shock and Vibration Digest* **19**: 11-18.

[18] Leissa A.W., 1987, Recent studies in plate vibrations: part II, Complicating effects, *The Shock and Vibration Digest* **19**: 10-24.

[19] Wang C.M., Wang L., Liew K.M., 1994, Vibration and buckling of super elliptical plates, *Journal of Sound and Vibration* **171**(3): 301-314.

[20] Altekin M., Altay G., 2008, Static analysis of point-supported super-elliptical plates, *Archive of Applied Mechanics* **78**(4): 259-266.

[21] Altekin M., 2008, Free linear vibration and buckling of super-elliptical plates resting on symmetrically distributed point-supports on the diagonals, *Thin-Walled Structures* **46**(10): 1066-1086.

[22] Altekin M., 2009, Free vibration of orthotropic super-elliptical plates on intermediate supports, *Nuclear Engineering and Design* **239**(6): 981-999.

[23] Altekin M., 2010, Bending of orthotropic super-elliptical plates on intermediate point supports, *Ocean Engineering* **37**(11): 1048-1060.

[24] Laura P.A., Rossit C., 1998, Thermal bending of thin, anisotropic, clamped elliptic plates, *Ocean Engineering* **26**(5): 485-488.

[25] Zhang D.G., 2013, Non-linear bending analysis of super elliptical thin plates, *International Journal of Non-Linear Mechanics* **55**: 180-185.

[26] Mc Nitt R.P., 1963, Free vibration of a damped semi-elliptical plate and a quarter-elliptical plate, *AIAA Journal* **29**(9): 1124-1125.

[27] Hasheminejad S.M., Vaezian S., 2014, Free vibration analysis of an elliptical plate with eccentric elliptical cut-outs, *Meccanica* **49**: 37-50.

[28] Irie T., Yamada G., 1979, Free vibration of an orthotropic elliptical plate with a similar hole, *Bulletin of JSME* **22**: 1456-1462.

[29] Ghaheri A., Keshmiri A., Taheri-Behrooz F., 2014, Buckling and vibration of symmetrically laminated composite elliptical plates on an elastic foundation subjected to uniform in-plane force, *Journal of Engineering Mechanics* **140** (7).

[30] Biswas P., 1975, Large deflection of a heated elliptical plate under stationary temperature, *Defense Science Journal* **26**: 41-46.

[31] Dastjerdi Sh., Lotfi M., Jabbarzadeh M., 2016, The effect of vacant defect on bending analysis of graphene sheets based on the Mindlin nonlocal elasticity theory, *Composites Part B* **98**: 78-87.

[32] Guminiak M., Szajek K., 2014, Static analysis of circular and elliptic plates resting on internal flexible supports by the boundary element method, *Journal of Applied Mathematics and Computational Mechanics* **13**(2): 21-32.