Providing a Simple Method for the Calculation of the Source and Target Reliability in a Communication Network (SAT)

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Abstract
The source and target reliability in SAT network is defined as the flawless transmission from the source node to all the other nodes. In some references, the SAT process has been followed between all the node pairs but it is very time-consuming in today’s widespread networks and involves many costs. In this article, a method has been proposed to compare the reliability in complex networks based on the spanning tree approach. Initially, a primary spanning tree called FST is formed and all the other spanning trees of the graph will be computed based on this primary spanning tree. After the computation of all the spanning trees of the network graph, the reliability of the network will be easy to determine by adding up the multiplications of the accuracy probability of the functions in all the edges within the spanning tree. The proposed method will not yield additional trees. Moreover, this algorithm has little complexity compared to the conventional methods and also requires little memory capacity.

Keywords:
The scale-free and the small world network, the spanning tree, reliability, graph

1 – Introduction
The network systems of the internet of things are always vulnerable to failures and different attacks. The network failures can include cases such as the failure of one object in the internet of things and the attacks may be caused by hackers, the penetrators’ activities or other malice. Failure in the internet network can cause widespread damages in industrial settings. For instance, if a thermostat fails to be in contact with the controller of the network, it does not perform the thermal cut-off on time and causes fire and fatalities. One of the most important features of these systems is the transformation in their topology. This can be caused by the nature of the node behaviors or the failure and flaws in their performance. The resistance in the communications in the face of the faults and failures and calculated responses to the attacks and problems are among the pivotal necessities in most networks and especially the internet of things. In the same vein, it is preferable for the communicative systems to show a high level of reliability, availability and resistance in response to failures, problems and attacks [1 and 2].

Network studies are not simple as in the past and the investigated networks are today called complex networks. Among the features of these networks, we may refer to the fact that they are neither completely random nor completely regular. The attempt to simulate these networks leads to the introduction of models such as the small world network and scale-free networks. The World Wide Web, the internet, aerial communicative networks and some systems on the ground like metabolism and protein networks from random networks are a case in point [3]. In mathematics, physics and sociology, the small world network refers to the graph in which a small number of nodes are close together and there is a short path between most of the nodes. In figure 1, a graph of a small world network is shown.

Fig 1. The graph of a small world network
The diagram of the graph in figure 2 is a non-scale network that means that each new node is connected to one or two existing nodes.

![The graph of a free-scale network](image)

2 – Research background

In research references, thorough investigations have been done on large and complex networks and their resistance to various attacks and possible errors. Different criteria and parameters have been proposed, each approaching these networks from a different angle and perspective. Below, some of these studies and their results are indicated and we will make an effort to introduce the implemented methods to raise the resistance of the complex networks such as the internet of things [8].

The theoretical and empirical results from the complex networks have divided them into two general classes based on the function of the probability distribution of nodes. P (k) is the probability distribution function referring to the probability that a node might be connected to K nodes. The first class is composed of those cases in which maximum P (k) occurs in K average and reduces the larger Ks exponentially and these networks are also called exponential networks. Most of the networks that are based on the random graph model are described by the small world model and are completely homogenous while each node has the same number of links that amounts to the K average [9].

\[ k=\langle k \rangle = E[k] \]  

In [4], the scale-free networks have been studied. Scale-free networks are classified into two general groups, namely the scale-free networks with local clustering defined by Albert model with the scale-free networks with high clustering properties defined as Kulin model. In this paper, it will be demonstrated that the absolute productiv-

ity in describing the response of the large networks outperforms the length of the path in response to the flaws and attacks. As we know, the length of the course in a graph is considered as the average of the shortest distance between two vertexes in the network graph. In [5], we investigate the effects of applying the bimodal distribution of the natural frequency in Kuramoto model on the scale-free, random and non-scale networks. In doing so, two models will be considered in which the pairwise fashion between the fluctuations is independent or normalized by the degree of the network vertexes. More time is required to reach a sustainable state for all the networks in a condition where the paired factor has not been normalized with regard to the vertexes degree. Under these two models, the scale-free and random networks will lose their synchrony with the increase in the inherent frequency and the bimodal distribution, while the small work network gets close to the convergent state by increasing this frequency and then the regularity of the system decreases followed by regular fluctuations. In the model where the coupling coefficient is adjusted to the vertex degree, more time is needed for reaching the sustainable state and the network frequency change is also more sensitive. With the increase in the inherent frequency in the bimodal distribution, the order index of the scale-free and random network drops below one and then irregular fluctuations appear with a small scope. Unlike the scale-free and random networks, with the increase in the inherent frequency in the bimodal distribution of the small world network, the order index rises, the system becomes synchronized, and the flaws of the system decrease. Then, with a rise in the frequency, the order index drops and then begins to show regular fluctuation within a remarkable scope. Ultimately, a high level of the source and target reliability has been reached in communication networks as in [5].

In their study entitled as “the effect of network structure on the large-scale innovation”, Schillin and Phelps showed that the features of the high clustering factor and the short length of the path have a positive effect on the efficiency of the network. In other words, the shorter the length of the path in a network, and the higher the clustering factor, the higher effects will be observed on the creation of new knowledge [6]. In their research titled “the role of network structure and effects in the diffusion of innovations”, Kim et al. investigated the networks and showed that if a network has the properties and characteristics of the small world network, the speed of the information radiation goes up in that network [7].

3 – Statement of the problem

Consider a graph in which the edges show the communicative or connective links and the nodes show a rotor
or base station. To calculate the reliability from a source node to a target node, the sum of the probabilities of the spanning trees of the graph can be used as explained below. A network can graphically consist of nodes or edges called G (v, b) denoting a graph with v vertex and b edge. Spanning tree G includes a set of edges while covering all the vertexes. In fact, all G vertexes exist in the spanning tree on condition that no distance is created and the tree is also connected [10].

Stage one:
A spanning tree is selected from the spanning trees of the network graph, creating the relation between the source node and the entire target node with the least edge as the first FST spanning tree. We put FST links in “S” set.

Stage two:
Put all the links of the network that are not in FST graph in T set.

Stage three:
Here, \( C_1^m \) link can create some flaws in FST. We select a link path from T set with the highest degree of link entry with the node of its previous graph to be positioned in the flawed FST. After selecting the edge of the graph, it must be included in the last process of this stage. All the successful spanning trees of the STI must be maintained.

There is the probable \( C_2^m \) doubly connected edge that disrupts the first FST tree. The doubly connected edges from the T set with the highest degree of link entry are selected for its previous graphic nodes as substitutes in FST. After selecting the links, they should be included in IV process. All the successful spanning trees of STI must be maintained.

The process will be repeated for three, four and d edges and if they did not form a ring, the successful spanning trees will be registered and maintained.

When the selected edge consists of T and C components (i.e. those which form the ring and round), the generation of flawed and unsuccessful spanning tree will not occur. This would minimize the number of defective spanning trees.

Stage four:
SAT reliability parameter is derived from the accumulation of all the sets in the separated spanning trees and the logical variables of each spanning tree must be replaced with relevant probable variables. Consider graph G (6, 9) in figure 4 as we intend to show the proposed method for the calculation of the reliability of the source node to the target node using this graph.

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The first spanning tree is shown in Figure 5. First, we delete link 2 and we obtain the defective tree as shown in Figure 6. This graph has been obtained after deleting link 2 in FST.

![Fig 6. The first flawed spanning tree.](image)

Now, given the shape of the defective tree in Figure 6, we see that only edge 1 can change the defective spanning tree into the complete spanning tree. The point that exists as an exception is that the edges of the input node should not be considered from the defective spanning tree in the formation of the complete spanning tree. Thus, edge 1 is not considered.

![Fig 7. The formation of the complete spanning tree from the defective one](image)

According to Figure 7, none of the edges of the T combination that will be selected can make a complete spanning tree because node 1 is separated from the graph after deleting edge 2. Therefore, it can be concluded that the deletion of link 2 cannot lead to any complete spanning tree and, for this reason, it has been placed in the third column in Table 1 and there is no word for it. In fact, all successful spanning trees are set in Table 1 (obtained from the deletion of the links of the first spanning tree). At the next stage, we omit link 3 and then complete the spanning tree by adding link 3 of T.

![Table 1. Successful spanning trees](table)

<table>
<thead>
<tr>
<th>Number</th>
<th>The first flawed spanning trees</th>
<th>The added link from T for making the complete spanning tree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23789</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>23789</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>23789</td>
<td>4</td>
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<tr>
<td>4</td>
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<td>23789</td>
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<td>23789</td>
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<td>23789</td>
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<td>1, 6</td>
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<tr>
<td>13</td>
<td>23789</td>
<td>1, 4</td>
</tr>
<tr>
<td>14</td>
<td>23789</td>
<td>4, 5, 6</td>
</tr>
<tr>
<td>15</td>
<td>23789</td>
<td>1, 5, 6</td>
</tr>
<tr>
<td>16</td>
<td>23789</td>
<td>4, 5, 1</td>
</tr>
<tr>
<td>17</td>
<td>23789</td>
<td>1, 4, 5, 6</td>
</tr>
<tr>
<td>18</td>
<td>23789</td>
<td>None</td>
</tr>
</tbody>
</table>

Given that each link in the network graph has probability \( P \), this can be the probability of the sustainability of the connection. The reliability of the network can be obtained as shown below through the sum of multiplications and the probabilities of the links in all successful spanning trees after their formation.

\[
R = \left( \bigcup_{ST} \right) = \sum P(ST) = \left( \left( P_{v_1,v_2}P_{v_2,v_3}P_{v_3,v_4}P_{v_4,v_5} + P_{v_1,v_2}P_{v_2,v_3}P_{v_3,v_4}P_{v_4,v_5} + \cdots \right) \right)
\]

\[
= P_{v_1,v_2}P_{v_2,v_3}P_{v_3,v_4}P_{v_4,v_5} + P_{v_1,v_2}P_{v_2,v_3}P_{v_3,v_4}P_{v_4,v_5} + \cdots
\]

\[
+ P_{v_1,v_2}P_{v_2,v_3}P_{v_3,v_4}P_{v_4,v_5} + \cdots
\]

\[
+ P_{v_1,v_2}P_{v_2,v_3}P_{v_3,v_4}P_{v_4,v_5} + \cdots
\]

\[
+ P_{v_1,v_2}P_{v_2,v_3}P_{v_3,v_4}P_{v_4,v_5} + \cdots
\]
In the above equation, the first term is related to the multiplication of the probabilities in the links of the first spanning tree and the rest of terms are the result of the multiplication of probabilities in the links of successful spanning trees in table 1. In this relation, q probability has been used for the omitted links (q = 1-p).

4 – Conclusion
In this article, a new method has been proposed for the calculation of reliability in complex networks based on spanning tree. The details of the method were explained by providing one example of the graph. In terms of computational complexity, the provided method is much better than the common methods as it involves less memory capacity and does not involve the calculation of all spanning trees.

5 – References
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