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An Alternative Stability Proof for Direct Adaptive Function Approximation Techniques Based Control of Robot Manipulators

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Abstract
This short note points out an improvement on the robust stability analysis for electrically driven robots given in the paper. In the paper, the author presents a FAT-based direct adaptive control scheme for electrically driven robots in presence of nonlinearities associated with actuator input constraints. However, he offers not suitable stability analysis for the closed-loop system. In other words, it does not consider the role of saturation function in both control design and stability analysis.

Keywords
Model Free, Robust Control, Robot Manipulator

1. Introduction

As pointed out in the paper [1], the FAT-based adaptive control scheme has a simpler structure and less computational burden compared with neuro-fuzzy control approaches to design a model-free controller. These advantages have been previously mentioned in [2-5]. The considerable point is that the respectable author does not give suitable stability analysis for the overall control system. The stability analysis is in a decentralized form without considering the role of actuator nonlinearities. The objective of this paper is to modify the previous results on the robust stability analysis of the work proposed by [1]. The overall closed-loop system composed by full actuated robotic manipulator for both n degrees of freedom and the proposed controller is proved to be robust, and BIBO stable, while the joint position/velocity tracking errors are asymptotically stable in agreement with Lyapunov's direct method.

This paper is organized as follows. Section 2 briefly presents modeling of the robotic system including the permanent magnet DC motors subjected to actuator saturation. In Section 3, direct adaptive controller proposed by [1] is reviewed considering to actuator input constraint. The stability analysis is also presented in this section. Finally, concluding remarks are drawn in section 4. In what follows, we shall use the following notation. We denote by $\|x\|$ the Euclidean norm of a vector $x \in \Re^n$. We use the notation $\underline{\lambda}(\square)$ and $\overline{\lambda}(\square)$ to indicate the smallest and largest Eigen values, respectively, of a positive definite bounded matrix. We say that $x(\cdot):[0,T] \to \Re^n$ is in $L_2[0,T]$ if $\int_0^T \|x\|^2 dt < \infty$, $x(\cdot)$ is in $L_\infty[0,\infty)$ if $\|x\| < \infty$ for all $t \in [0,\infty)$.

2. Dynamic Modeling

Consider an n-link manipulator driven by geared permanent magnet DC motors with voltages being inputs to amplifiers as [1].

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau}_l \tag{1}$$

$$\mathbf{J}\mathbf{r}^{-1}\ddot{\mathbf{q}} + \mathbf{B}\mathbf{r}^{-1}\dot{\mathbf{q}} + \mathbf{r}\boldsymbol{\tau}_l = \boldsymbol{\tau}_m \tag{2}$$

$$\mathbf{R}\mathbf{I}_a + \mathbf{L}\dot{\mathbf{I}}_a + \mathbf{k}_b\mathbf{r}^{-1}\dot{\mathbf{q}} = \mathbf{v}(t) \tag{3}$$

Where, the parameters are defined exactly similar to the research [1]. Following the same procedure as [1], we define a second-order nonlinear differential equation of integrated actuator and manipulator, called "available model" as

$$\ddot{q} = f(t) + v(t) \tag{4}$$

where the presented variables $\ddot{q}$, and $v(t)$ are the i th element of the vector $\ddot{\mathbf{q}}$, and $\mathbf{v}(t)$, respectively; and $f(t)$ is referred as the lumped uncertainty. For practical situation, the actuator input is subjected to some constraints, called motor saturation limits. This occurs usually between output of the controller and the PWM module [6-7]. For the development in this paper, we assume that the relation between the actual actuator input $\mathrm{v}(t)$ and control signal produced by the controller $\mathrm{u}(t)$ is given by

$$\mathrm{v}(t)=\mathrm{sat}(\mathrm{u}(t)) \tag{5}$$

Where $\mathrm{sat}(\mathrm{u}(t)) \in \Re$ is the saturation function. According to [8-9], the hard saturation function can be divided into a linear function $\mathrm{u}(t)$ and a dead-zone function $\mathrm{dzn}(\mathrm{u}(t), u_{\max})$. Thus, the control input applied to the system through the actuator is expressed as follows:

$$\mathrm{sat}(\mathrm{u}(t))=\mathrm{u}(t) - \mathrm{dzn}(\mathrm{u}(t), u_{\max}) \tag{6}$$

Now, substituting (5) into (4), and using (6), it follows that

$$\ddot{q} = \mathrm{u}(t) - \mathrm{dzn}(\mathrm{u}(t), u_{\max}) + f(t) \tag{7}$$

Remark 1: The control input given by equation (5) indicates that the motor voltage is bounded, that is

$$|\mathrm{v}(t)| \le \mathrm{u}_{\max} \tag{8}$$

Where $\mathrm{u}_{\max}$ is a positive constant representing the maximum permitted voltage of the motor. As a result, the variables I_a, $\dot{I}_a$, and $\dot{q}$ are upper bounded. Proof is the same as [9] in the scalar form.

3. Revisiting [1] Considering Actuator Voltage Input Constraint

Following the same notation as [1], using function approximation technique, we propose a control law in the form of

$$\mathrm{u}(t) = \hat{P}^T \varphi(t) + v_c \tag{9}$$

Where $\hat{P}$ is the estimation of weighting vector P used into a function approximator $P^T \varphi(t)$ which approximates the following function based on the universal approximation theorem

$$P^T \varphi(t) + \varepsilon_m = \ddot{q}_d + k_1 e + k_2 \dot{e} - f(t) \tag{10}$$

Where $\varphi(t) \in \Re^N$ denotes basis functions' vector fixed by the designer, the number N represents the number of basis functions used, ε_m is reconstruction error; q_d is the desired joint position, k_1, and k_2 are positive scalar gains which are selected as control design parameters, and e is the joint position tracking error expressed by

$$e = q_d - q \tag{11}$$

In order to obtain the adaptive control law, we form the tracking system from (7), (9), and (10) as

$$\ddot{e} + k_1 e + k_2 \dot{e} = \tilde{P}^T \varphi(t) + \varepsilon_m - v_c + \mathrm{dzn}(\mathrm{u}(t), u_{\max}) \tag{12}$$

Where $\tilde{P} = P - \hat{P}$. Introducing A, B, and E as

$$A = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \tag{13}$$

The error equation (12) can then be written in the following state space form

$$\dot{E} = AE + B(\tilde{P}^T \varphi(t) + \varepsilon_m - v_c + \mathrm{dzn}(\mathrm{u}(t), u_{\max})) \tag{14}$$

3.1 Stability analysis

Before stating the stability analysis, the following lemma is given. First, we present the following two assumptions, which are requiring in determining the sufficient conditions on the control parameters.

Assumption 1: The desired trajectory and its time derivative are in L_∞ space, $(q_d, \dot{q}_d \in L_\infty)$.

Assumption2: The reconstruction error ε_m is bounded, i.e. $|\varepsilon_m| < \varepsilon$ with known ε.

Now, we are ready to present the following lemma.

Lemma 1. $\left|\mathrm{dzn}(\mathrm{u}(t))\right|$ satisfies the following inequality:

$$\left|\mathrm{dzn}(\mathrm{u}(t),u_{\max})\right| \le \frac{\delta u_{\max}}{(1-\delta)} \tag{15}$$

Where δ is a constant which always has a value smaller than 1.

Proof: Suppose that $\mathrm{u}(t)$ exists within $[-\max\{\mathrm{u}(t)\},\max\{\mathrm{u}(t)\}]$, and δ is $\max\left\{1-\frac{u_{\max}}{\mathrm{u}(t)}\right\}$. Then,

$$\left|\mathrm{dzn}(\mathrm{u}(t),u_{\max})\right| \le \delta\left|\mathrm{u}(t)\right| \tag{16}$$

Is satisfied by Figure 1. This result, together (9), (10) and (12) gives

$$\left|\mathrm{dzn}(\mathrm{u}(t),u_{\max})\right| \le \delta\left|\ddot{q}-f(t)\right| + \delta\left|\mathrm{dzn}(\mathrm{u}(t),u_{\max})\right| \tag{17}$$

Now, according to (4), (5), and (8), we have:

$$\left|\mathrm{dzn}(\mathrm{u}(t),u_{\max})\right| \le \frac{\delta u_{\max}}{(1-\delta)}$$

This completes the proof

To carry out the stability analysis of the closed-loop system formed by dynamic Equation (14), the following positive definite function is proposed:

$$V(E,\tilde{P}) = \frac{1}{2}E^T P_0 E + \frac{1}{2\gamma}\tilde{P}^T\tilde{P} \tag{18}$$

Where γ is a positive gain related to the adaption laws; P_0, and Q are the unique symmetric, positive definite matrices satisfying the matrix Lyapunov equation

$$A^T P_0 + P_0 A = -Q \tag{19}$$

It must be noted that, (18) is not a Lyapunov function since it does not include all the system states. Now, differentiating (18) along the trajectory of the uncertain system (14), rearranging with some manipulation, this leads to

$$\dot{V}(E,\tilde{P}) = -\frac{1}{2}E^T QE + E^T P_0 B\varepsilon_m + E^T P_0 B\mathrm{dzn}(\mathrm{u}(t),u_{\max}) + \tilde{P}^T\varphi(t)B^T P_0 E - E^T P_0 B v_c - \frac{1}{\gamma}\tilde{P}^T\dot{\hat{P}} \tag{20}$$

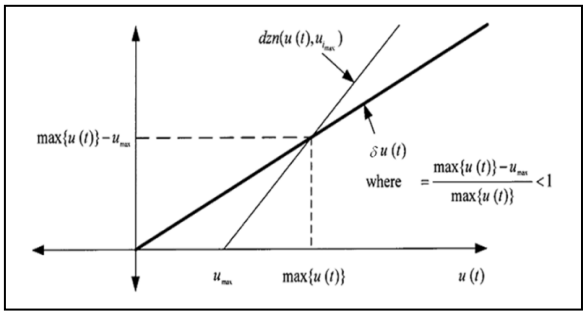

Figure1. The linear bound of dead-zone function

If the update law is given by

$$\dot{\hat{P}} = \gamma\varphi(t)B^T P_0 E \tag{21}$$

Thus, we have

$$\dot{V}(E,\tilde{P}) \le -\frac{1}{2}\underline{\lambda}(Q)\|E\|^2 - E^T P_0 B v_c + \left|E^T P_0 B\right|\left(\left|\varepsilon_m\right| + \left|\mathrm{dzn}(\mathrm{u}(t), u_{\max})\right|\right) \tag{22}$$

Now, according to Lemma 1 and assumption 2 we have

$$\dot{V}(E,\tilde{P}) \le -\frac{1}{2}\underline{\lambda}(Q)\|E\|^2 - E^T P_0 B v_c + \left|E^T P_0 B\right|\left(\varepsilon + \frac{\delta u_{\max}}{(1-\delta)}\right) \tag{23}$$

In order to make $\dot{V}(E,\tilde{P}) \le 0$, the robust control term v_c should be determined so that the inequality

$$\left|E^T P_0 B\right|\left(\varepsilon + \frac{\delta u_{\max}}{(1-\delta)}\right) - E^T P_0 B v_c \le 0 \tag{24}$$

Is satisfied. Toward this end, v_c is selected as

$$v_c = \rho sign(E^T P_0 B) \tag{25}$$

Where

$$\varepsilon + \frac{\delta u_{\max}}{(1-\delta)} < \rho \tag{26}$$

As a result, (23) can be reduced to

$$\dot{V}(E,\tilde{P}) \le -\frac{1}{2}\underline{\lambda}(Q)\|E\|^2 \tag{27}$$

So far, we have proved that E and $\tilde{P}$ are uniformly bounded, i.e. $E, \tilde{P} \in L_\infty$. Since, it is easy to have

$$\frac{1}{2}\int_0^\infty E^T Q E dt \le -\int_0^\infty \dot{V} dt = V_0 - V_\infty < \infty \tag{28}$$

We may conclude $E \in L_2$. Therefore, boundedness of $\dot{E}$ can be obtained by observing (14), since the right hand side of Equation (14) is bounded. This will further give convergence of E to zero asymptotically. Since $e = q_d - q$ and $\dot{e} = \dot{q}_d - \dot{q}$ thus boundedness of e, and $\dot{e}$ follows boundedness of q, and $\dot{q}$ according to assumption 1. Extending this result to all joints implies the boundedness of system states $\mathbf{q}$, and $\dot{\mathbf{q}}$. This result, together with remark 1 implies that the robotic system has the Bounded Input-Bounded Output (BIBO) stability, since all of system's states are bounded.

4. Conclusions

This paper improves stability results of the robust adaptive controller proposed by [1] considering actuator voltage input constraint. It is shown that the joint position and velocity tracking errors are asymptotically stable in agreements with Lyapunov direct method, while the other signals in the system remain bounded.

5. References

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