Appendix II

<table>
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<th>MACHINE SPECIFICATIONS</th>
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<tr>
<td>Rated torque, Nm</td>
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<tr>
<td>Rated current, A</td>
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<tr>
<td>P. No. of pole pairs</td>
</tr>
<tr>
<td>(R_1, R_2, \Omega)</td>
</tr>
<tr>
<td>(L_{dc}, L_{eq}), mH</td>
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<tr>
<td>(\lambda_m), Wb</td>
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<tr>
<td>J. Rotor inertia constant, Kg.m(^2)</td>
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<td>B. Viscous coefficient, Nm/rad/sec.</td>
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Reference


The loss minimization control of IPM motor drives, introduced by Morimoto is simulated here for comparison [4]. Fig. (7) shows the speed response of both methods for two operating points. It can be seen that since the PI controllers of the Morimoto’s method are tuned in nominal operating point, dynamics of other operating points are not desirable. But in the case of SMC, a change of operating point does not affect the desirable performance of the motor drive.

In order to show the robustness of SMC against the parameter variations, another simulation is carried out where the motor parameters are assumed to vary over a certain pattern due to ambient and operating conditions. Efficiency and total electrical losses of the motor are shown in Figs. (11-12) for varying speed (at rated torque) and varying torque (at rated speed) respectively. In these figures, three different pairs (η and \( P_{\text{Loss}} \)) of plots are shown. The first pair, shown by dash-dotted line and used as a basis for comparison, is related to an ideal response of the controller in confronting to the parameter variations. It is obtained by assuming that the controller knows the exact value of the machine parameters which never happen in practice. The second pair of plot, shown by solid line, is the result of the SMC and the third one, shown by dashed line, is related to the method introduced by S. Morimoto. It can be seen that parameter variations have undesirable effect on the motor efficiency except in near the nominal operating conditions in both methods. However, it is seen that the SMC is more robust against parameter variations resulting in higher motor efficiency over a wide operating range.

**Appendix I**

The matrix components of equation (13) are:

\[
f_1 = \frac{3P}{2} \left[ \left( L_{d} - L_{q} \right) \omega \left( L_{d} i_{d} - L_{q} i_{q} \right) \right] \left( R_{s} R_{d} \left( L_{q}^2 - L_{d}^2 \right) \right) i_{d} + \left( L_{d} R_{d} \right) i_{d} \right] i_{q} \right\} \left( \alpha - \frac{1}{L_{q}} \frac{R_{s} R_{d}}{L_{q} \left( L_{d} + R_{s} \right)} \right) i_{d} \right\} \right) + \left( R_{s} R_{d} \left( L_{q}^2 - L_{d}^2 \right) \right) \right\}
\]

\[
f_2 = 2R \left( L_{d} - L_{q} \right) \left( R_{s} R_{d} \left( L_{q}^2 - L_{d}^2 \right) \right) i_{d} + \left( R_{s} R_{d} \left( L_{q}^2 - L_{d}^2 \right) \right) \right\} \left( \alpha - \frac{1}{L_{q}} \frac{R_{s} R_{d}}{L_{q} \left( L_{d} + R_{s} \right)} \right) i_{d} \right\} \right) \]

\[
f_3 = 2L_{q} \left( L_{d} - L_{q} \right) \left( \frac{R_{s} R_{d}}{L_{q} \left( L_{d} + R_{s} \right)} \right) i_{d} + \left( R_{s} R_{d} \left( L_{q}^2 - L_{d}^2 \right) \right) \right\} \left( \alpha - \frac{1}{L_{q}} \frac{R_{s} R_{d}}{L_{q} \left( L_{d} + R_{s} \right)} \right) i_{d} \right\} \right) \]

\[
f_4 = 2L_{q} \left( L_{d} - L_{q} \right) \left( \frac{R_{s} R_{d}}{L_{q} \left( L_{d} + R_{s} \right)} \right) i_{d} + \left( R_{s} R_{d} \left( L_{q}^2 - L_{d}^2 \right) \right) \right\} \left( \alpha - \frac{1}{L_{q}} \frac{R_{s} R_{d}}{L_{q} \left( L_{d} + R_{s} \right)} \right) i_{d} \right\} \right) \]

**VI. Conclusion**

In this paper, a high performance model-following nonlinear sliding mode controller based on a feedback linearization control system for an IPM synchronous motor has been described. This controller minimizes the motor losses besides the tracking of a reference speed. Moreover, the desirable robustness of the controlled IPM synchronous motor drive against the parametric uncertainties is improved resulting in a more energy saving over a wide motor operating range.
V. Simulation Results

The performance of an IPM motor under the proposed model-following nonlinear sliding mode control system has been investigated by extensive simulation. The IPM synchronous motor specifications are listed in Appendix II. The overall motor drive block diagram is shown in Fig. (2). Many simulation runs have been carried out to examine the proposed SMC scheme. Firstly, an unloaded motor, $T_1 = 0$, is simulated. The motor is started by an exponential speed command reference with the final value of machine rated speed as in Fig. (3). Then at the time $t=1$ (sec) a rated load torque is applied to the motor. Reference Speed command is reduced exponentially to the half rated speed at $t=1.5$ (sec). Subsequently, at $t=2.5$ (sec) the rated load is removed from the motor.

As it is expected, SMC rejects the external load disturbance. Also it can be seen that the chattering associated with the sliding mode exists with low amplitude in Fig. (4) which shows the load and the motor torque plots.

The $d$-$q$ axis currents are also shown in Fig. (5). The simulation results confirm desirable motor performance under the proposed efficiency optimization controller.

Figs. (6-9) show the phase plot of the SMC. In Figs. (6) and (7) the two errors of $e_1$ and $e_2$ with respect to time are plotted. Also the effect of $\lambda_1$ and $\lambda_2$ values are shown in these figures. Sliding surfaces $S_1$ and $S_2$ are shown in Figs. (8) and (9). It is seen that the two errors oscillate and finally converge the zero point. This confirms the reachability of SMC the controlled system.
\[
\begin{bmatrix}
\dot{y}_{m1} \\
\dot{y}_{m2}
\end{bmatrix} = \begin{bmatrix}
-a_{m1} & 0 \\
0 & -a_{m2}
\end{bmatrix} \begin{bmatrix}
y_{m1} \\
y_{m2}
\end{bmatrix} + \begin{bmatrix}
a_{m1} & 0 \\
0 & a_{m2}
\end{bmatrix} \begin{bmatrix}
y_{l_{ref}} \\
y_{l_{ref}}
\end{bmatrix}
\] (18)

where \(a_{m1}\) and \(a_{m2}\) are the designed positive constants with \(a_{m1}\) determining the reference torque performance and \(a_{m2}\) determining the reference loss minimization condition. Also we have:

\[
y_{l_{ref}} = T_{eq_{ref}}, y_{2_{ref}} = 0
\] (19)

The tracking errors between the plant of (16) and reference model are:

\[
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix} = \begin{bmatrix}
y_1 - y_{m1} \\
y_2 - y_{m2}
\end{bmatrix}
\] (20)

and its error dynamics are derived as follows:

\[
\begin{bmatrix}
e_{1} \\
e_{2}
\end{bmatrix} = \begin{bmatrix}
f_1 & 0 & 1 \\
f_2 & 1 & 0
\end{bmatrix} \begin{bmatrix}
ev_{1} \\
ev_{2}
\end{bmatrix} + \begin{bmatrix}
\varphi_1 \\
\varphi_2
\end{bmatrix}
\] (21)

where \(\varphi_1\) and \(\varphi_2\) are assumed bounded parameter uncertainties and the linear feedback is:

\[
\begin{bmatrix}
ev_{d} \\
ev_{q}
\end{bmatrix} = \begin{bmatrix}
\dot{\varphi}_1 + a_{m2} y_{m2} - a_{m1} y_{2_{ref}} \\
\dot{\varphi}_2 + a_{m1} y_{m1} - a_{m2} y_{1_{ref}}
\end{bmatrix}
\] (22)

Now, the sliding mode design technique can be used for the error system (21) to make the errors \(e_1\) and \(e_2\) reach zero. That is the developed torque and loss minimization condition track their reference values. The control algorithm consists of three steps:

- Calculating coordinate controller, \(\nu'_d\) and \(\nu'_q\), according to the sliding mode method.
- Calculating \(\dot{\varphi}_d\) and \(\dot{\varphi}_q\) using feedback linearization control of (22).
- Calculating \(\nu_d\) and \(\nu_q\) using feedback linearization control of (16).

### B. Sliding Mode Surface

In order that \(e_1\) and \(e_2\) converge to zero, two sliding surfaces are introduced as:

\[
s_1 = e_1 + \lambda_1 \int e_1 dt
\] (23)

\[
s_2 = e_2 + \lambda_2 \int e_2 dt
\] (24)

where the parameters \(\lambda_1\) and \(\lambda_2\) are positive constants which determine the convergence rate. A Lyapunov function is then proposed as:

\[
V = \frac{1}{2} s_1^2 + \frac{1}{2} s_2^2 \Rightarrow V = s_1 \dot{s}_1 + s_2 \dot{s}_2
\] (25)

By differentiating sliding surfaces and substituting from (22), we will have:

\[
\dot{s}_1 = \dot{f}_1 + v'_d + \varphi_1 + \lambda_1 e_1
\] (26)

\[
\dot{s}_2 = \dot{f}_2 + v'_q + \varphi_2 + \lambda_2 e_2
\] (27)

then:

\[
\dot{V} = S_1 \dot{f}_1 + v'_d \varphi_1 + \lambda_1 e_1 + S_2 \dot{f}_2 + v'_q \varphi_2 + \lambda_2 e_2
\] (28)

Assuming \(\nu'_d\) and \(\nu'_q\) as:

\[
\nu'_d = -f_1 - \lambda_2 e_1 - Q_s sgn(s_1), Q_1 > |\varphi_1|
\] (29)

\[
\nu'_q = -f_2 - \lambda_1 e_2 - Q_s sgn(s_2), Q_2 > |\varphi_2|
\] (30)

and substituting them in (28):

\[
\dot{V} = S_1 [-Q_s sgn(s_1) + \varphi_1] + S_2 [-Q_s sgn(s_2) + \varphi_2]
\]

\[
\geq [S_1 - |\varphi_1|] [S_1 - Q_s sgn(s_1) + \varphi_2] \leq 0
\] (31)

Hence, using the proposed controller (29) - (30), the reachability of sliding mode control of the controlled system (21) is guaranteed. After reaching the surfaces \(S_1\) and \(S_2\), the system output errors will converge to zero, that is:

\[
y_1 = T_e \rightarrow T_{eq_{ref}}, y_2 \rightarrow y_{2_{ref}} = 0
\] (32)

The proposed sliding mode control is robust with respect to matched and mismatched uncertainties. In addition, with the use of the feedback linearization control, the complete decoupled control of torque and indirectly flux can be obtained.

The chattering associated with sliding mode is the main drawback of the sliding mode application. The sliding mode control chattering is substantially reduced if in equations (28) and (29), \(sgn(s_1)\) and \(sgn(s_2)\) are replaced by the following saturation function or by \(\tan^2\) function [15]:

\[
\text{Sat}(\frac{s}{\phi}) = \begin{cases} 
+1 & \text{if } s > \phi \\
\frac{s}{\phi} & \text{if } -\phi \leq s \leq \phi \\
-1 & \text{if } s < -\phi
\end{cases}
\] (33)

where \(\phi\) is a positive parameter.
Conventional notation is used for machine parameters and variables. In steady state, the electrical power losses of the machine can be expressed as:

\[ P_L = \frac{3}{2} R_i \left( i_{qT}^2 + i_{dT}^2 \right) \frac{2}{3} R_x \left( i_{qT}^2 + i_{dT}^2 \right) - L_d L_{qT} \omega_c^2 \]

\[ + \frac{3}{2} \frac{R_i}{R_x} \left( R_s + R_x \right) \left( L_m + L_{d1} i_{dT} + L_{q1} i_{qT} \right) \]

\[ + 3 \frac{R_i}{R_x} \left( \frac{L_m}{R_x} - L_q \right) i_{dT}^2 \omega_c \]

\[ + 3 \frac{R_i}{R_x} \left( \frac{L_m}{R_x} - L_q \right) i_{qT}^2 \omega_c \]

\[ + \frac{R_i}{R_x} \left( \frac{L_m}{R_x} - L_q \right) \left( i_{dT}^2 + i_{qT}^2 \right) \omega_c \]

III. Loss Minimization Condition

In this section, a loss minimization condition for an IPM synchronous motor is achieved. By differentiating (3) and (4) with respect to \( i_{dT} \) and \( i_{qT} \) the following equations are obtained:

\[ \frac{\partial T_e}{\partial i_{dT}} = \frac{3}{2} R_i \left( \frac{\partial i_{dT}}{\partial i_{dT}} \right) \left( \frac{R_i}{R_x} + R_x \right) \left( \frac{L_m}{R_x} - L_q \right) \left( i_{dT}^2 + i_{qT}^2 \right) \]

\[ \frac{\partial P_L}{\partial i_{dT}} = 3 R_i \left( i_{dT} + \frac{i_{dT}}{i_{dT}} \right) + 3 \frac{R_i}{R_x} \left( \frac{L_m}{R_x} - L_q \right) \left( i_{dT}^2 + i_{qT}^2 \right) \]

\[ + 3 \frac{R_i}{R_x} \left( \frac{L_m}{R_x} - L_q \right) \left( i_{dT}^2 + i_{qT}^2 \right) \omega_c \]

In order to reach the loss minimization condition in every operating point, the above differentials must be equal to zero i.e.:

\[ \frac{\partial T_e}{\partial i_{dT}} = 0, \quad \frac{\partial P_L}{\partial i_{dT}} = 0 \]

Then:

\[ \frac{\partial i_{dT}}{\partial i_{dT}} = \frac{\left( L_m + \left( L_m - L_q \right) i_{dT} \right) i_{dT}}{L_m + \left( L_m - L_q \right) i_{dT}} \]

\[ \frac{\partial i_{qT}}{\partial i_{qT}} = -P \]

\[ \frac{\partial i_{qT}}{\partial i_{qT}} = \frac{Q}{Q} \]

where:

\[ P = R_s i_{dT} + \frac{R_x}{R_s} \left( L_m - L_q \right) i_{dT} \]

\[ + \frac{R_i}{R_x} \left( \frac{L_m}{R_x} - L_q \right) \left( L_m + L_{d1} i_{dT} \right) L_{d1} \omega_c \]

\[ Q = R_s i_{qT} + \frac{R_x}{R_s} \left( L_m - L_q \right) i_{qT} \]

\[ + \frac{R_i}{R_x} \left( \frac{L_m}{R_x} - L_q \right) \left( L_m + L_{q1} i_{qT} \right) L_{q1} \omega_c \]

\[ + \frac{R_i}{R_x} \left( R_s + R_x \right) \left( \frac{L_m}{R_x} - L_q \right) \]

By equating the right hand side of (8) and (9) and rearranging the result, the mentioned condition is obtained as:

\[ \frac{\alpha_i^2}{R_x^2} \left( R_s + R_x \right) \left( L_m + \left( L_m - L_q \right) i_{dT} \right) i_{dT} \]

\[ + \frac{\alpha_i^2}{R_x^2} \left( R_x + R_s \right) \left( L_m + \left( L_m - L_q \right) i_{dT} \right) \]

IV. Sliding Mode Controller

In this Section a sliding mode controller is designed to provide both high performance motor drive operation and loss minimization.

A. Coordinate System

In order to control the developed torque and efficiency, the system outputs are chosen as:

\[ y_1 = T_e = \frac{3}{2} \left( R_s i_{dT} + \left( L_m - L_q \right) i_{qT} \right) \]

\[ y_2 = -\frac{\alpha_i^2}{R_x^2} \left( R_s + R_x \right) \left( L_m + \left( L_m - L_q \right) i_{dT} \right) \]

\[ - R_s \lambda_m i_{dT} \]

\[ + \frac{\alpha_i^2}{R_x^2} \left( R_x + R_s \right) \left( L_m + \left( L_m - L_q \right) i_{dT} \right) \]

\[ + \frac{\alpha_i^2}{R_x^2} \left( R_s + R_x \right) \left( L_m + \left( L_m - L_q \right) i_{dT} \right) \]

where \( f_1 \) and \( f_2 \) and \( g_1, g_2 \) are given in Appendix I. Furthermore, using the nonlinear feedback linearization control method to decouple the control inputs, the resulting system is [14]:

\[ \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{y}_d \\ \tilde{y}_q \end{bmatrix} \]

and the linear feedback is defined as:

\[ \begin{bmatrix} \tilde{v}_d \\ \tilde{v}_q \end{bmatrix} = \begin{bmatrix} g_1 v_d + g_2 v_q \\ g_1 v_d + g_2 v_q \end{bmatrix} \]

Owing to (16) being a nonlinear system, the dynamics are hard to regulate by a constant state feedback gain [13]. Therefore, for the dynamic system of (16), a model-following nonlinear sliding mode controller is proposed to track a desired reference model. The reference model is designed in a linear form as:
Loss Minimization Sliding Mode Control of IPM Synchronous Motor Drives

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Abstract: In this paper, a nonlinear loss minimization control strategy for an interior permanent magnet synchronous motor (IPMSM) based on a newly developed sliding mode approach is presented. This control method sets force the speed control of the IPMSM drives and simultaneously ensures the minimization of the losses besides the uncertainties exist in the system such as parameter variations which have undesirable effects on the controller performance except at near nominal conditions. Simulation results are presented to show the effectiveness of the proposed controller.

Index Terms: Permanent magnet motors, speed control, loss minimization control, sliding mode.

I. Introduction

Interior permanent magnet synchronous (IPMS) motors are widely used in high performance drive applications such as robotics, aerospace and electric vehicles (EV) [1,2]. These applications require demanding still control methods [4]. Thus much effort has been directed towards the efficiency optimization control (EOC) of the IPMS motors by minimizing machine copper and iron losses [5-8]. Model-based EOC methods are very fast and do not produce torque ripples, but they are not robust against machine parameter variations [5-7]. The stator resistance may vary due to the skin effects, temperature variations, etc. Core losses also vary due to the variations of motor flux and speed and become an important issue at high speeds [8]. Also permanent magnet (PM) flux may vary due to temperature variations and excessive flux weakening. Machine inductance is known to depend on air-gap flux [9]. The adverse effect of motor parameter variations on minimum loss operation of IPMS motors is analyzed in [10,11] and it is shown that these parameter variations except at near nominal conditions have undesirable effects on the controller performance. Thus, it is vital to compensate the variation of the mentioned parameters in the high performance IPMS motor drives especially when an EOC is used.

Sliding mode control (SMC) has been studied by many researchers due to its favorable advantages, such as insensitivity to parameter variations and external load disturbances [12-13]. Only the bounds of the uncertainties are needed. The robustness of this controller is guaranteed, but the worst drawback is the chattering, which limits the applications of SMC. The chattering phenomenon is greatly considered as motion which oscillates about the sliding surface. There are two possible mechanisms which produce such a motion. First, the presence of switching device non-idealities such as delays; second, the presence of parasitic dynamics (actuator and sensor dynamics) in series with the plant. Using a discrete-time control design approach, or high sampling rate, we can reduce the chattering due to the switching device non-idealities. Several methods, observer-based sliding mode control, sliding mode based on disturbance compensation and boundary layer sliding mode control have been used to reduce the chattering due to parasitic dynamics. The most common and simplest approach to reduce the effects of chattering has been the boundary layer [14].

In this paper, a newly designed sliding mode controller for an IPMS motor drive is introduced to both minimize the machine losses and track a reference speed command. Finally, extensive simulation results are presented to show robust, efficient and high performance motor drive operation under the proposed control system.

II. Machine Model

Under certain assumptions a widely used model for an IPM synchronous motor including copper and iron losses in a synchronously rotating reference frame is presented in Fig. (1) [5,10].

The model equations are given as follows:

\[
\begin{align*}
\frac{di_e}{dt} &= \frac{R_e}{L_e(R_s + R_e)}(-R_e i_e + v_q) + \frac{L_1}{L_e} \omega \cdot \omega_i e
\end{align*}
\]

\[
\begin{align*}
\frac{di_q}{dt} &= \frac{R_s}{L_1(R_s + R_e)}(-R_e i_e + v_q) - \frac{L_1}{L_e} \omega \cdot \omega_i e + \omega_m
\end{align*}
\]

\[
T_e = \frac{3P}{2} \left[ \lambda_m + \left( L_{d_e} - L_{e_q} \right) i_{e_q} \right]
\]