Investigation of blood flow as third order non-Newtonian fluid inside a porous artery in the presence of a magnetic field by an analytical method

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Received: 26 September 2017    Accepted: 24 November 2017

Abstract: In this research various nonlinear fluid models have been introduced and the balloon movement in the porous arteries, including third-order non-Newtonian fluid, is described under the influence of the magnetic field. In order to solve the nonlinear equations governing the desired artery, an analytical method of approximation collocation and least squares are proposed. The effect of various parameters such as Brownian motion, thermophoresis and pressure gradient are shown on the distribution of velocity and fluid temperature. According to the studies, increase the thermophoresis parameter and Brownian motion are increases the temperature. Also, by increasing the values of the electromagnetic parameter, fluid velocity decreases. Although many papers have investigated the flow of intra-arterial blood in the presence of MHD, it is clear that the governing equations of these articles unfortunately reveal obvious errors by extracting the governing equations. In this paper, the equations have been revised and corrected. It can also be seen by increasing the thermophoresis and Brownian parameters, increases the temperature of the nanofluid, but reduces velocity of the nanofluid and also increase the porosity parameter, the velocity of fluid flow between the balloon and the vessel wall decreases.

Keywords: Non-Newtonian fluid; Porous artery; Magnetic field; Thermophoresis force.

1. Introduction
One of the most important causes of cardiovascular disease is blockage of the coronary arteries of the heart causes by the accumulation of cholesterol fat in the inner wall of the arteries and prevents blood flow. Gradual enlargement of the plaques increases the severity of the disease gradually. Coronary angioplasty or coronary artery bypass graft surgery is a method used to open the closed arteries of the heart. In angioplasty a small balloon is placed temporarily in the closed...
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the temperature decreases. As the number of
ix tract increases the thickness of the
magnetic boundary layer decreases. The
suction and magnetic parameters reduce the
frictional coefficient of the skin's wall.
Agboud and Saouli [11] developed an
entropy analysis for viscoelastic flow on an
elastic plate. Also the effects of magneto
metric parameters, springs and Prandtl
numbers on speed and temperature have been
investigated. The generated entropy number
increases with increasing Prandtl number,
magnetic parameter, surface temperature and
Reynolds number. Mamaloukas et al. [12]
studied the effects of the cross-magnetic
magnetic field on the Maxwell fluid model on
an elastic surface. To solve the equation the
fourth order Runge-Kutta method was used.
Examples of elastic parameter effects and
magnetism parameters on velocity. The
results show that by increasing the magneto
metric parameter, the velocity increases and
decreases with increasing elastic parameter.
Ellahi et al. [13] the effects of slip on
nonlinear currents of a third order non-ionic
fluid were investigated between two center
cylinders. It was concluded that by increasing
the atmospheric pressure gauge the velocity
slip increases the pressure and barometric
pressure of the pressure gradient but
decreases with increasing the third-order fluid
flow parameter. Hatami et al. [14] simulated
the transfer of gold nanoparticles in the blood
(third order non-genomic fluid) under the
influence of the electromagnetic field in the
porous veins. It is shown that the increase of
the thermophoresis parameter increases the
temperature profile, also decreases with the
increase of the electromagnetic parameter of
the fluid velocity profile.

According to investigations conducted by
the aforementioned researchers, In this
research, we try to consider the motion of an
intravenous balloon with blood clots using
the analytical method by considering the
blood as a third order non-Newtonian fluid
and also applying a magnetic field. Also, by
extraction of equations in the next section, it
has been determined that all articles [14-17]
have obvious errors in the definition of some
parameters. Unfortunately, some parameters
are mistakenly removed from the equations
that have been modified in the present paper.

2. Problem description and governing
equations
The flow of calm stable and incompressible
blood is considered in the bloodstream.
Blood is considered as a third-order non-
Newtonian nanofluid. Base fluid
nanoparticles are hydrodynamic and thermal
equilibrium. Also, nanoparticles have the
same size and shape. The geometry and
coordinate system of the problem are shown
in Fig. (1).
It is assumed that the balloon is moving in the vessel at a constant speed and the space between the balloon and the vessel is porous due to artery obstruction. The fluid is an electric conductor under the influence of a uniform magnetic field. The effects of thermal emission of nanoparticles and Brownian motion are also considered in the transfer equations. Given the above assumptions, the mass, momentum, and energy and concentration equations are as follows [15]:

\[ \rho_f \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \nabla \cdot \mathbf{T} - \frac{\mu \nabla T}{k} \left( 1 + \lambda \frac{\partial \mathbf{V}}{\partial t} \right) + \rho g + J \times \mathbf{B}_0 \]  

(1)

\[ (\rho c)_f \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = K \nabla^2 T + (\rho c)_p \left[ D_b \nabla \phi + \frac{D_T}{T_w} \nabla^2 T \right] \]  

(2)

\[ \left( \frac{\partial \phi}{\partial t} + \mathbf{V} \cdot \nabla \phi \right) = D_b \nabla^2 \phi + \frac{D_T}{T_w} \nabla^2 T \]  

(3)

In the above equations \( \mathbf{V} \) is the velocity vector and, \( T \) temperature, \( \phi \) Concentration of nanoparticles, \( \mathbf{T} \) Stress tensor, \( (\rho c)_p \) heat capacity of nanoparticles, \( (\rho c)_f \) Base fluid density, \( \rho = \phi \rho_p + (1-\phi) \rho_f [1 - \beta_t (T - T_w)] \]  

(4)

\( \mu \) The nanofluid viscosity is considered as a function of temperature according to the Vogel’s model [16]. This model has a higher accuracy than the fixed viscosity model:

\[ \mu = \mu_0 e^{\left( \frac{H - T_0}{T + \theta_0} \right)} \]  

(5)

\( B_0 \) Is the magnetic field and \( J \) is the current density that follows Ohm's law [19].

\[ J \times \mathbf{B} = -\sigma \mathbf{B}_0^2 \mathbf{V} \]  

(6)

That \( \sigma \) is the conductivity of the nanofluid. Here, it is assumed that the balloons with radius \( R \) at constant velocity
$V_0$ and the temperature $T_0$ in the vessel move with the radius $2R$ and the concentration of the nanoparticles is on the wall of the balloon $\phi_w$.

As a result, boundary conditions depend on the equations of velocity, temperature and concentration as follows:

$$ r = R \Rightarrow V = V_0, \quad T = T_0, \quad \phi = \phi_0 $$
$$ r = 2R \Rightarrow V = 0, \quad T = T_w, \quad \phi = \phi_w \quad (7) $$

The stress tensor for a third order fluid is defined as:

$$ \sigma = (\nabla V) + (\nabla V)' $$

That $-pI$ is part of the stress due to the constraints due to the incompressibility. $\mu$ is the shear adhesion constant and $a_1, a_2, b_3$ are the constants of the material. And $A_1, A_2$ tensors are defined as follows:

$$ \overline{V} = \frac{V}{V_0}, \overline{r} = \frac{r}{R}, \overline{\mu} = \frac{\mu}{\mu_0}, \overline{\vartheta} = \frac{T - T_w}{T_0 - T_w}, \overline{\phi} = \frac{\phi - \phi_w}{\phi_0 - \phi_w} \quad (10) $$

By substituting high dimensionless parameters in Equation (1) to (3) and simplify the governing equations are reduced:

$$ \frac{d\mu}{dr} + \frac{\mu}{r} \frac{dV}{dr} + \frac{A}{r} \frac{dV}{dr} + 3A \frac{dV}{dr} + 3A \frac{dV}{dr} + \frac{dV}{dr} = P\mu V + M\overline{V} + C - Br \overline{\phi} + Gr \overline{\theta} - G_i \overline{\phi} - B_i \quad (11) $$

$$ N_i \frac{\partial \theta}{\partial r} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{1}{r} \frac{\partial \theta}{\partial r} + N_i \left( \frac{\partial \theta}{\partial r} \right)^2 = 0 \quad (12) $$

$$ N_i \left( \frac{\partial \phi}{\partial r} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right) + N_i \left( \frac{\partial \theta}{\partial r} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) = 0 \quad (13) $$

In the above equations $P$ Porosity parameters, $M$ Magnetic parameters, $A$ third order fluid flow parameter, $C$ Pressure gradient parameter, $N_i$ thermophoresis Parameter, $N_o$ Brownian parameter, $Br$ Brownian diffusion parameters, $Gr$ Grashof number, And $B_i, G_i$ constant coefficients are given as follows:
\[ A = \frac{2\beta V_0^2}{\mu_0 R_0^2}, \quad C = \frac{(\partial \hat{P}) R^2}{V_0 \mu_0}, \quad P = \frac{\varphi R^2}{K}, \quad M_2 = \frac{G_0 B_i^2 R^2}{\mu_0} \]

\[ Br = \frac{(\rho_p - \rho_f) \mu_0 V_0}{R^2 (\phi_m - \phi_w) g}, \quad Gr = \frac{(\theta_m - \theta_w) \beta_f R^2 (1 - \phi_w) g}{\mu_0 V_0} \]

\[ Bt = \frac{\rho_f (1 - \phi_w) + \rho_f \phi_w}{\mu_0 V_0} \]

\[ Gt = \frac{\rho_f (\phi_m - \phi_w) R^2 (\theta_m - \theta_w) \beta_f g}{\mu_0 V_0} \]

The boundless boundary conditions for equations (11) to (13) are obtained as follows:

\[ r = 1 \Rightarrow V = V_0, \quad \theta = 1, \quad \phi = 1 \]
\[ r = 2 \Rightarrow V = 0, \quad \theta = 0, \quad \phi = 0 \quad (15) \]

It is clear from the extraction of equations that, unfortunately, in all papers [14-17], Fields \( N_t \) and \( N_b \) are incorrectly changed in equation (13) and sentences that affect the parameters of \( G_t \) and \( B_t \) are erroneously deleted in all of these papers And their effect is ignored. The main purpose of this paper is to solve this problem with the corrected governing equations, which is presented in the next section.

3. Collocation (CM) and Least Squares (LSM)

One of the approximate analytical methods for solving differential equations is the Weighted Residual Methods (WRMs). In the WRMs, the weight functions are those functions that are arbitrarily selected and used in this method. But the type of arbitrary functions that can be used can affect the solution. The WRMs is classified according to the type of weighting function used in different ways. These methods can be summarized as follows: Collocation method, Galerkin method, Least squares method. In this paper used the collocation and least squares method to solve the problem. Suppose we have the following differential equation:

\[ D(u(x)) = p(x) \quad (16) \]

The answer to this differential equation (the estimation function) is assumed to be a linear combination of arbitrary functions (base functions):

\[ u = \tilde{u} = \sum_{i=1}^{n} c_i \phi_i \quad (17) \]

The placement of the estimated function within the differential equation gives us the remainder or the error:

\[ E(x) = R(x) = D(\tilde{u}(x) - p(x)) \neq 0 \quad (18) \]

Now the Dirac Delta function is defined according to its effect on other functions as follows:

\[ \int_{-\infty}^{\infty} \delta(x) dx = 1 \quad (19) \]

The consequences of this definition are as follows:

\[ \int_{\Omega} \delta(x - \xi) f(x) dx = \begin{cases} f(\xi) & \xi \in \Omega \\ 0 & \xi \notin \Omega \end{cases} \quad (20) \]

In the collocation method we use the property defined for the Dirac delta function Equation (20). Thus, we consider the weighted functions used in Dirac's delta functions \( \delta(x - x_i) \) that \( x_i \) as arbitrary point
coordinates within the definition of the differential equation. It is obvious that the number of unknown coefficients in the estimation function is considered to be a point. In this case we will have (if the point is needed, in other words, the unknown coefficient in the function is estimated):

\[
\int_{\Omega} \delta(x - x_i) R(x) \, dx = 0
\]

\[
\int_{\Omega} \delta(x - x_2) R(x) \, dx = 0
\]

\[
\int_{\Omega} \delta(x - x_3) R(x) \, dx = 0
\]

\[
\vdots
\]

\[
\int_{\Omega} \delta(x - x_\text{n}) R(x) \, dx = 0
\]

In this way, there are three independent equations that can be used to obtain the unknown coefficients of the estimation function.

In some cases, we need to obtain the best and most optimal estimation function for the differential equation. To do this, we need to get the estimated function with the minimum error in the range. In this case, the integral above represents the sum of the sum of the remaining squares or the same errors within the definition of the differential equation. In order to obtain the optimal response, the coefficients of the base functions must be obtained in such a way that the sum of the squares of the errors is minimal. To do this, we deduce from the sum of the squares of errors the differential of each of the coefficients of the basic functions and make it equal to zero. In other words, we minimize the sum of squares of errors compared to the basic function coefficients. This method is known as the least squares (LSM) method.

\[
\frac{d}{dc_i} \int_{\Omega} R^2 \, dx = 0
\]  

(22)

4. Results

The nonlinear differential equations (11) to (13) with respect to boundary conditions (15) are analyzed analytically using the residual weight method and numerically using the Runge-Kutta method for different values of porosity parameters, magnet parameter, third order fluid parameter, pressure gradient parameter, thermophoresis parameter, Brownian parameter and Brownian diffusion parameter are solved. In order to validate the results of the present study, the results were first compared to the distribution of velocity, temperature and concentration of the collocation method with the analytical results obtained from reference [15] in Figures (2- 4).
In the case of considering all the parameters, one corresponds to all the graphs. Also Figures (5-7) show the comparison of collocation and least squares methods with numerical results. The numerical values of these methods and their errors are shown in tables (1) to (3) for different values. Given the shapes and tables, a good match is observed in all modes.
### Table 1.
**Comparison of CM and LSM with Numerical method for θ**

<table>
<thead>
<tr>
<th>r</th>
<th>CM</th>
<th>LSM</th>
<th>NUM</th>
<th>error CM</th>
<th>error LSM</th>
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<td>0.8655360185</td>
<td>0.8652995783</td>
<td>0.8654433988</td>
<td>9.26197E-05</td>
<td>0.00014328</td>
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<td>1.1</td>
<td>0.7418783125</td>
<td>0.7414742248</td>
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<td>0.000318218</td>
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<td>1.3</td>
<td>0.6274312719</td>
<td>0.6270291904</td>
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<td>7.40761E-05</td>
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</tr>
<tr>
<td>1.4</td>
<td>0.5208872313</td>
<td>0.5206458976</td>
<td>0.5208160218</td>
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</tr>
<tr>
<td>1.5</td>
<td>0.4211842544</td>
<td>0.4211819407</td>
<td>0.421153207</td>
<td>6.89337E-05</td>
<td>6.662E-05</td>
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<td>1.6</td>
<td>0.3274639244</td>
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<tr>
<td>1.7</td>
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<td>1.8</td>
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<td>1.9</td>
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<td>5.51372E-05</td>
<td>0.000111528</td>
</tr>
</tbody>
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### Table 2.
**Comparison of CM and LSM with Numerical method for φ**

<table>
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<tr>
<th>r</th>
<th>CM</th>
<th>LSM</th>
<th>NUM</th>
<th>error CM</th>
<th>error LSM</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.000134</td>
<td>0.0002</td>
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<td>1.3</td>
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### Table 3.
**Comparison of CM and LSM with Numerical method for V**

<table>
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<tr>
<th>r</th>
<th>CM</th>
<th>LSM</th>
<th>NUM</th>
<th>error CM</th>
<th>error LSM</th>
</tr>
</thead>
<tbody>
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<td>0.013612</td>
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<td>1.3</td>
<td>0.561966417</td>
<td>0.5757561022</td>
<td>0.5553183575</td>
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<td>0.020438</td>
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<td>1.4</td>
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<td>0.4406129683</td>
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<td>1.5</td>
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<td>0.0024</td>
<td>0.01676</td>
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</table>

2  0  0  0  0  0  0
Figs (8), (9) and (10) show the effect of the thermophoresis parameter on the distribution of temperature, concentration and velocity. It is observed that with increasing thermophoresis parameters the temperature increases. Also, according to Figs. (9) and (10), it is clearly observed that with the increase of the thermophoresis parameter, the concentration and velocity of nanofluid decreases.

The effect of Brownian motion on the temperature, concentration and velocity distribution is shown in Figures (11-13). By increasing the Brownian parameter, such as the thermophoresis parameter, the temperature of the nanofluid increases (Fig. 11) but as the parameter increases, the velocity decreases (Fig. 12) It is also seen in (Fig. 13) that the Brownian parameter has no effect on the concentration of the nanoparticle.
Figure (14-17) shows the effect of porosity, non-Newtonian fluid parameter, magnetic field and pressure gradient on the nanofluid velocity distribution. With increasing porosity and gradient of pressure, the velocity increases which is obvious but decreases with increasing magnetic field. Because this parameter generates Lorentz force and this friction creates friction against the direction of flow and reduces the velocity.
5. Discussion
The recent studies of [14-17] is improved in this paper considering the effects of the parameters of $Gt$ and $Bt$ on governing equation, thus, not restricted to the cases $Gt = 0$ and $Bt = 0$ of [14-17]. Also the results indicate that the temperature distribution is strongly depending on the thermophoresis and Brownian numbers.

Finally, it is worthy to mention that the analytical solutions presented in this paper may provide insight into the physical phenomena of the problem at hand and also they could be used as benchmarks or validation tests for the numerical schemes. these methods solve the equations directly and do not need any perturbation, linearization, simplifications or small parameter versus Homotopy Perturbation Method (HPM) and Parameter Perturbation Method (PPM).

6. Conclusion
In this research, by applying the method of collocation (CM) and least squares (LSM), with effect of different parameters on the distribution of velocity temperature and concentration of nanofluid in the porous artery in the presence of a magnetic field has been investigated. According to the studies, the following results are obtained:

1. With increasing thermophoresis parameters, increases the temperature of the nanofluid if the parameter increases, the velocity and concentration of the nanofluid are reduced.
2. With Increasing the Brownian parameter increases the temperature of the nanofluid but this parameter does not have any effect on concentration, but reduces velocity.
3. If the porosity parameter is increased, the flow velocity decreases between the balloon and the wall of the vein.
4. If the MHD parameter is increased, decreases the flow velocity between the balloon and the wall of the vein.
5. If the third-order non-Newtonian parameter is increased, increases the flow velocity between the balloon and the wall of the vein.
Nomenclature:

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Stress tensor</td>
<td>$T$</td>
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<tr>
<td>Velocity</td>
<td>$V$</td>
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<tr>
<td>Dimensionless number Brownian</td>
<td>$Br$</td>
</tr>
<tr>
<td>Viscosity constants</td>
<td>$H,Y$</td>
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<tr>
<td>Pressure gradient</td>
<td>$C$</td>
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<tr>
<td>Brownian diffusion coefficient</td>
<td>$D_b$</td>
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<td>Thermophoresis coefficient</td>
<td>$D_t$</td>
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<td>Electrical Field</td>
<td>$E$</td>
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<td>Acceleration of gravity</td>
<td>$g$</td>
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<td>Grashov number</td>
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<td>Electrical current density</td>
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<td>$R$</td>
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<td>Reference speed</td>
<td>$V_0$</td>
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<td>The modules of matter</td>
<td>$\alpha,\beta$</td>
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<tr>
<td>Coefficient of thermal expansion</td>
<td>$\beta_t$</td>
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<tr>
<td>Third order non-Newtonian parameter</td>
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<td>Volume fraction</td>
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<td>Temperature</td>
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<td>$Bi$</td>
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References

Investigation of blood flow as third order non-Newtonian fluid inside porous arteries in presence of magnetic field using analytical methods. Physica E: Low-dimensional Systems and Nanostructures 70 (2015) 146-156.


