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Free Vibration of Functionally Graded Cylindrical Shell Panel With and Without a Cutout

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ABSTRACT

The free vibration analysis of the functionally graded cylindrical shell panels with and without cutout is carried out using the finite element method based on a higher-order shear deformation theory. A higher-order theory is used to properly account for transverse shear deformation. An eight noded degenerated isoparametric shell element with nine degrees of freedom at each node is considered. The stiffness and mass matrices are derived based on the principle of minimum potential energy. The stiffness and mass matrices of the element are evaluated by performing numerical integration using the Gaussian quadrature. The effect of volume fraction exponent on the fundamental natural frequency of simply supported and clamped functionally graded cylindrical shell panel without a cutout is studied for various aspect ratios and arc-length to thickness ratios. Results are presented for variation of the fundamental natural frequency of the cylindrical shell panel with cutout size for simply supported and clamped boundary conditions.

Keywords : Functionally graded materials; Free vibration; Finite element method; Higher-order shear deformation theory; Cylindrical shell panel with a cutout.

1 INTRODUCTION

RECENTLY, a new concept involving tailoring of the internal microstructure of the composite materials is developed to achieve the required function. The term Functionally Graded Materials (FGM) was coined by the Japanese researchers. These are heterogeneous in nature and thus the presence of different materials optimizes the responses of the structures undergoing severe loadings, reduces local stress concentrations, preservation of the structural strength and ductility. FGMs are developed for general use as structural components in extremely high temperature environments. FGMs in the form of shells find application in aerospace, automobile, medicine, sport, energy, sensors, defense, and optoelectronics. The free vibration of functionally graded shells is an important engineering problem. Many researchers studied the free vibration of functionally graded shells. Loy et al. [1] investigated the free vibration of simply supported FGM cylindrical shells. This is extended to cylindrical shells with different support conditions by Pradhan et al. [2]. The governing equations based on the classical shell theory are solved using Rayleigh-Ritz method. Yang and Shen [3] investigated the free vibration and parametric resonance of shear deformable functionally graded (FG) cylindrical shell panels. Patel et al. [4] carried out the free vibration

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analysis of functionally graded elliptical cylindrical shells using a higher-order theory where the analysis is carried out using finite element formulation based on a higher order theory. The influence of non-circularity, radius-thickness ratio, material profile index on free vibration frequencies and mode shape characteristics of shells are studied. Pradyumna and Bandyopadhyay [5] conducted the free vibration analysis of functionally graded curved panels using a higher-order finite element formulation. The effects of panel geometry parameters and boundary conditions are studied. Matsunaga [6] has studied free vibration and stability of functionally graded circular cylindrical shells according to a 2D higher-order deformation theory. Modal transverse shear and normal stress are calculated by integrating 3D equations of motion in thickness direction satisfying stress boundary conditions at outer and inner surfaces. Zhao et al. [7] have presented thermo elastic and vibration analysis of functionally graded cylindrical shells. In this study, they have analyzed the static response and free vibration of functionally graded shells using element free Kp-Ritz method with properties varying along the thickness direction. Comparisons reveal that numerical results agree well with classical and finite element methods. Tornabene and Erasmo [8] studied the free vibration of four parameters functionally graded parabolic panels and shells of revolution based on the first-order shear deformation theory. Numerical results presented include the influence of the parameters of the power-law distribution on the mechanical behavior of shell structures. Kiani et al. [9] presents thermoelastic free vibration and dynamic behavior of functionally graded doubly curved panels based on first-order shear deformation theory. Results are presented for spherical panels, cylindrical panels and hyperbolic paraboloidals. Qu et al. [10] presented the general formulation for free, steady-state and transient vibration analysis of functionally graded shells of revolution subjected to arbitrary boundary conditions. Numerical examples are given for the free vibration of functionally graded cylindrical, conical and spherical shells. Fadaee et al. [11] studied the free vibration analysis of Levy-type functionally graded spherical shell panel using a new exact closed-form solution. The strain displacement relations of Donnell and Sanders theories are used to obtain the exact solution. The effects of various stretching-bending couplings on the frequency parameters are discussed. Malekzadeh et al. [12] investigated the free vibration of functionally graded cylindrical shell panels with a cutout under thermal environment using the three-dimensional Chebyshev-Ritz method. The effects of volume fraction index, different types of temperature distributions through the thickness, the size of cutout and geometrical parameters on the free vibration are studied. Ebrahimi and Najafizadeh [13] presented free vibration analysis of 2-Dimensional functionally graded cylindrical shells using equations of motion based on Love's first approximation classical shell theory. Su et al. [14] presented a unified accurate solution for vibration analysis of arbitrary functionally graded spherical shell segments with general end restraints. The method is formulated by the Ritz procedure on the basis of the first-order shear deformation theory. The effects of the boundary conditions, power-law exponents, and shell segments on the free vibrations of the spherical shells are also investigated and parameter effects on frequency behaviors are illustrated. Ye et al. [15] studied the free vibration of laminated functionally graded spherical shells with general boundary conditions and arbitrary geometric parameters. The study is based on the 3D shell theory of elasticity and energy based Rayleigh-Ritz procedure. Numerous vibration results for several laminated FG spherical shells with various boundary conditions are presented for different geometric parameters and power-law exponents. Su et al. [16] presented the free vibration analysis of functionally graded open shells including cylindrical, conical and spherical ones with arbitrary subtended angle and general boundary conditions. The formulation is derived by the modified Fourier series in conjunction with Rayleigh-Ritz method according to the first-order shear deformation shell theory. Parametric studies are carried out for FGM open shells with respect to the boundary conditions, material profiles and geometrical parameters. Tornabene et al. [17] studied the free vibration of free-form doubly curved shells made of functionally graded materials using higher-order equivalent single layer theories. The natural frequencies and mode shapes of several structures are presented. Akbari et al. [18] considered free vibration analysis of functionally graded open conical panels based on first-order shear deformation theory. Parametric studies are conducted to study the influences of boundary conditions, semi-vertex angle, subtended angle , power law index and thickness to radius ratio on natural frequencies and associated mode shapes. Bahadori and Najafizadeh [19] investigated free vibration analysis of 2-Dimensional functionally graded axi-symmetric cylindrical shell on Winkler-Pasternak elastic foundation by first-order shear deformation theory and using Navier differential quadrature solution methods. This mainly focuses on dynamic behavior of moderately thick functionally graded cylindrical shell based on first-order shear deformation theory. Tornabene et al. [20] have presented numerical and exact models for free vibration analysis of cylindrical and spherical shell panels. The paper shows a comparison between classical 2D and 3D finite elements, classical and refined 2D generalized differential quadrature (GDQ) methods and an exact three dimensional solution. Xie et al. [21] studied the free vibration of four parameters functionally graded spherical and parabolic shells of revolution with arbitrary boundary conditions. The first-order shear deformation theory is adopted to account for the transverse shear effect and rotary inertia. Mirzaei and Kiani [22] studied the free vibration characteristics of functionally graded carbon nanotube reinforced composite cylindrical panels using first-order shear deformation theory. Numerical results are presented which show that natural frequencies of the panel are dependent on volume fraction of carbon nanotubes and their distribution through the thickness. Kiani [23] investigated the free vibration of carbon nanotube reinforced composite plate on point supports using Lagrangian multipliers. It is shown that volume fraction of carbon nanotubes and graded pattern influence the vibration characteristics of plates. Kiani [24] studied the free vibration characteristics of carbon nanotube reinforced composite spherical panels based on first-order shear deformation theory. Numerical results are presented which reveal the influence of boundary conditions, geometric parameters of the panel, volume fraction and graded pattern of carbon nanotubes.

From the review of literature, it is obvious that most of the researchers considered the free vibration of functionally graded cylindrical and spherical shells without a cutout except by Malekzadeh et al. [12] who considered the free vibration of cylindrical shell panels with a cutout. In their paper, results are presented only for clamped functionally graded cylindrical shell panels with a cutout subjected to temperature distribution through the thickness. Hence, the objective of this investigation is to analyze the free vibration analysis of cylindrical shell panel with and without cutout for simply supported and clamped boundary conditions using the finite element method based on a higher-order shear deformation theory. A higher-order theory is used to properly account for transverse shear deformation. An eight noded degenerated isoparametric shell element is used with nine degrees of freedom at each node. The effect of cutout size on the fundamental natural frequencies of functionally graded cylindrical shell panel is studied for simply supported and clamped boundary conditions.

2 GOVERNING EQUATIONS

Consider a functionally graded shell panel of uniform thickness as shown in Figs. 1and 2. The displacements along the local coordinate axes *x, y* and *z* at any point in the shell are assumed as:

$$u=u_0+z\,\theta_y+z^2u_0^*+z^3\theta_y^* \qquad v=v_0-z\,\theta_x+z^2v_0^*-z^3\theta_x^* \qquad w=w_0 \tag{1}$$

The strains along the local coordinate axes *x,y* and *z* are given by

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x}=\varepsilon_{x0}+zK_x+z^2\varepsilon_{x0}^*+z^3K_x^*\\
\varepsilon_y &= \frac{\partial v}{\partial y}=\varepsilon_{y0}+zK_y+z^2\varepsilon_{y0}^*+z^3K_y^*\\
\gamma_{xy} &= \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\gamma_{xy0}+zK_{xy}+z^2\gamma_{xy0}^*+z^3K_{xy}^*\\
\gamma_{xz} &= \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=\phi_x+z\,\gamma_{xz0}^*+z^2\phi_x^*\\
\gamma_{yz} &= \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}=\phi_y+z\,\gamma_{yz0}^*+z^2\phi_y^*
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
&\varepsilon_{x0}=\frac{\partial u_0}{\partial x},\varepsilon_{y0}=\frac{\partial v_0}{\partial y},\gamma_{xy0}=\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x},K_x=\frac{\partial \theta_y}{\partial x},K_y=-\frac{\partial \theta_x}{\partial y},K_{xy}=\frac{\partial \theta_y}{\partial y}-\frac{\partial \theta_x}{\partial x},\\
&\varepsilon_{x0}^*=\frac{\partial u_0^*}{\partial x},\varepsilon_{y0}^*=\frac{\partial v_0^*}{\partial y},\gamma_{xy0}^*=\frac{\partial u_0^*}{\partial y}+\frac{\partial v_0^*}{\partial x},K_x^*=\frac{\partial \theta_y{}^*}{\partial x},K_y^*=-\frac{\partial \theta_x^*}{\partial y},K_{xy}^*=\frac{\partial \theta_y^*}{\partial y}-\frac{\partial \theta_x^*}{\partial x},\\
&\phi_x=\theta_y+\frac{\partial w_0}{\partial x},\phi_y=-\theta_x+\frac{\partial w_0}{\partial y},\gamma_{xz0}^*=2u_0^*,\gamma_{yz0}^*=2v_0^*,\phi_x^*=3\theta,\phi_y^*=-3\theta_x^*
\end{aligned} \tag{3}$$

The stress-strain relations of a functionally graded shell at a distance *z* from the mid-surface with respect to *x, y* and *z* axes are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

or

$$\{\sigma\} = \left[Q_{ij}\right]\{\varepsilon\} \qquad (i, j = 1,2,6) \tag{4}$$

$$\begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

or

$$\{\tau\} = \left[Q_{ij}\right]\{\gamma\} \qquad (i, j = 4,5) \tag{5}$$

where

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}$$
$$Q_{12} = \nu Q_{11}$$
$$Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1+\nu)}$$
$$E = (E_c - E_m)V_f + E_m$$
$$\nu = (\nu_c - \nu_m)V_f + \nu_m$$
$$\rho = (\rho_c - \rho_m)V_f + \rho_m$$
$$V_f = \left(\frac{z}{t} + \frac{1}{2}\right)^n$$

The constitutive relationship for the shell may be written as:

$$\{F\} = [D]\{\chi\}$$
$$\{F\} = \left\{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, N_x^*, N_y^*, N_{xy}^*, M_x^*, M_y^*, M_{xy}^*, Q_x, Q_y, S_x, S_y, Q_x^*, Q_y^*\right\}^T$$
$$\{\chi\} = \left\{\varepsilon_{x0}, \varepsilon_{y0}, \gamma_{xy0}, K_x, K_y, K_{xy}, \varepsilon_{x0}^*, \varepsilon_{y0}^*, \gamma_{xy0}^*, K_x^*, K_y^*, K_{xy}^*, \phi_x, \phi_y, \gamma_{xz}^*, \gamma_{yz}^*, \phi_x^*, \phi_y^*\right\}^T$$
$$[D] = \begin{bmatrix} [A_{ij}][B_{ij}][C_{ij}][D_{ij}] & [0] & [0] & [0] \\ [B_{ij}][C_{ij}][D_{ij}][E_{ij}] & [0] & [0] & [0] \\ [C_{ij}][D_{ij}][E_{ij}][F_{ij}] & [0] & [0] & [0] \\ [D_{ij}][E_{ij}][F_{ij}][G_{ij}] & [0] & [0] & [0] \\ [0]\;[0]\;[0]\;[0] & [A_{pq}] & [B_{pq}] & [C_{pq}] \\ [0]\;[0]\;[0]\;[0] & [B_{pq}] & [C_{pq}] & [D_{pq}] \\ [0]\;[0]\;[0]\;[0] & [C_{pq}] & [D_{pq}] & [E_{pq}] \end{bmatrix} \tag{6}$$

where

$$\begin{bmatrix} N_x & M_x & N_x^* & M_x^* \\ N_y & M_y & N_y^* & M_y^* \\ N_{xy} & M_{xy} & N_{xy}^* & M_{xy}^* \end{bmatrix} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \left[1 \quad z \quad z^2 \quad z^3\right] dz \tag{7}$$

$$\begin{bmatrix} Q_x & S_x & Q_x^* \\ Q_y & S_y & Q_y^* \end{bmatrix} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} \begin{bmatrix} 1 & z & z^2 \end{bmatrix} dz \tag{8}$$

$$\left(A_{ij}, B_{ij}, C_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij} \right) = \int_{-\frac{t}{2}}^{\frac{t}{2}} \left[Q_{ij} \right] \left(1, z, z^2, z^3, z^4, z^5, z^6 \right) dz \qquad (i, j = 1,2,6)$$

$$\left(A_{pq}, B_{pq}, C_{pq}, D_{pq}, E_{pq} \right) = \int_{-\frac{t}{2}}^{\frac{t}{2}} \left[Q_{ij} \right] \left(1, z, z^2, z^3, z^4 \right) dz \qquad (p, q = 4,5)$$

3 FINITE ELEMENT FORMULATION

An eight noded degenerated isoparametric shell element [25, 26] with nine degrees of freedom at each node is considered in the present analysis. The stiffness matrix and mass matrix of the element are derived using the principle of minimum potential energy. The geometry of the element is defined by the global co-ordinates *X, Y* and *Z*.

$$X = \sum_{i=1}^{8} N_i X_i, Y = \sum_{i=1}^{8} N_i Y_i, Z = \sum_{i=1}^{8} N_i Z_i \tag{9}$$

where N_i are the shape functions of the element. The displacements of any point in the element are expressed as:

$$\begin{aligned}
U_0 &= \sum_{i=1}^{8} N_i \left(l_1 U_{0i} + m_1 V_{0i} + n_1 W_{0i} \right) \\
V_0 &= \sum_{i=1}^{8} N_i \left(l_2 U_{0i} + m_2 V_{0i} + n_2 W_{0i} \right) \\
W_0 &= \sum_{i=1}^{8} N_i \left(l_3 U_{0i} + m_3 V_{0i} + n_3 W_{0i} \right) \\
\theta_x &= \sum_{i=1}^{8} N_i \left(E_{11i} \theta_{xi} + E_{12i} \theta_{yi} \right) \\
\theta_y &= \sum_{i=1}^{8} N_i \left(E_{21i} \theta_{xi} + E_{22i} \theta_{yi} \right) \\
U_0^* &= \sum_{i=1}^{8} N_i \left(l_1 U_{0i}^* + m_1 V_{0i}^* \right) \\
V_0^* &= \sum_{i=1}^{8} N_i \left(l_2 U_{0i}^* + m_2 V_{0i}^* \right) \\
\theta_x^* &= \sum_{i=1}^{8} N_i \left(E_{11i} \theta_{xi}^* + E_{12i} \theta_{yi}^* \right) \\
\theta_y^* &= \sum_{i=1}^{8} N_i \left(E_{21i} \theta_{xi}^* + E_{22i} \theta_{yi}^* \right)
\end{aligned} \tag{10}$$

where

$$E_{11i} = l_1 l_{1i} + m_1 m_{1i} + n_1 n_{1i},$$
$$E_{12i} = l_1 l_{2i} + m_1 m_{2i} + n_1 n_{2i},$$
$$E_{21i} = l_2 l_{1i} + m_2 m_{1i} + n_2 n_{1i},$$
$$E_{22i} = l_2 l_{2i} + m_2 m_{2i} + n_2 n_{2i}.$$

3.1 Element stiffness matrix

The shell strains of the element may be expressed in a matrix form

$$\{\chi\} = [B]\{\delta^e\} \tag{11}$$

where $[B]$ is known as strain – displacement matrix and

$$\{\delta_e\} = \{U_{01}, V_{01}, W_{01}, \theta_{x1}, \theta_{y1}, U_{01}^*, V_{01}^*, \theta_{x1}^*, \theta_{y1}^* ...... U_{08}, V_{08}, W_{08}, \theta_{x8}, \theta_{y8}, U_{08}^*, V_{08}^*, \theta_{x8}^*, \theta_{y8}^*\}^T$$

The non-zero coefficients of $[B]$ are available in ref.[27]. The element stiffness matrix is given by

$$\left[K^e\right] = \int_{-1}^{1}\int_{-1}^{1} [B]^T [D][B]|J| d\xi d\eta \tag{12}$$

where $|J|$ is the determinant of the Jacobian matrix

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

In which

$$\frac{\partial x}{\partial \xi} = l_1 \frac{\partial X}{\partial \xi} + m_1 \frac{\partial Y}{\partial \xi} + n_1 \frac{\partial Z}{\partial \xi},$$
$$\frac{\partial y}{\partial \xi} = l_2 \frac{\partial X}{\partial \xi} + m_2 \frac{\partial Y}{\partial \xi} + n_2 \frac{\partial Z}{\partial \xi},$$
$$\frac{\partial x}{\partial \eta} = l_1 \frac{\partial X}{\partial \eta} + m_1 \frac{\partial Y}{\partial \eta} + n_1 \frac{\partial Z}{\partial \eta},$$
$$\frac{\partial y}{\partial \xi} = l_2 \frac{\partial X}{\partial \eta} + m_2 \frac{\partial Y}{\partial \eta} + n_2 \frac{\partial Z}{\partial \eta}.$$

The element stiffness matrix is evaluated using 2 × 2 reduced integration technique of Gauss Quadrature.

3.2 Element mass matrix

The element mass matrix is given by

$$\left[M^e\right] = \int_{-1}^{1}\int_{-1}^{1}[N]^T\,[P][N]\left|J\right| d\,\xi d\,\eta. \tag{13}$$

The matrices $[N]$ and $[P]$ are given by

$$[N] = \sum_{i=1}^{8}\begin{bmatrix} l_1N_i & m_1N_i & n_1N_i & 0 & 0 & 0 & 0 & 0 & 0 \\ l_2N_i & m_2N_i & n_2N_i & 0 & 0 & 0 & 0 & 0 & 0 \\ l_1N_i & m_1N_i & n_1N_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{11i}N_i & E_{12i}N_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{21i}N_i & E_{22i}N_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & l_1N_i & m_1N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & l_2N_i & m_2N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_{11i}N_i & E_{12i}N_i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_{21i}N_i & E_{22i}N_i \end{bmatrix}$$

$$[P] = \begin{bmatrix} I_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_4 \end{bmatrix}$$

where

$$\left(I_1, I_2, I_3, I_4\right) = \int_{-t/2}^{t/2} \rho(1, z^2, z^4, z^6)dz$$

The element stiffness matrix is evaluated using 3×3 reduced integration technique of Gauss Quadrature.

3.3 Solution process

The element stiffness and mass matrices are assembled to obtain the respective global matrices $[K]$ and $[M]$. The natural frequencies are obtained from the condition

$$\left|[K]-[M]\right| = 0 \tag{14}$$

This is a generalized eigen value problem and is solved by using the subspace iteration method [28].

4 RESULTS AND DISCUSSION

The analysis presented in the previous sections is applicable for the free vibration of various types of functionally graded shells. In the present investigation, free vibration of functionally graded cylindrical shell panel with and without a cutout is studied. The results are presented for functionally graded cylindrical shell panel with and without a cutout ($R_X / a = 5$) for simply supported and clamped boundary conditions. The following boundary conditions are used in the present investigation.

Simply supported:

$U_{0i} = V_{0i} = W_{0i} = \theta_{Xi} = U_{0i}^* = V_{0i}^* = \theta_{Xi}^* = 0$ along edges AB and CD (Fig.1)

$U_{0i} = V_{0i} = W_{0i} = \theta_{yi} = U_{0i}^* = V_{0i}^* = \theta_{yi}^* = 0$ along edges BC and AD (Fig.1)

Clamped:

$U_{0i} = V_{0i} = W_{0i} = \theta_{Xi} = \theta_{yi} = U_{0i}^* = V_{0i}^* = \theta_{Xi} = \theta_{yi}^* = 0$ along edges AB,BC,CD,AD (Fig.1)

The following material properties are used in the investigation.

$$E_m = 70GPa,\ \nu_m = 0.3,\ \rho_m = 2707Kg / m^3,\ E_c = 151GPa,\ \nu_c = 0.3,\ \rho_c = 3000Kg / m^3$$

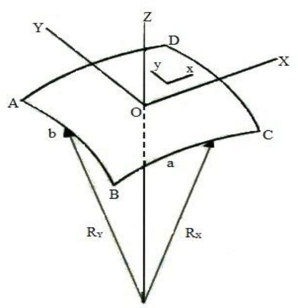

Fig.1
Shell geometry.

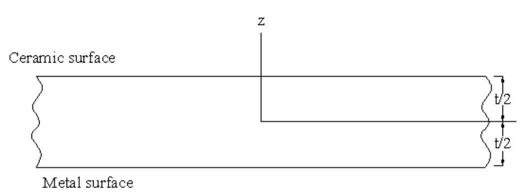

Fig.2
Material variation through thickness.

Convergence study has been conducted and the results are shown in Table 1. In the present investigation, a discretization consisting of 60 elements (Fig.3) is adopted. The accuracy of the present finite element analysis is verified by considering the following problems.

Free vibration of functionally graded cylindrical shell panel without a cutout for clamped boundary condition (Fig.4). The following material properties are considered for this problem.

$$E_c = 322.2715GPa,\ E_m = 207.7877GPa,\ \nu_c = 0.24,\ \nu_m = 0.31776,\ \rho_c = 2370Kg / m^3,\ \rho_m = 8166Kg / m^3$$

The non-dimensional fundamental frequencies obtained using the present finite element analysis are compared (Table 2) with those in reference [5].

Free vibration of functionally graded cylindrical shell panel with a cutout for clamped boundary condition (Fig.3). The following material properties are considered for this problem.

$$E_c = 322.2715GPa,\ E_m = 207.7877GPa,\ \nu_c = 0.24,\ \nu_m = 0.31776,\ \rho_c = 2370Kg / m^3,\ \rho_m = 8166Kg / m^3$$

The non-dimensional fundamental frequencies obtained using the present finite element analysis are compared (Table 3) with those in reference [12].

Free vibration of simply supported isotropic homogeneous square plate with a square cut-out. The natural frequency parameter obtained using present FEM (Tables 4 and 5)) is compared with that available in literature.

Free vibration of clamped isotropic homogeneous plate with a square cut-out. The fundamental natural frequency parameter obtained using present FEM (Table 6) is compared with that available in literature.

Free vibration of CSCS rectangular isotropic homogeneous plate with a rectangular cut-out. The fundamental natural frequency parameter obtained using present FEM (Table 7) is compared with that available in literature.

From the results presented in Tables 2-7, it is evident that the present finite element analysis is accurate and reliable.

Table 1
Convergence study: Non-dimensional fundamental natural frequency $\omega_n' = \omega_n a^2 \sqrt{\rho_m t / D_m}$ of cylindrical shell panel with a cutout (n=2). $D_m = E_m t^3 / 12(1-\nu_m^2)$

Number of elements	ω_n'
32	78.460
48	77.256
60	76.785
80	76.696

Table 2
Comparison of non-dimensional frequency parameter $\omega_n' = \omega_n a^2 \sqrt{\rho_m t / D_m}$ for a clamped functionally graded cylindrical shell panel. $D_m = E_m t^3 / 12(1-\nu_m^2)$

Source	$n = 0$	$n = 0.2$	$n = 2.0$	$n = 10.0$
Present FEM	74.2803	60.458	40.4876	35.0498
Reference [5]	74.518	57.479	40.750	35.852

Table 3
Comparison of non-dimensional fundamental frequency $\omega_n' = \omega_n a^2 \sqrt{\rho_c t / D_c}$ for clamped cylindrical shell panel with a cutout (*R/a*=5,*a/b*=1, *n*=0). $D_c = E_c t^3 / 12(1-\nu_c^2)$

Cutout to panel ratio (*e/a* or *f/b*)	Present FEM	Reference [12]
0.1	33.886	34.677
0.3	38.330	39.287
0.5	57.689	58.619

Table 4
Comparison of natural frequency parameter $\omega_n' = \omega_n a^2 \sqrt{\rho t / D}$ of simply supported isotropic homogeneous square plate with a square cutout. *a/t*=100 , ν=0.3, $D = Et^3 / 12(1-\nu^2)$

Source	*e/a*= *f/b*=0.3	*e/a*= *f/b*=0.5
Present FEM	19.6339	23.5856
Mirzaei and Kiani [29]	19.6490	23.5641
Liew et al. [30]	19.3910	23.4410
Lam et al. [31]	19.3570	23.2350

Table 5
Comparison of natural frequency parameter $\omega_n^{'} = \omega_n a^2 \sqrt{\rho\, t / D}$ of simply supported isotropic homogeneous square plate with a square cutout. a/t=100, ν=0.3, e/a=f/b = 0.4, $D = Et^3 / 12(1-\nu^2)$

Mode type	Source	$\omega_n^{'}$
SS	Present FEM	20.8919
	Mirzaei and Kiani [29]	20.9151
	Liew et al. [30]	20.7240
AS	Present FEM	41.4867
	Mirzaei and Kiani [29]	42.1561
	Liew et al. [30]	41.9070
AA	Present FEM	72.7916
	Mirzaei and Kiani [29]	71.9878
	Liew et al. [30]	71.4990

SS: Double symmetric AS: Anti-symmetric /symmetric AA: Double anti-symmetric

Table 6
Comparison of fundamental natural frequency parameter $\omega_n^{'} = \omega_n a^2 \sqrt{\rho\, t / D}$ of square clamped isotropic homogeneous plate with a square cutout. a/t=100 , ν=0.3, $D = Et^3 / 12(1-\nu^2)$

Source	*e/a= f/b*			
	0.1	0.2	0.3	0.4
Present FEM	36.5129	37.2569	41.1020	65.6090
Mirzaei and Kiani [29]	36.3141	37.2017	40.9624	65.3050
Malekzadeh et al. [12]	36.7943	37.9162	41.6279	66.5457
Mundkur et al. [32]	36.5045	38.1073	41.7912	65.7150

Table 7
Comparison of fundamental natural frequency parameter $\omega_n^{'} = \omega_n a^2 \sqrt{\rho\, t / D}$ of CSCS isotropic homogeneous rectangular plate with a rectangular cutout. a/t=100, ν=0.3, e/a=f/b= 1/3 , a/b=9/8 $D = Et^3 / 12(1-\nu^2)$

Source	$\omega_n^{'}$
Present FEM	34.3089
Mirzaei and Kiani [29]	31.2803
Liew et al. [30]	32.4250
Lam et al. [31]	34.04
Aksu et al. [33]	33.22[e], 33.83

e: From experiment

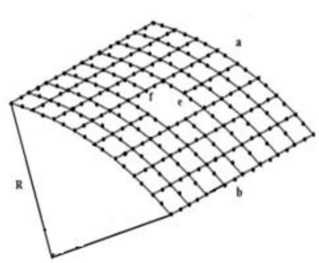

Fig.3
Cylindrical shell panel with a cutout.

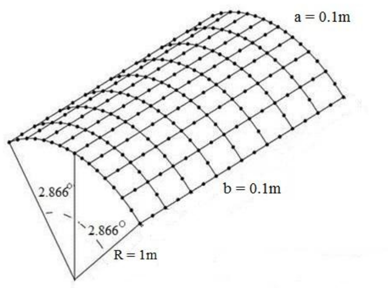

Fig.4
Cylindrical shell panel without a cutout.

The effect of volume fraction exponent on the fundamental natural frequency of simply supported and clamped cylindrical shell panels without a cutout is shown in Tables 8 and 9. Fig.5-10 show the variation of the fundamental natural frequency (ω_n^* / ω_n) of simply supported functionally graded cylindrical shell panel with cutout size for aspect ratios 1, 2, 0.5 and arc length to thickness ratios 10 and 100 and for values of volume fraction exponent 0, 0.2, 0.5, 1.0, 2.0, 5.0, 10.0. ω_n^* is the fundamental natural frequency with a cutout; ω_n is the fundamental natural frequency without a cutout. Fig.11-16 show the variation of the fundamental natural frequency (ω_n^* / ω_n) of clamped functionally graded cylindrical shell panel with cutout size for aspect ratios 1, 2, 0.5 and arc length to thickness ratios 10 and 100 and for values of volume fraction exponent 0, 0.2, 0.5, 1.0, 2.0, 5.0, 10.0. Typical mode shapes are presented in Figs.17 and 18. From the results presented in Tables 8 and 9 and Figs. 5-16, the following are observed.

The fundamental natural frequency of simply supported and clamped functionally graded cylindrical shell panels without a cutout decreases with the increase in the volume fraction exponent (Tables 8 and 9).

Table 8
Effect of volume fraction exponent '*n*' on the fundamental natural frequency (*Hz*) of simply supported functionally graded cylindrical shell panel without cutout.

(*a/b*)	(*a/t*)	*n*						
		0	0.2	0.5	1.0	2.0	5.0	10.0
1	10	1609.534	1537.922	1466.8113	1400.939	1342.1388	1282.0129	1243.3613
	100	216.7255	207.1075	197.6846	189.409	182.5507	175.8648	170.1958
2	10	2968.82	2840.345	2712.067	2591.119	2477.883	2356.766	2284.351
	100	432.0886	414.885	397.9604	382.6503	369.407	353.928	341.069
0.5	10	734.977	705.7786	677.245	651.210	627.953	600.5683	579.011
	100	108.959	103.977	99.0289	94.6222	91.2112	88.0855	85.4358

Table 9
Effect of volume fraction exponent '*n*' on the fundamental natural frequency (*Hz*) of clamped functionally graded cylindrical shell panel without cutout.

(*a/b*)	(*a/t*)	*n*						
		0	0.2	0.5	1.0	2.0	5.0	10.0
1	10	2090.397	1994.918	1897.1597	1804.506	1722.501	1644.789	1600.451
	100	288.201	273.939	259.869	247.702	238.6089	230.9478	224.592
2	10	4025.685	3850.88	3667.308	3484.774	3309.485	3136.088	3051.112
	100	646.82	614.925	583.612	556.1658	535.3598	517.4432	503.273
0.5	10	1018.937	969.533	920.382	876.232	841.074	810.1365	788.497
	100	130.736	124.31	117.9669	112.479	108.2054	104.685	101.8077

The fundamental natural frequency of simply supported functionally graded cylindrical shell panel with *a/b* = 1 and *a/t* = 10 decreases up to cutout size to panel size ratio 0.1. Thereafter, it increases with the increase in cutout size to panel size ratio for various values of volume fraction exponent (Fig. 5).

The fundamental natural frequency of simply supported functionally graded cylindrical shell panel with *a/b* = 1 and *a/t* = 100 decreases with the increase in cutout size to panel size ratio up to 0.5 and then it increases with the increase in cutout size to panel size ratio for various values of volume fraction exponent (Fig.6).

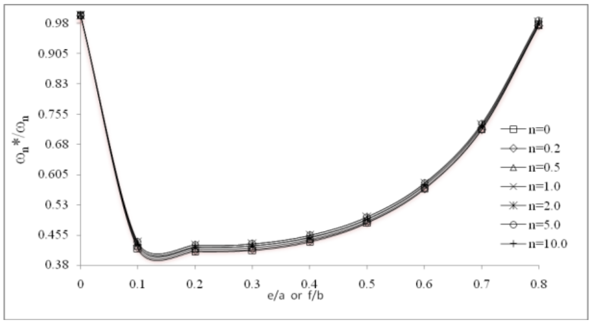

Fig.5
Variation of fundamental natural frequency of simply supported functionally graded cylindrical shell panel (*a/b* = 1, *a/t* = 10).

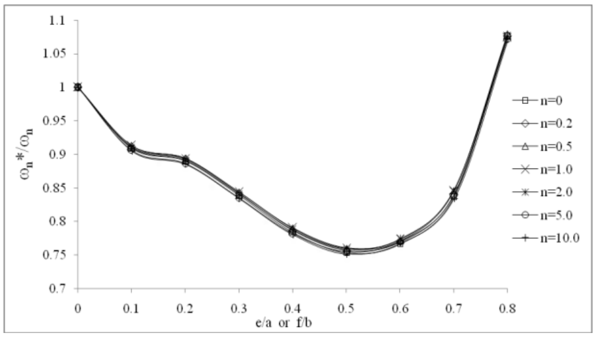

Fig.6
Variation of fundamental natural frequency of simply supported functionally graded cylindrical shell panel (*a/b* = 1, *a/t* = 100).

The fundamental natural frequency of simply supported functionally graded cylindrical shell panel with *a/b* = 2, 0.5 and *a/t* = 10, 100 generally decreases up to cutout size to panel size ratio 0.3 and then it increases with the increase in cutout size to panel size ratio for various values of volume fraction exponent (Figs.7-9).

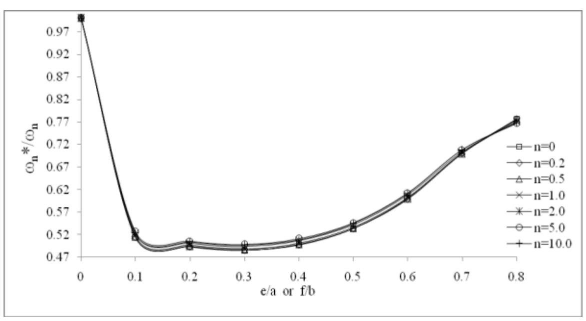

Fig.7
Variation of fundamental natural frequency of simply supported functionally graded cylindrical shell panel (*a/b* = 2, *a/t* = 10).

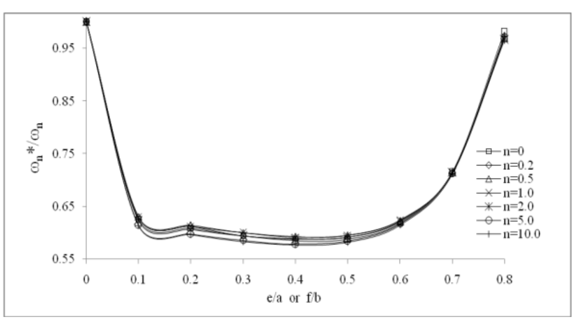

Fig.8
Variation of fundamental natural frequency of simply supported functionally graded cylindrical shell panel (*a/b* = 2, *a/t* = 100).

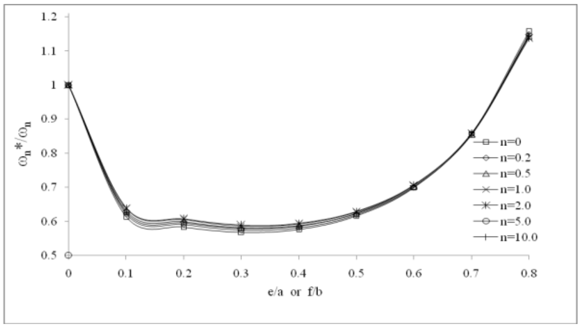

Fig.9
Variation of fundamental natural frequency of simply supported functionally graded cylindrical shell panel (a/b = 0.5, a/t = 10).

The fundamental natural frequency of simply supported functionally graded cylindrical shell panel with a/b = 0.5 and a/t = 100 increases up to cutout size to panel size ratio about 0.1. Thereafter, it decreases with the increase in cutout size to panel size ratio about 0.55. It increases again with further increase in cutout size to panel size ratio for various values of volume fraction exponent (Fig. 10).

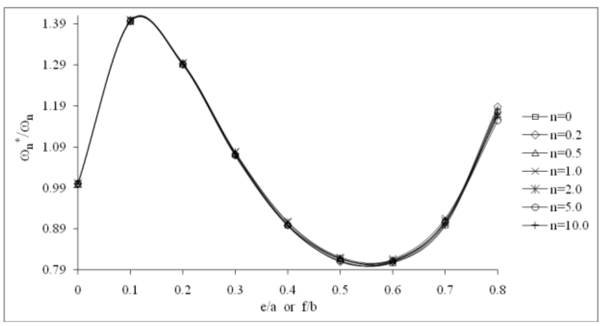

Fig.10
Variation of fundamental natural frequency of simply supported functionally graded cylindrical shell panel (a/b = 0.5, a/t = 100).

The fundamental natural frequency of clamped functionally graded cylindrical shell panel with a/b = 1, 2, 0.5 and a/t = 10, 100 decreases up to cutout size to panel size ratio 0.1 and then increases with the increase in cutout size to panel size ratio for various values of volume fraction exponent (Figs. 11-15) except for a/b=0.5 and a/t=100.

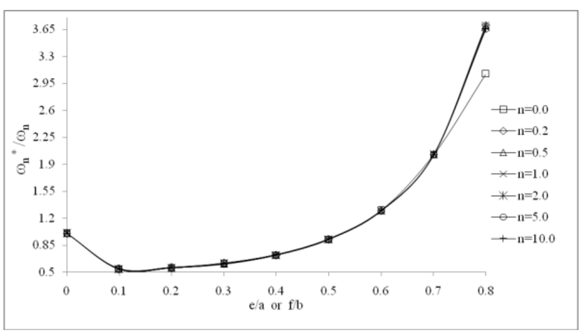

Fig.11
Variation of fundamental natural frequency of clamped functionally graded cylindrical shell panel (a/b=1, a/t = 10).

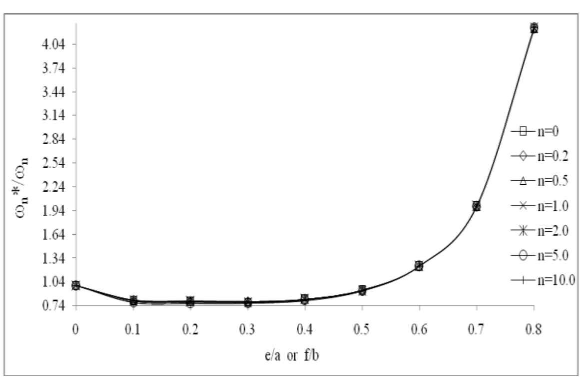

Fig.12
Variation of fundamental natural frequency of clamped functionally graded cylindrical shell panel (a/b=1, a/t = 100).

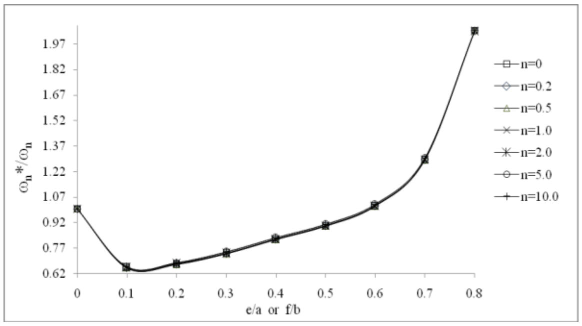

Fig.13
Variation of fundamental natural frequency of clamped functionally graded cylindrical shell panel (*a/b*=2, *a/t* = 10).

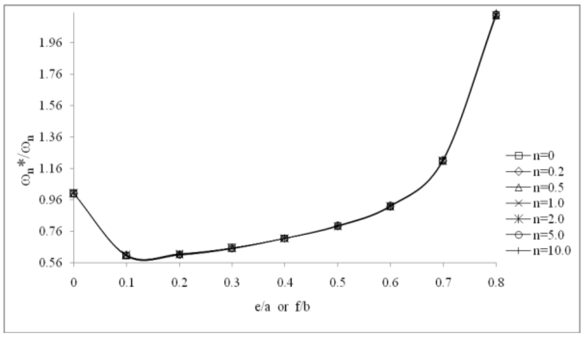

Fig.14
Variation of fundamental natural frequency of clamped functionally graded cylindrical shell panel (*a/b*=2, *a/t* = 100).

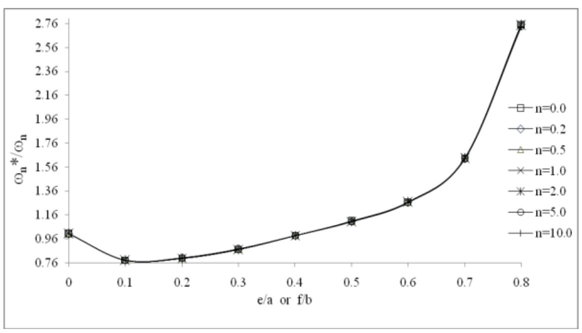

Fig.15
Variation of fundamental natural frequency of clamped functionally graded cylindrical shell panel (*a/b*=0.5, *a/t* = 10).

The fundamental natural frequency of clamped functionally graded cylindrical shell panel with *a/b* = 0.5 and *a/t* = 100 increases up to cutout size to panel size ratio about 0.1. Thereafter, it decreases with the increases in cutout size to panel size ratio about 0.35. It increases again with further increase in cutout size to panel size ratio for various values of volume fraction exponent (Fig. 16).

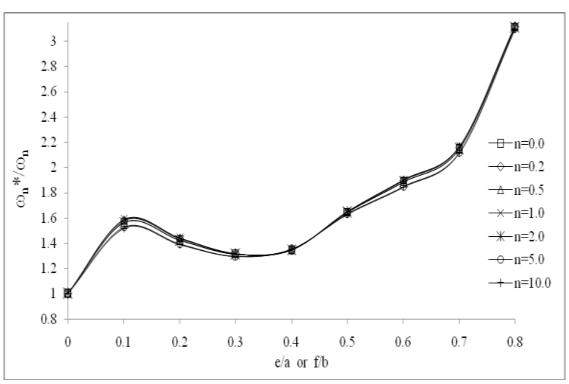

Fig.16
Variation of fundamental natural frequency of clamped functionally graded cylindrical shell panel (*a/b*=0.5, *a/t* = 100).

5 CONCLUSIONS

In this investigation, the free vibration of functionally graded cylindrical shell panel with and without a cutout is studied using finite element method based on a higher-order shear deformation theory. An eight-noded degenerated isoparametric shell element with nine degrees of freedom at each node is considered. Results are presented for fundamental natural frequencies of the cylindrical shell panel with and without a cutout for simply supported and clamped boundary conditions. From the results presented, the following conclusions can be made.

1) The fundamental natural frequency of simply supported and clamped functionally graded cylindrical shell panels without a cutout decreases with increase in volume fraction exponent for various values of aspect ratio, arc-length to thickness ratio.
2) The fundamental natural frequency of simply supported and clamped functionally graded cylindrical shell panel generally decreases initially and then increases with the increase in cutout size for various values of volume fraction exponent except for the case a/b=0.5 and a/t=100.
3) In the case of simply supported and clamped functionally graded cylindrical shell panels with a/b=0.5 and a/t=100, the fundamental natural frequency increases initially, then decreases and then increases with the increase in cutout size for various values of volume fraction exponent.

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