A High Performance Feedback Active Noise Control System

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Abstract
In many active noise control (ANC) applications, an online secondary path modeling method that uses a white noise as a training signal is required. This paper proposes a new feedback ANC system. Here we modified both the FxLMS and the VSS-LMS algorithms to raised noise attenuation and modeling accuracy for the overall system. The proposed algorithm stops injection of the white noise at the optimum point and reactivate the injection during the operation, if needed, to maintain performance of the system. Preventing continuous injection of the white noise increases the performance of the proposed method significantly and makes it more desirable for practical ANC systems. Computer simulation results shown in this paper indicate effectiveness of the proposed method.

Keywords: Active noise control, Secondary path, Feedback ANC, White noise.

1. Introduction

An active noise control (ANC) system is based on a destructive interference of an anti-noise, which have equal amplitude and opposite phase replica primary noise, with unwanted noise (primary noise). Following the superposition principle, the result is cancellation or reduction of both noises [1].

The effect shown by the secondary path transfer function, the path leading from the noise controller output to the error sensor measuring the residual noise, generally causes instability to the standard least mean square (LMS) algorithm. Resolving the instability problem requires using FxLMS algorithm [1]. The FxLMS algorithm uses estimation of the secondary path to compensate the problem raised by the transfer function. In many applications the secondary paths are usually time varying or non-linear, which leads to a poor performance or system instability. Hence, online modelling of secondary path is required to ensure convergence of the ANC algorithm [2-7].

The proposed system is based on modified versions of FxLMS and variable step size (VSS) LMS algorithm. Here we adapt the FxLMS and VSS-LMS algorithms with reference signal and generated white noise power variation, respectively.

To increase performance of the algorithm we stop the VSS-LMS algorithm at the optimum point. This means stopping the injection of the white noise. Not continually injection of the white noise makes the system more desirable especially in ANC headphones applications.
Additionally a sudden change in secondary path during the operation makes the algorithm to reactivate injection of the white noise to adapt with the changes. Considering the above features in the proposed method assists obtaining a better convergence rate and modelling accuracy, which results in a robust system.

The rest of the paper is organized as follows. In Section 2, the feedback ANC system is briefly described. Section 3 introduces our proposed method. In section 4, simulation results are illustrated, and finally in Section 5 conclusions are drawn.

2. Feedback ANC Systems

The block diagram of a feedback FxLMS ANC system is shown in Figure 1 [8]. Here, \( P(z) \) is the primary path and \( S(z) \) represents the secondary path. In this figure \( \hat{S}(z) \) is estimation of the secondary path \( S(z) \).

As Figure 1 shows, the reference signal \( x(n) \) is a summation of two signals \( e(n) \) and \( \hat{y}(n) \) [1]:

\[
x(n) = \hat{d}(n) = e(n) + \sum_{m=0}^{M-1} \hat{s}_m y(n - m) ,
\]

where \( \hat{s}_m \) represents coefficients of the \( M \) th order FIR filter \( \hat{S}(z) \). The secondary signal \( y(n) \) is generated as:

\[
y(n) = w(n)^T x(n) ,
\]

where \( w(n) \) and \( x(n) \) are the coefficient and signal vectors of length \( L \), order of the FIR filter \( \hat{W}(z) \), at time \( n \). These coefficients are updated by the FxLMS algorithm as follows:

\[
w_l(n + 1) = w_l(n) + \mu_x x(n - 1) e(n)
\]

\[
l = 0,1,...,L - 1 , \quad \mu > 0 ,
\]

where \( \mu_x \) is the step size, and

\[
\hat{x}'(n) = \hat{x}(n) \ast x(n) ,
\]

is the filtered reference signal. For a deep study on feedback FxLMS algorithm the reader may refer to [1].

![Figure 1. Block diagram of feedback ANC system using FxLMS algorithm [8].](image-url)
3. Proposed Method

Figure 2 shows block diagram of the proposed ANC system. The proposed method is an extension of the technique recently developed by authors [6, 7].

Here we suggest a new version of the FxLMS algorithm to increase noise attenuation. In (3) $\mu_w$ is usually set to a low value. This prevents the system to diverge when power of the reference signal $x(n)$ is increased. However, once the power decreases the low value of $\mu_w$ reduces the noise attenuation and convergence rate of the adaptive filter ($W(z)$). Thus, if $\mu_w$ could be increased when the power decreases, and vice versa, the system performance would be risen significantly.

Thereby we modified (3) as follows:

$$w(n+1) = w(n) + \frac{1}{P_x(n)} \mu_w(n) f(n) \hat{x}(n),$$

where $P_x(n)$ is given as:

$$P_x(n) = \sqrt{P_x(n-1) + (1-\gamma)(x(n)-e(n))^2}, \quad 0.9 < \gamma < 1.$$  \hspace{1cm} (6)

The residual error signal $e(n)$ of this algorithm is expressed as:

$$e(n) = d(n) - y'(n) + v'(n)$$

\[y'(n) = s(n) * y(n), \quad v'(n) = s(n) + v(n),\]  \hspace{1cm} (7)

where $v(n)$ is an internally generated white Gaussian noise, which is injected at the output of the control filter $W(z)$.

As the figure shows, $\hat{v}(n)$ generates the error signal for both the modelling filter $\hat{s}(z)$ and the control filter $W(z)$ by subtracting from $e(n)$:

$$f(n) = [d(n) - y'(n) + v'(n)] - \hat{v}(n).$$  \hspace{1cm} (8)

Coefficients of the modelling filter $\hat{s}(z)$ in VSS-LMS algorithm [3] are updated as follows:

$$\hat{s}(n+1) = \hat{s}(n) + \mu_s(n) f(n) v(n),$$

where $\mu_s(n)$ is the step-size parameter of the modelling process given as:

$$\mu_s(n) = p(n) \mu_{s_{\text{max}}} + (1-p(n)) \mu_{s_{\text{min}}}. $$  \hspace{1cm} (10)

In this equation $p(n) = \frac{P_f(n)}{P_e(n)}$, where $P_f(n)$ and $P_e(n)$ are the power of error signals $f(n)$ and $e(n)$. These powers are estimated as:

$$P_f(n) = \lambda P_f(n-1) + (1-\lambda) f^2(n)$$

$$P_e(n) = \lambda P_e(n-1) + (1-\lambda) e^2(n), \quad 0.9 < \lambda < 1.$$  \hspace{1cm} (11)
where $\mu_{\text{min}}, \mu_{\text{max}}$, and $\lambda$ are experimentally determined. These values are selected so that the adaptation is neither too slow nor it becomes unstable. The step size $\mu$ can be correspondingly changed with power of the generated white noise. When power of the generated white noise increases, the secondary path modelling accuracy raises. Hence we adapt (9) with generated white noise power variation as follows:

$$\hat{s}(n+1) = \hat{s}(n) + \sqrt{P_v(n)}\mu(n)f(n)v(n)$$

where $P_v(n)$ represents power of the generated white noise $v(n)$, given as:

$$P_v(n) = \gamma P_v(n-1) + (1-\gamma)v(n)^2, \quad 0 < \gamma < 1$$

Apart from the above modifications, the main point which results in an increased noise attenuation and convergence rate is due to preventing continuous injection of white noise during system operation. Thereby, the modelling algorithm must be stopped at the point where the modelling filter accuracy is sufficiently high, called the optimum point.

Here, the VSS-LMS algorithm is briefly described to show the way the optimum point is obtained. During the process of this algorithm, $\mu$ is increased as the error signal $f(n)$ decreases and vice versa. Hence, the modelling filter, $\hat{S}(z)$, converges to a good estimation when $f(n)$ decreases. This happens when $\mu$ increases as high as $\mu_{\text{max}}$. Thus, the injection of the white noise is stopped at the optimum point which is measured using:

$$\frac{1}{k} \sum_{n=1}^{k} \left( \frac{\mu_{\text{max}} - \mu}{\sum_{n=1}^{k} P_v(n)} \right) < \alpha ,$$

(14)

where $k$ is the number of iteration time and $M$ is the length of the $W(z)$. As the number of iteration increases, equation (14) gets closer to zero. In this equation $\alpha$ is a parameter obtained experimentally where it is $1 \times 10^{-6} < \alpha < 1 \times 10^{-4}$.

At this point, $\hat{S}(z)$ converges to a good estimation of $S(z)$. As can be seen from Figure 2, this condition validity is monitored at the performance monitoring stage.

In some practical cases the secondary path may suddenly change. This event derives system to diverge. To prevent this effect we have to update $\hat{S}(z)$.

The proposed algorithm is design in the way that it monitors the secondary path changes by the following expression:

$$20\log_{10}[|f(n)|] < 0 ,$$

(15)

If the validity of the above equation does not satisfy, the system reactivates the VSS-LMS algorithm and injects white noise to remodel $\hat{S}(z)$. The same as before, the injection is stopped at the optimum point using (14).
4. Simulations and Performance Results

In this section the proposed ANC system is simulated using Matlab version 7.1. In this simulation, we have used the primary path $P(z)$ and secondary path $S(z)$ of the experimental data provided in [9]. Using these data, $P(z)$ and $S(z)$ are considered as FIR filters with tap-weight lengths 48 and 16 respectively. Rate of the sampling frequency in this simulation was 2 KHz.

To evaluate the performance of the proposed system, we could not find any feedback ANC system with online secondary modelling method to compare with. Thus we implement the existing feedforward ANC systems [3-5] on feedback structure. Except the Eriksson’s method which has been presented on feedback structure in [8].

In this simulations performance of the proposed method is compared with that of Akhtar’s [3], Zhang’s method [5] and Eriksson’s method [4]. Extensive experiments have been performed to find suitable values for a fast and stable performance of the ANC system. Simulations parameters for all of the four methods are set for the most proper situation as described in Table 1.

Length of FIR filter $\hat{S}(z)$ for modelling the secondary path, and length of the adaptive filter $W(z)$ used for the noise cancellation have been chosen 16 and 32, respectively. We have set length of the third filters, $H(z)$, and the delay $\Delta$ in Zhang’s methods to 16. In this evaluation, a white noise with variance of 0.05 has been used as training signal in secondary path modelling for all of the four methods.

We have performed simulations for two separate cases. Comparative results of modelling accuracy and noise reduction for the system are illustrated in Case1. Case2 indicates effectiveness of the proposed algorithm in maintaining its performance against sudden changes of the secondary path behaviour.

It is important to be noted that a white noise with SNR of 30 dB is added to all of the reference noises used in these Cases.

To show the convergence rate and modelling accuracy of the system the relative modelling error is used as defined below:
\[
\Delta S(\text{dB}) = 10 \log_{10} \left( \frac{\sum_{i=0}^{M-1} |s_i(n) - \hat{s}_i(n)|^2}{\sum_{i=0}^{M-1} |s_i(n)|^2} \right)
\]  

(16)

To signify performance of the system on noise reduction the following equation is used:

\[
R = -10 \log_{10} \left( \frac{\sum x^2(n)}{\sum d^2(n)} \right)
\]  

(17)

All the results shown in each case have been obtained as an average 10 different experiments. To set the initial value for \( \hat{s}(t) (i_0) \), off-line secondary path modeling is performed. The off-line modeling is stopped when the modeling error (13) is reduced to -5 dB.

It is interesting to be noted that in both cases the other approaches obtain approximately the same noise reduction (17), while their modelling error (16) are different (see Figures 3, 4 and 6).

4.1. Case 1

For the first experiment the reference noise is a multi-component periodic signal as defined below:

\[
x = 0.89 \sin(2\pi ft) + 0.85 \cos(14\pi ft) + 1.1 \sin(4\pi ft) + 0.79 \cos(8\pi 4ft), f = 23Hz.
\]  

(18)

Figure 3 shows the comparative results on the basis of the modelling accuracy (16) and noise reduction (17).

In the second experiment we use an engine noise at 3700 rpm as reference signal. The comparative results for the approaches are illustrated using (16) and (17) in Fig. 4.

Fig. 3b and Fig. 4b shows that once injection of the white noise is stopped at the optimum point for the proposed algorithm, the noise reduction ratio is accelerated positively.

**Table 1. Simulation parameters for the four approaches.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akhtar’s method</td>
<td>((\mu_u, \mu_{\text{max}}, \mu_{\text{sam}}, \lambda))</td>
<td>(3 \times 10^{-5}, 2.5 \times 10^{-4}, 7.5 \times 10^{-4}, 0.99)</td>
</tr>
<tr>
<td>Zhang’s method</td>
<td>((\mu_u, \mu_z, \beta))</td>
<td>(4 \times 10^{-4}, 1 \times 10^{-2}, 1 \times 10^{-2})</td>
</tr>
<tr>
<td>Eriksson’s method</td>
<td>((\mu_u, \mu_z))</td>
<td>(3 \times 10^{-5}, 1 \times 10^{-2})</td>
</tr>
<tr>
<td>Proposed method</td>
<td>((\mu_u, \mu_{\text{max}}, \mu_{\text{sam}}, \lambda, \gamma, \alpha))</td>
<td>(7 \times 10^{-7}, 4 \times 10^{2}, 9 \times 10^{1}, 0.99, 0.999, 2.1 \times 10^{-9})</td>
</tr>
</tbody>
</table>
Figure 3. Comparison results for the proposed method in Case 1 with the other existing approaches. (a) Noise reduction versus iteration time \( n \), (b) Relative modeling error versus iteration time \( n \).

4.2. Case 2

In this case we assume that the secondary path transfer function suddenly changes during the operation. Figure 5 shows the magnitude response of the original and changed secondary path. In this figure, the solid line represents secondary path at the start point, \( n = 0 \), and the dashed line represents the changed path at iteration \( n = 20000 \). In this experiment the reference signal is a narrowband signal comprising frequencies of 100, 200, 300, and 400 Hz with variance of 2. Figure 5 shows the curve on the basis of the relative modellung error and noise reduction.

5. Conclusions

This paper has proposed a new technique for on-line secondary path modelling in feedback ANC systems. Preventing continuous injection of the white noise increases the performance of the proposed method significantly and makes it more desirable for practical ANC systems. Computer simulations results demonstrate that the proposed method has achieved a high performance in noise attenuation.
Figure 5. Magnitude response of secondary path. (Solid line: Original path, Dashed line: Changed path at $n=20,000$)

Figure 6. Comparison results for the proposed method in Case2. (a) Noise reduction versus iteration time $(n)$, (b) Relative modeling error versus iteration time $(n)$.

References