Chaotic Time Series Prediction Using Optimal Fuzzy Systems Based on Sequential Quadratic Programming- Case Study: Gold Price

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Abstract

This paper presents a fuzzy approach to the prediction of highly nonlinear time-series. The optimized Mamdani-type fuzzy system denoted SQP-FLC is applied for the input-output modeling of measured data. In order to tune fuzzy membership functions, a sequential quadratic programming (SQP) method is employed. The proposed method is evaluated and validated on a highly complex time series, daily gold price data. The time series is primarily investigated for its chaotic properties. Correlation dimension and autocorrelation function (ACF) for the time series are discussed. Accordingly, time delay and embedding dimension are computed. Month selection in each stage is based on computed correlation coefficients. Thus, for the proposed fuzzy predictor, 3, 5, and 7 dynamics are selected and the time series are verified. The simulation results for one-step-ahead prediction of daily gold price in 2010, compared with methods of ANFIS and GA-FLC, demonstrate comparably better performance of the proposed SQP-FLC until the higher significant dynamics of the chaotic trend is taken into account.

Keywords: Chaotic Time Series, Complex Systems, Mamdani-Type Fuzzy Modeling, Optimization, Sequential Quadratic Programming

1. Introduction

Prediction of time series is of high interest to the researchers. They show unexpected behaviors which always needs much experience and expert systems to predict their future data. One of the most noticeable time series is the gold price trend where precise gold price prediction can be important for investors, economists, politicians or any agent who sees gold as an indicator of the future performance of the world's economy.

Looking back to previous works on the prediction of gold price data reveals that the parameters of interest exhibit some temporal dependence. These suggest that nonlinearity is a regular feature of the data that should be modeled and used in forecasting, although variations in parameter values may need to be incorporated [1]-[3].

In the classical study of prediction, the stochastic models such as AutoRegressive (AR), Moving Average (MA), Fractional AutoRegressive Integrated Moving Average ARIMA), Autoregressive Conditional Heteroskedasticity GRACH, etc. are used in papers for prediction of various types of time series. Forecasting the electricity market and prices has been a pertinent field of studying the regression models [4]-[6]. On the

other hand, the use of intelligent systems, capable of learning input-output structures of a highly complex system, such as neural networks , fuzzy systems [7], [8], optimization algorithms and their combinations in order to strengthen their features, has greatly been extended in more accurate prediction of nonlinear or chaotic oriented time series [9]-[12].

Fuzzy systems have shown great abilities in modeling and identification of complex nonlinear systems. TSK-type and Mamdani-type fuzzy controllers are the most well-known in studies and researches. The degrees of freedom of a fuzzy logic controller (FLC) are related to its structure and the associated parameters. The structure is defined by the fuzzy input and output variables and the number and shape of the fuzzy sets for each variable whereas the parameters are related to the distribution of the membership functions on the universe of discourse and the rule base [13].

Thus, a new way of modeling gold price time series is introduced, where the dependency of the signal to its history can be used to forecast, at least in a short-term basis, and where nonlinear models, such as fuzzy controller, can provide successful predictions. It can be seen that the most important characteristic of fuzzy systems is their capacity to learn nonlinear functions based on a finite number of observations. That means that the structure draws regularities from the training data, which would subsequently be validated to new sets of data for prediction. As the models re-train the membership functions or rule base of the fuzzy model, they have biased on the obtained characteristics of the training data set, hence approaching the dynamics of the time series. To encounter the choice of the training set and its size, the selection of the tuning parameters, and the time at which the learning must be enough, are only a few general issues, where experimenting plays an important role. In order to deal with the choice of the training set parameters, optimization algorithms are employed mostly. Evolutionary algorithms are mostly used in training the rule base or membership function parameters of fuzzy logic controllers [14], [15]. In [16], Li and Du use genetic algorithm (GA) for learning the rule base of FLCs accompanied by regulating the input/output scaling in their structures. GA-based fuzzy modeling is also worked for TSK models in [17] and rapid Learning of B-spline fuzzy controllers is discussed in [18]. A more developed type of fuzzy system auto-tuning using extended evolutionary algorithms is addressed in the work of Anzar, et al. [19].

In [20], Gao discusses the Mamdani-type fuzzy controllers as universal fuzzy controllers along with representing a constructive procedure to obtain universal fuzzy controllers. It also introduces dynamical fuzzy models and their ability in approximation which is the basis of modeling a complex time-series in this paper.

This paper is organized as follows. In section 2, fundamentals of chaotic time series prediction are introduced, gold price data 0f 2010 is chosen for experimental work and its chaotic characteristic is discussed. Section 3 introduces proposed Mamdani-type fuzzy prediction structure. Optimization and tuning of fuzzy models based on a nonlinear optimization algorithm, sequential quadratic programming (SQP), are discussed in section 4. Finally, the experimental results of gold price prediction are represented in section 5.

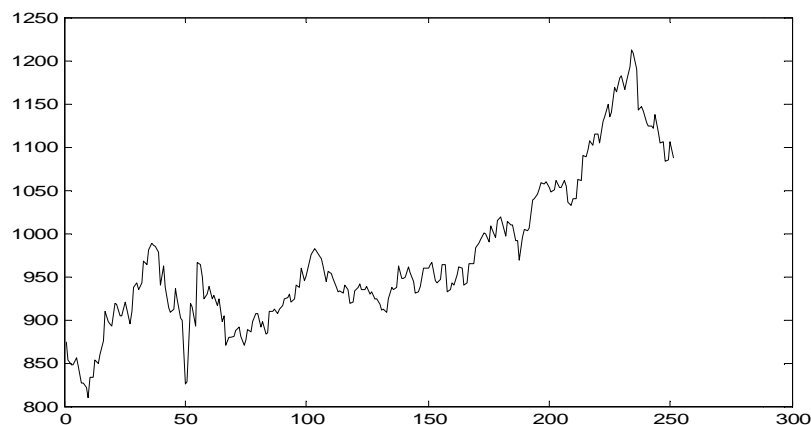## 2. Time series of gold price data

Time series represent unexpected behaviors which make them not to easily predict their future data samples. They could be viewed as complex nonlinear systems. In the case, input-output representation of nonlinear systems works well to these class of models.

In stochastic modeling of the time series, they can be shown as linear-in-parameter functions such as previous data samples [4]-[5].

Taylor in [21] has worked on forecasting time series with seasonal cycles using parsimonious smoothing method. This kind of behavior can be seen in load data as he dealt with. However, gold price time series has a non-cyclic behavior which makes the prediction more and more difficult. Rapidly varying mean and variance gold price series are shown on figure 1. In this paper, the focus is on 2010 gold price data which is inherently more nonlinear in comparison with nearby annual data such as 2009 shown in figure 2. The daily gold price of 2010 is depicted in figure 3 as well. Prices, in this paper, are collected from World Gold Council, taken from Timothy Green's Historical Gold Price Table. It should be noted that London prices are converted to US dollars.
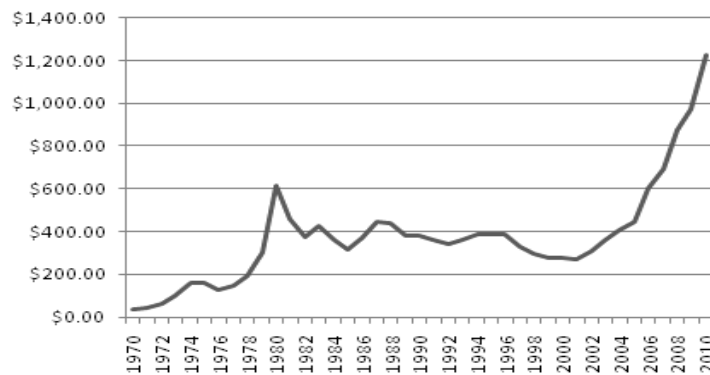
Figure 1. Average annual gold price data from 1970 to 2010 (in USD)

Figure 2. Daily gold price data of 2009 (in USD)

Figure 3. Daily gold price data of 2010 (in USD)

Mainly, the concept of state space is a helpful technique in confronting the time series. Reconstructing the phase space from a time series of a single variable $\{x(t), t = 1,2,..N\}$ suggested by Taken [22], the information on the dynamics of chaotic time series is extracted. A vector at time t in this phase space is constructed with the help of a time lag t as follows:

$$x^{(m)}(t) = \big(x(t), x(t + t), ..., x(t + (m - 1)t)\big) \tag{1}$$

This construction produces a trajectory in m-dimensional space. The minimum reconstruction dimension to unfold the system is called the embedding dimension, which is noted by m. The exact embedding dimension is not known, however, a reasonable attractor map can theoretically be found in a lower embedding dimension of the phase space when chaos exists in systems. In recent years, authors have tried to propose an appropriate method finding the true time delay and embedding dimension of a time series [23], [24]. Autocorrelation function (ACF) where first reaches zero or its minimum value gives time delay. ACF at time delay t is defined as below:

$$\rho_t = E[(x_t - m)(x_{t+t} - m)]/\{E[(x_t - m)].E[(x_{t+t} - m)]\}^{1/2} \tag{2}$$

where m is the expectation of the time series over N. Behavior of ACF as a function of t reveals the dynamics of the chaotic time series.

The minimum reconstruction dimension to detect the system's behavior is called the embedding dimension. The most effective and common method in finding the optimal embedding dimension is false nearest neighbors (FNN). In this method, it is evaluated where a false cut in the state trajectories, obtained by projecting the attractor in a lower dimensional space, stops [25], [26].

Considering delay vectors as:

$$X_i(t) = [y(t), y(t - t), ..., y(t - (m - 1)t)]^T \tag{3}$$

The r-neighbor of a delay vector in equation (3) is defined as:

$$X_r^{NN}(t) = [y(t_r), y(t_r - t), ..., y(t_r - (m - 1)t)]^T, r = 1,2,...,5 \tag{4}$$

The distance between the two neighbor vectors is defined by Euclidean norm:

$$R_m^2 = \sum_{i=0}^{m-1} [y\,(t-i\,t) - y\,(t_r - i\,t)]^2 \tag{5}$$

Increasing the dimension of the space from m to $m+1$, the two added elements of the augmented vectors are respectively $y(t-mt)$ and $y(t_r - mt)$. Then, the distance between the augmented neighbor vectors is as:

$$R_{m+1}^2 = R_m^2 + [y(t-mt) - y(t_r - mt)]^2 \tag{6}$$

So, the difference of the distances rather by the m-dimensional distance is computed as:

$$\sqrt{\frac{R_{m+1}^2 - R_m^2}{R_m^2}} = \frac{|y(t-mt) - y(t_r - mt)|}{R_m} \tag{7}$$

Once the above quantity is upper than a threshold, about 10-15, the regarded neighbor delay vector is selected.

Time delay provides the best separation of neighboring trajectories within the minimum embedding space. The quality of a phase space portrait depends on the time delay and therefore, a reasonable value of the time delay is desired. Thus, for the gold price time series, the first cutoff (zero correlation value) of the ACF occurs at day 82 while the first cutoff (corresponding to 95% confidence level) of the ACF occurs at day 79. Therefore, according to figures 4 and 5, time delay equal to 79 is selected and embedding dimension is evaluated for 3, 5 and 7 in applying the optimal fuzzy method.
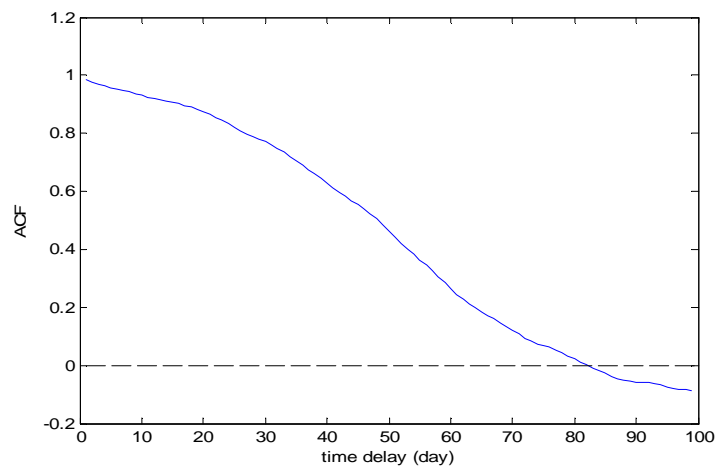
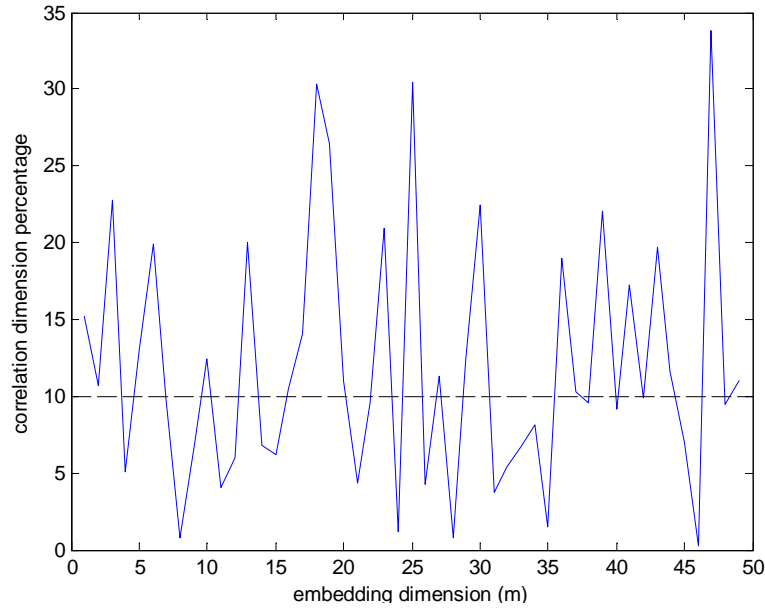*Figure 4, ACF plot for gold price data of 2010*

Figure 5, correlation dimension for gold price data of 2010

3. Proposed optimal Fuzzy scheme for TTME series prediction

3.1 *Mamdani-type Fuzzy system in nonlinear input-output modeling*

Model of the Mamdani fuzzy controller is based on rule base as its expert system and input-output membership functions accompanied with fuzzification and defuzzification parts.

In order to design a Mamdani fuzzy logic with a compact rule base, a rule R_i is written as:

If X_1 is A_{1j_1} and X_2 is A_{2j_2} and ... Then U is $w_{ik}B_k$ (8)

A_{1j_i} and B_k , $l = 1, ..., n$, $j_1 = 1, ..., n_1$, $k = 1, ..., m$, are respectively, fuzzy sets associated with the n fuzzy input variables, each partitioned into n_j fuzzy sets, and the fuzzy output variable, partitioned into m fuzzy sets. w_{ik} is a binary variable that determines the consequence of the rule, where subscript i refers to the rule and subscript

k refers to the output fuzzy set, $i = 1, ..., r$, $k = 1, ..., m$, with $r = n_1 \times n_2 \times ... \times n_n$, the

total number of candidate rules.

Thus, if $w_{ik} = 0 \, \forall k$, , $k = 1, ..., m$, rule i has no consequence and will not be

included in the controller rule base. In this case,

$$\sum_{k=1}^{m} w_{ik} = 0 \, \forall i = 1, \ldots, r$$ (9)

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And if $w_{ik} = 0$ for some k, then the consequence of i is B_k. In the case,

$$\sum_{k=1}^{m} w_{ik} = 1 \forall i = 1, \dots, r \tag{10}$$

Since every time $w_{ik} = 1$, a rule is included in the rule base, the total number of rules included in the controller rule base is given by:

$$J = \sum_{k=1}^{r} \sum_{k=1}^{m} w_{ik} \tag{11}$$

The crisp output u of the fuzzy controller is computed with the center of gravity

defuzzification formula

$$u = \frac{\sum_{k=1}^{m} m_k \, m_{B_k}(u)}{\sum_{k=1}^{m} m_{B_k}(u)} \tag{12}$$

with m_k, the center of fuzzy set B_k.
Using the product maximum inference mechanism

$$\mu_{B_k}(u) = \max\left(\mu_{R_1} w_{1k}, \mu_{R_2} w_{2k}, \dots, \mu_{R_r} w_{nk}\right) \tag{13}$$

and

$$\mu_{R_1} = \mu_{A_{1j_1}}(x_n) \mu_{A_{2j_2}}(x_n) \dots \mu_{A_{nj_n}}(x_n) \tag{14}$$

After multiplication by w_{ik}, the brackets of the left hand side of equation (14) will contain only the rules with consequence B_k.

$$\mu_{B_{lj}}(x) = \max\left(0, \min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right)\right) \tag{15}$$

When the fuzzy controller is introduced in a closed loop, the stability of the closed loop system depends on the triplets (a,b,c) associated with the input variables fuzzy sets, the centers of the output variable fuzzy sets, d_k, and the binary variables, w_{ik}, defining the rule base. The controller design or fuzzy modeling problems can be expressed as finding all these parameters in order to achieve stability, some performance design criteria, and compactness of the rule bases and in our case, exact approximation of complex chaotic time series.

One way of representing dynamical systems is the input/output representation which is used a lot in papers. For a time-series of which there is no more information other than some data series, identifying the dynamics is a major problem.

In the case, Mamdani-type fuzzy systems are capable of modeling such these complex dynamics due to their inherent nonlinear structure. Combining with an optimization of the rule base and membership function parameters, they behave as a training system to match its parameters with the data evaluation as well.

Regarding complex behaviors of chaotic time series, modeling and identification of these systems need more attention to include the higher significant dynamics of the system. Here, we have proposed to consider the model as a fuzzy logic system of Mamdani-type. The proposed structure of fuzzy identification for gold price time series is depicted in figure 4.
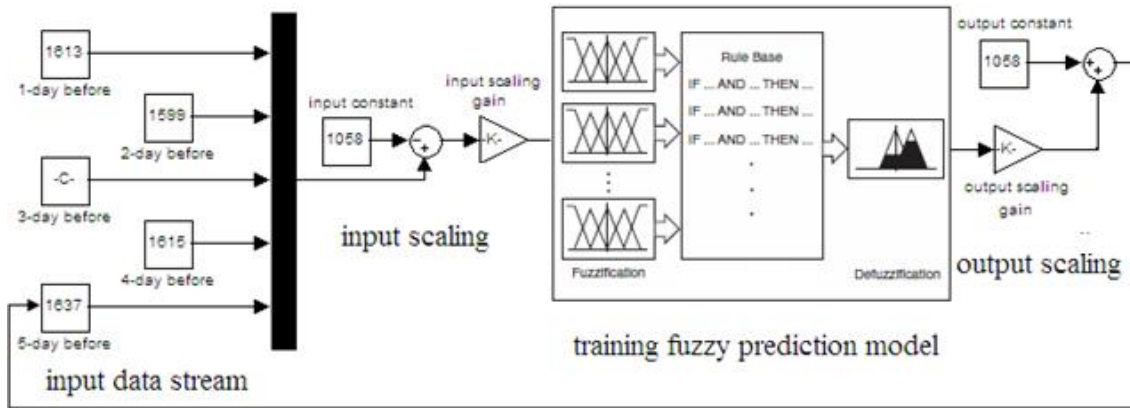
Figure 4, Structure of the proposed 5-input Mamdani-type fuzzy representation of time series

3.2 Sequential quadratic programming optimization method

Training the fuzzy model with evaluation data makes the system to represent true modeling of the dynamics hidden in the time series until the performance is satisfied. However, this training procedure needs an optimization of the system to be truly trained. As the fuzzy system is a type of nonlinear system, it could model complex systems well. On the other hand, optimization algorithm could handle with a nonlinear cost function, In this paper, a quadratic programming (QP) is used which solves quadratic programming sub-problems at each iteration. Updating an estimate of the Hessian of the Lagrangian, is considered at each iteration.

SQP methods are the state of the art in nonlinear programming methods in terms of efficiency, accuracy, and percentage of successful solutions, over a large number of test problems. A General Problem (GP) description is stated as:

$$\min_x f(x) \tag{16}$$

subject to

$$G_i(x) = 0 \, , i = 1, \dots, m_e \tag{17}$$

$$G_i(x) \leq 0 \, , i = m_e + 1, \dots, m \tag{18}$$

where x is the vector of length n design parameters, f(x) is the objective function, which returns a scalar value, and the vector function G(x) returns a vector of length m containing the values of the equality and inequality constraints evaluated at x. In our work, x is all the parameters of the inputs-output 3-membership functions of the Gaussian shape. Each membership function of the Gaussian form needs two parameters to be optimized. Thus, vector x in our problem is of dimension $dim = 3 \times 2 \times (n + 1)$ where n is the number of fuzzy inputs.

Consequently, n is of dimension 24, 36 and 48 for 3, 5 and 7 inputs respectively.

There are both lower bound and upper bound inequality constraints on the parameter design x such that

$$G_i(x) = x \leq 1, i = 1, \ldots, dim \tag{19}$$

$$G_i(x) = x \geq 0, i = 1, \ldots, dim \tag{20}$$

Given the problem description in General Problem, the principal idea is the formulation of a QP sub-problem based on a quadratic approximation of the Lagrangian function as:

$$L(x, \lambda) = f(x) + \sum_{i=1}^{dim} \lambda_i g_i(x) = f(x) + \sum_{i=1}^{dim} \lambda_i x_i \tag{21}$$

The algorithm performs a line search using a merit function similar to that proposed by [27], [28]. The QP sub-problem is solved using an active set strategy similar to that described in [29]. QP sub-problem is then:

$$\min_{d, R^n} \frac{1}{2} d^T H_k d + \nabla f(x_k)^T d \tag{22}$$

$$\nabla g_i(x_k)^T d + g_i(x_k) \leq 1, i = 1, \ldots, \dim \tag{23}$$

$$\nabla g_i(x_k)^T d + g_i(x_k) \geq 0, i = 1, \ldots, \dim \tag{24}$$

where H and $\nabla f(x_k)$ are the Hessian and gradient of the lagrangian function.

The SQP implementation introduced by Nocedal in [29] consists of three main stages including:
  a) Updating the Hessian Matrix
  b) Quadratic Programming Solution
  c) Line Search and Merit Function

At each major iteration a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function, H, is calculated.

The method used in is an active set strategy (also known as a projection method). It has been modified for both Linear Programming (LP) and Quadratic Programming (QP) problems.

The solution procedure involves two phases. The first phase involves the calculation of a feasible point (if one exists). The second phase involves the generation of an iterative sequence of feasible points that converge to the solution. Indeed, virtually all QP algorithms are active set methods [30], [31].

Objective function, f(x), in our problem is defined by three well-known norms as Mean Absolute Error (MAE), Median Absolute Deviation (MAD), and Mean Square Error (MSE) by

$$f_{MAE}(x) = \frac{1}{n} \sum_{i=1}^{n} |e_i| = \frac{1}{n} \sum_{i=1}^{n} |y_i - d_i| \tag{25}$$

$$f_{MAD}(x) = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - d_i}{y_i} \right| \tag{26}$$

$$f_{MSE}(x) = \frac{1}{n} \sum_{i=1}^{n} (y_i - d_i)^2 \tag{27}$$

where y_i and d_i are the true and predicted value in each iteration.

4. Simulation results

The procedure of the proposed fuzzy prediction, made up of two stages, is applied on time series data of world gold price. Data, as expressed in section 2, is inherited from World Gold Council. London prices converted to US dollars for consistency. In the first stage, the fuzzy system is learnt by the daily price data of some months in 2010. Then, the model is validated by some months after than those before. To select months for identification and prediction stages, we have used the correlation analysis method. Correlation coefficients of the daily gold price data of months in 2010 are represented in table 1.

TABLE 1, Correlation coefficients of daily gold price data of months of 2010

2010	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Jan.	1	-0.27	-0.44	0.290	0.52	-0.18	0.26	-0.21	-0.33	0.93	-0.32	-0.31
Feb.	-0.27	1	0.79	-0.41	-0.18	0.42	-0.32	0.35	-0.33	-0.23	0.86	-0.18
Mar.	-0.44	0.79	1	-0.44	-0.47	0.11	-0.37	0.22	-0.38	-0.41	0.92	-0.17
Apr.	0.29	-0.41	-0.44	1	-0.17	-0.18	0.86	-0.51	0.42	0.35	-0.32	0.38
May	0.52	-0.18	-0.47	-0.17	1	-0.06	-0.22	0.30	-0.36	0.44	-0.36	-0.49
Jun.	-0.18	0.42	0.11	-0.18	-0.06	1	-0.25	-0.14	0.21	-0.13	0.21	0.06
Jul.	0.26	-0.32	-0.37	0.86	-0.22	-0.25	1	-0.44	0.37	0.32	-0.25	0.38
Aug.	-0.21	0.35	0.22	-0.51	0.30	-0.14	-0.44	1	-0.45	-0.33	0.16	-0.42
Sep.	-0.33	-0.33	-0.37	0.42	-0.36	0.21	0.38	-0.45	1	-0.29	-0.25	0.79
Oct.	0.93	-0.23	-0.41	0.35	0.44	-0.13	0.32	-0.33	-0.29	1	-0.29	-0.25
Nov.	-0.32	0.86	0.92	-0.32	-0.36	0.21	-0.25	0.16	-0.25	-0.29	1	-0.08
Dec.	-0.31	-0.18	-0.17	0.38	-0.49	0.06	0.38	-0.4	0.79	-0.25	-0.08	1

It should be noted that months chosen for training the fuzzy system should be of adjacent months due to the fact that there are made apart of a whole data series and the trained system shouldn't mixed up with another data series of non-adjacent parts.

The model considers a one-step-ahead forecasting method with one output variable given by the available gold price, y_t. The input variables are the lagged gold price by one to three (five or seven, in other cases) periods of $y_{t-1}, y_{t-2}, y_{t-3}$ ($y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4}, y_{t-5}$) or ($y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4}, y_{t-5}, y_{t-6}, y_{t-7}$). The model functional form is expressed in equation (28):

$$y_t = f\left(y_{t-1}, y_{t-2}, ..., y_{t-7}\right) \tag{28}$$

where the function f is defined in equations (25)-(27).

Considering the computed correlation coefficients in table I, the data of February and March are selected for training the fuzzy model and data series of June and August are carried out for validating the model. It should be mentioned that days off are omitted each month due to the unavailability of the data. Thus, the number of 58 time series observations is selected for evaluating the proposed structure. Also, two datasets of 22 and 23 data are selected and compared for prediction step.

The Mamdani-type fuzzy system is treated in three cases to include 3, 5 and 7 previous daily data samples to determine whether the most important dynamics of the

system are taken into account. The selected evaluation criteria are MAD, MAE and MSE in each case. The value of the minimized cost functions shows at least 7 data samples of previous days are needed to model the supposed gold data exactly. In other words, dynamics of order at least 7 should be regarded as the approximation model of our time series in this paper.

Comparing the performance of the proposed model, the time series prediction is also carried out via two mostly accepted methods in the area, ANFIS and GA based fuzzy model (GA-FLC). Amongst soft computing models, these comparing methods referred in [9], [12], [14], [17] have been paid much attention for their simplicity in usage and ability in obtaining precise prediction performance. Proposed fuzzy scheme,SQP-FLC, is then compared in different cases. Predicted daily gold price of August 2010 compared with true data and the error, difference of predicted and real data, for 3-input, 5-input, and 7-input fuzzy models are shown in figures 5-7. Similarly, prediction is carried out for data of June in figures 8-10.
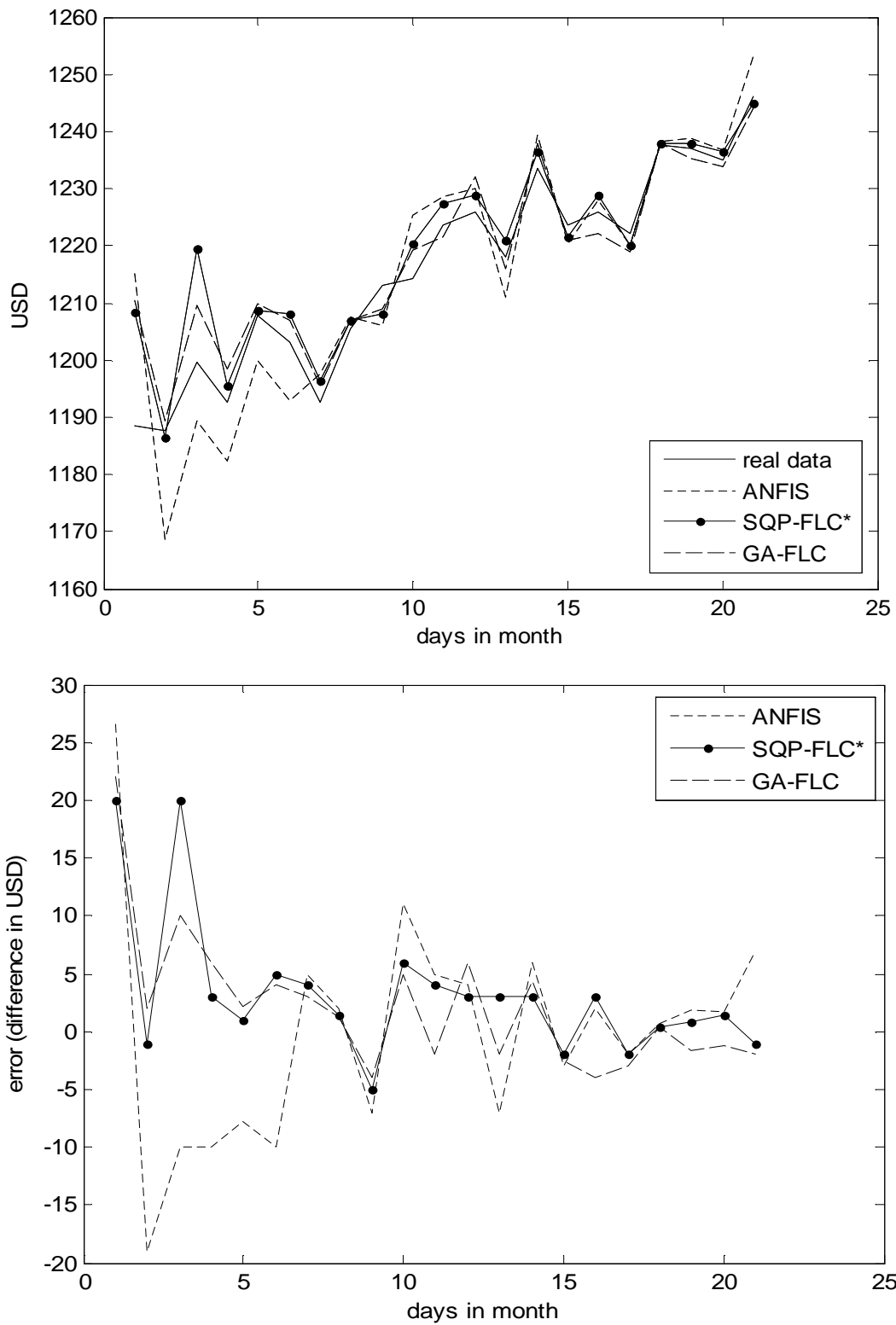
Figure 5, Predicted daily gold price of August 2010 compared with true data (above) and the relative error percentage (below) for 3-input fuzzy model
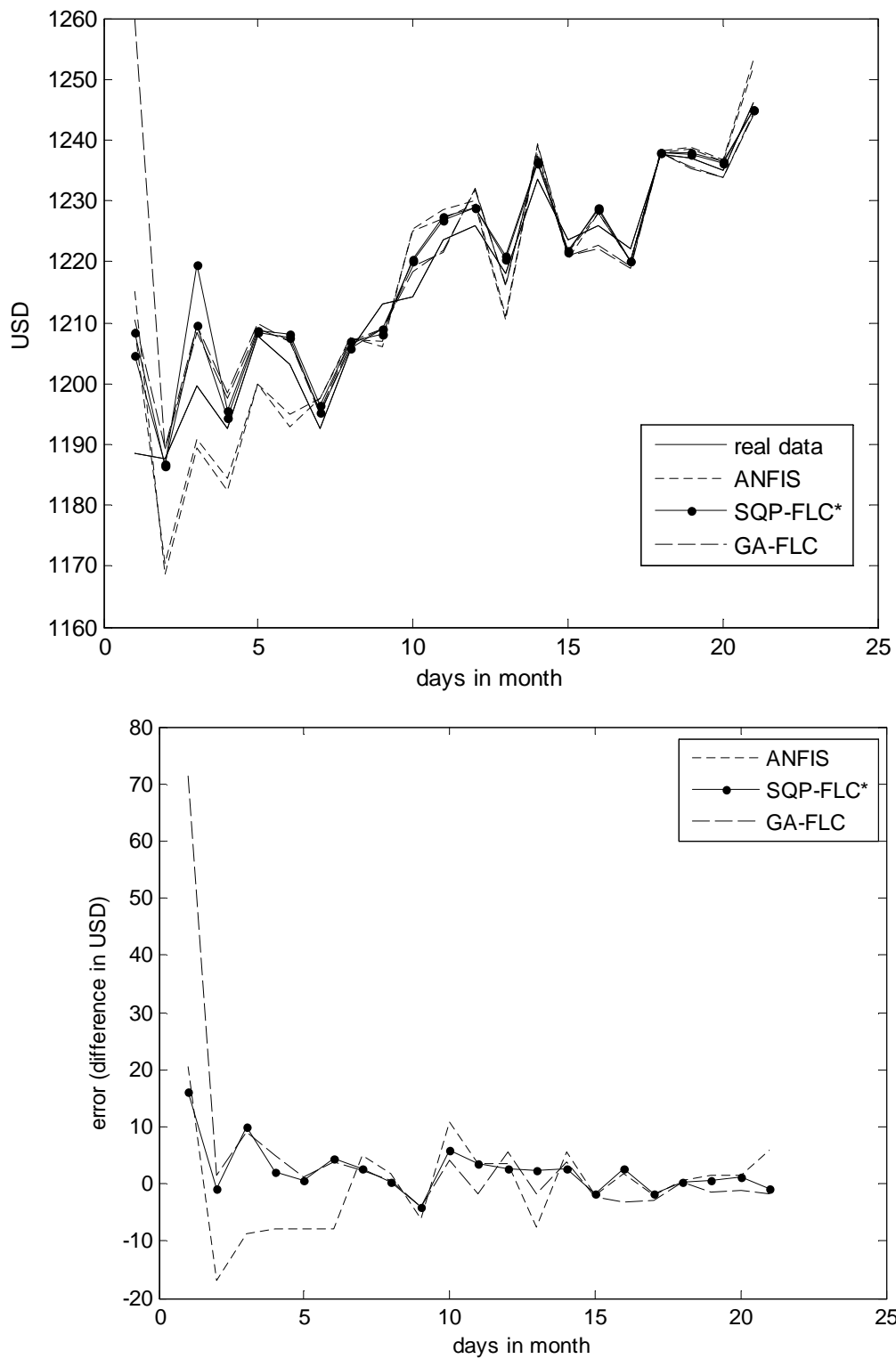
Figure 6, Predicted daily gold price of August 2010 compared with true data (above) and the relative error percentage (below) for 5-input fuzzy model
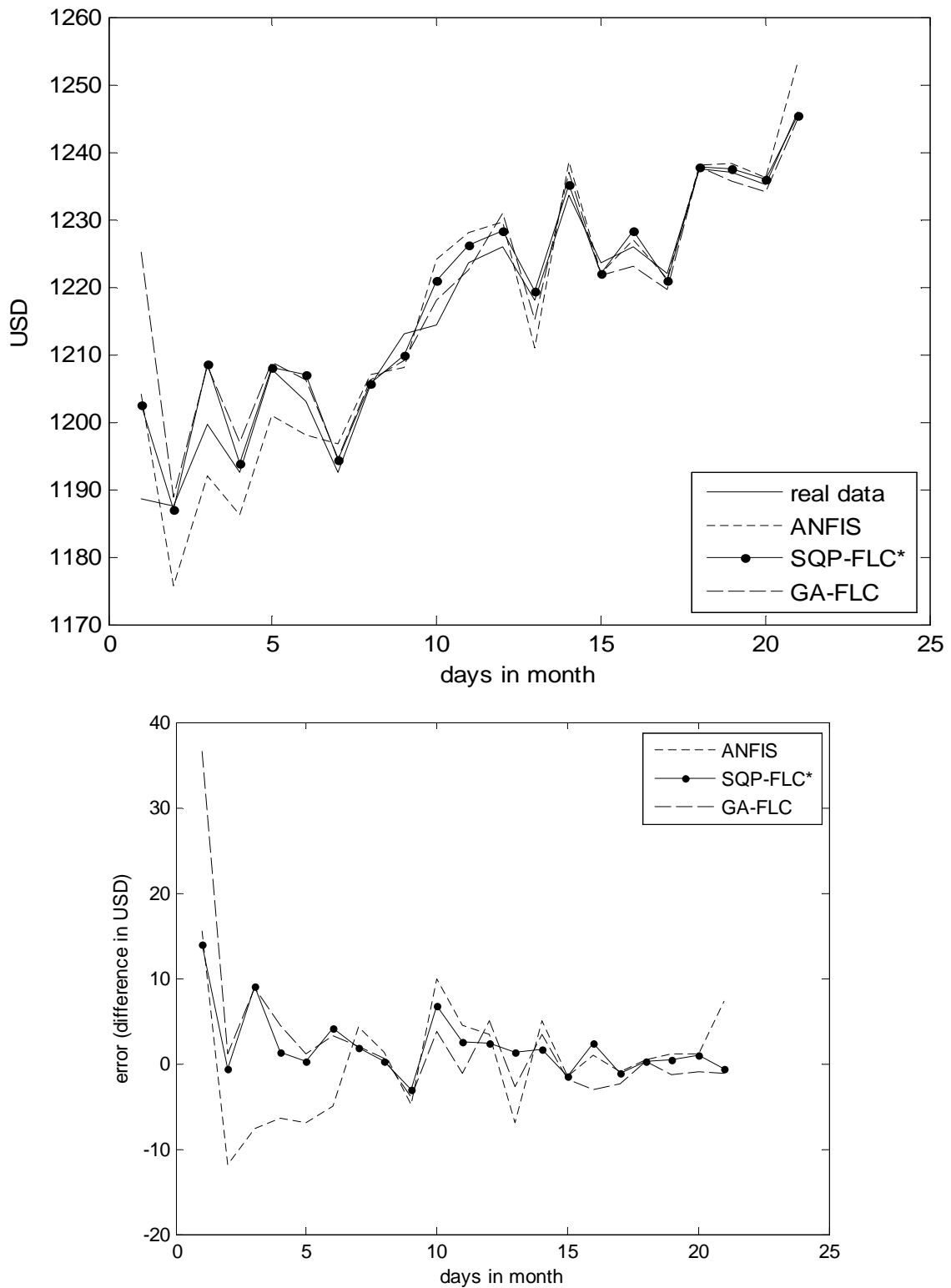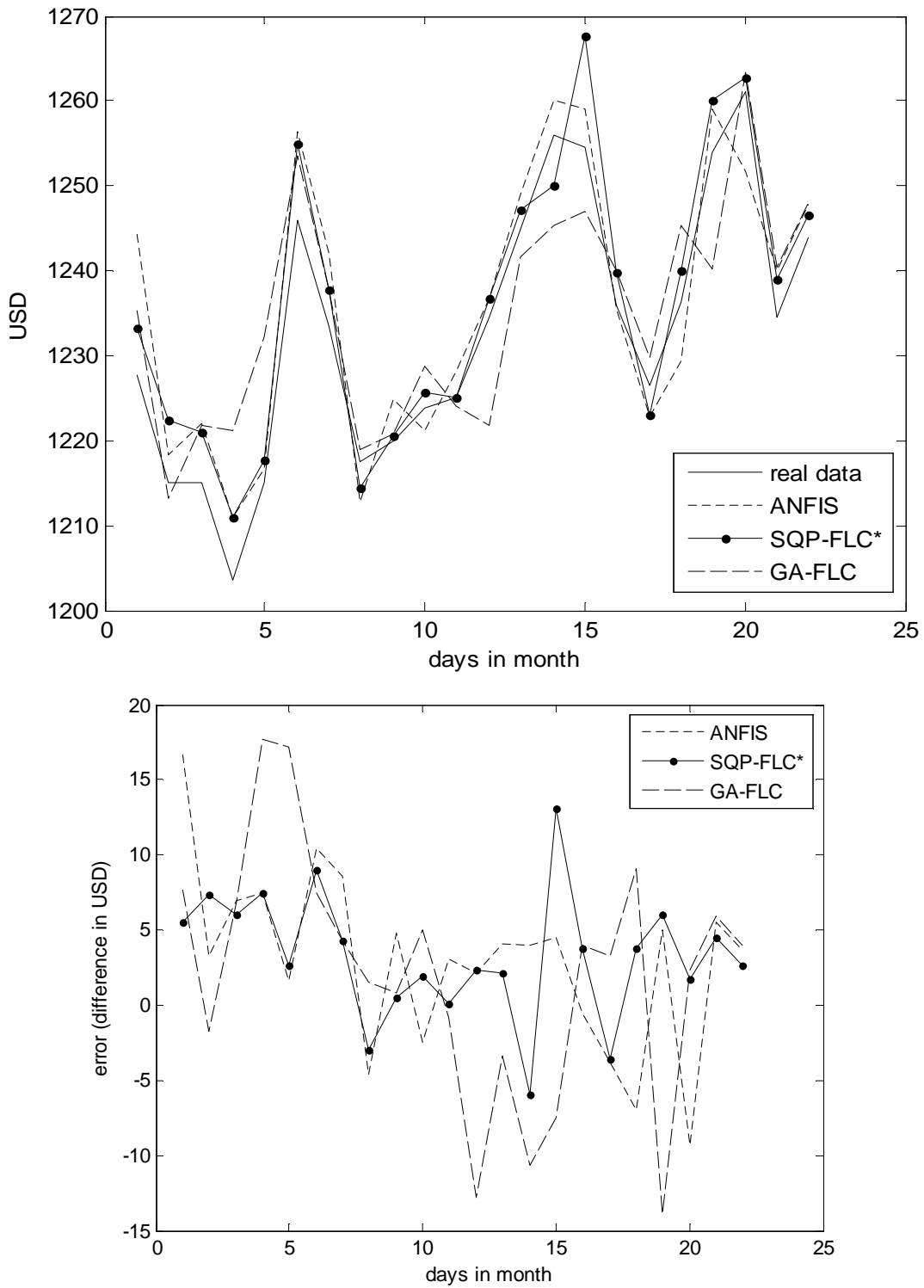
Figure 7, Predicted daily gold price of August 2010 compared with true data (above) and the relative error percentage (below) for 7-input fuzzy model

Figure 8, Predicted daily gold price of June 2010 compared with true data (above) and the relative error percentage (below) for 3-input fuzzy model.
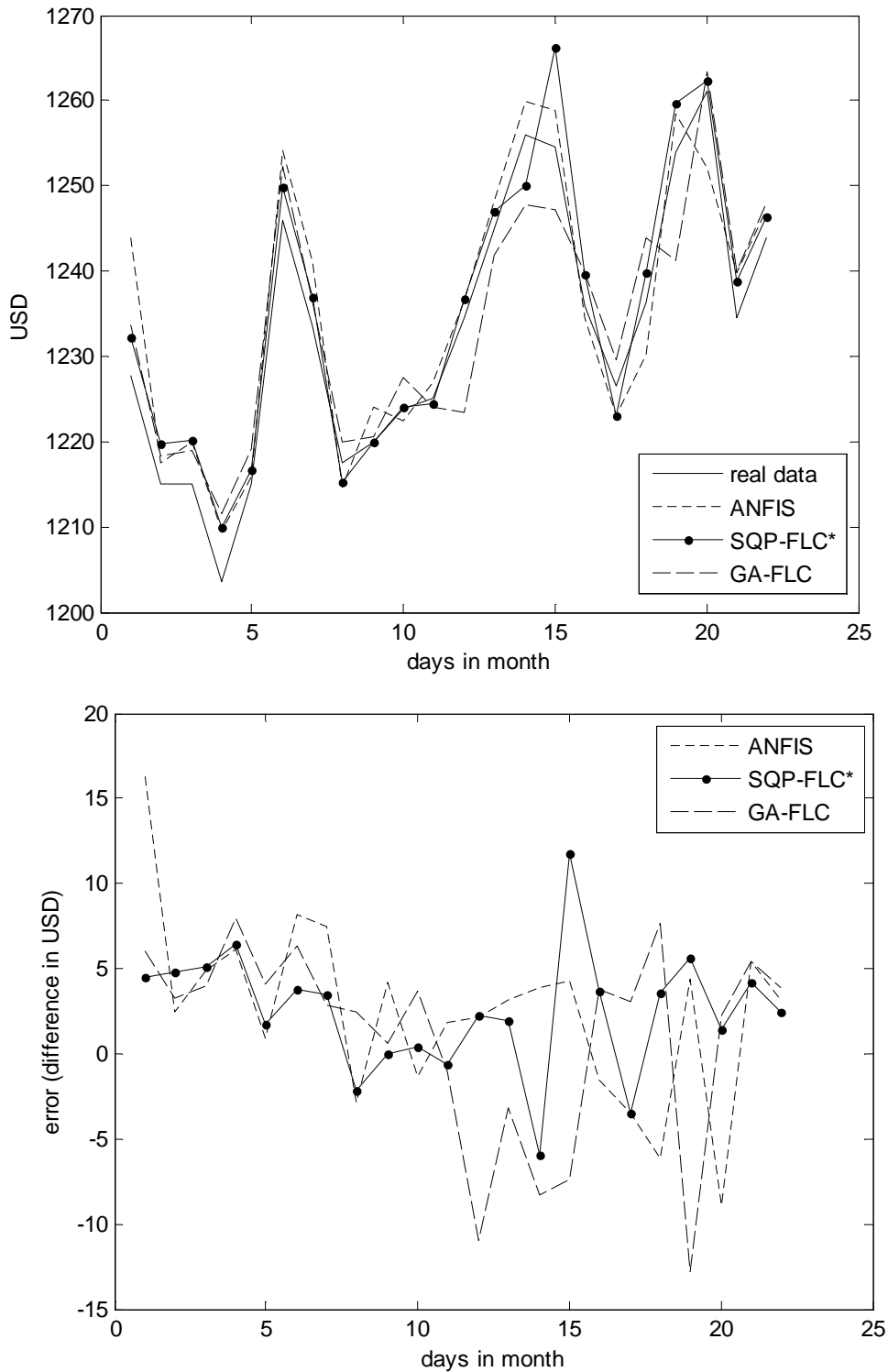
Figure 9, Predicted daily gold price of June 2010 compared with true data (above) and the relative error percentage (below) for 5-input fuzzy model
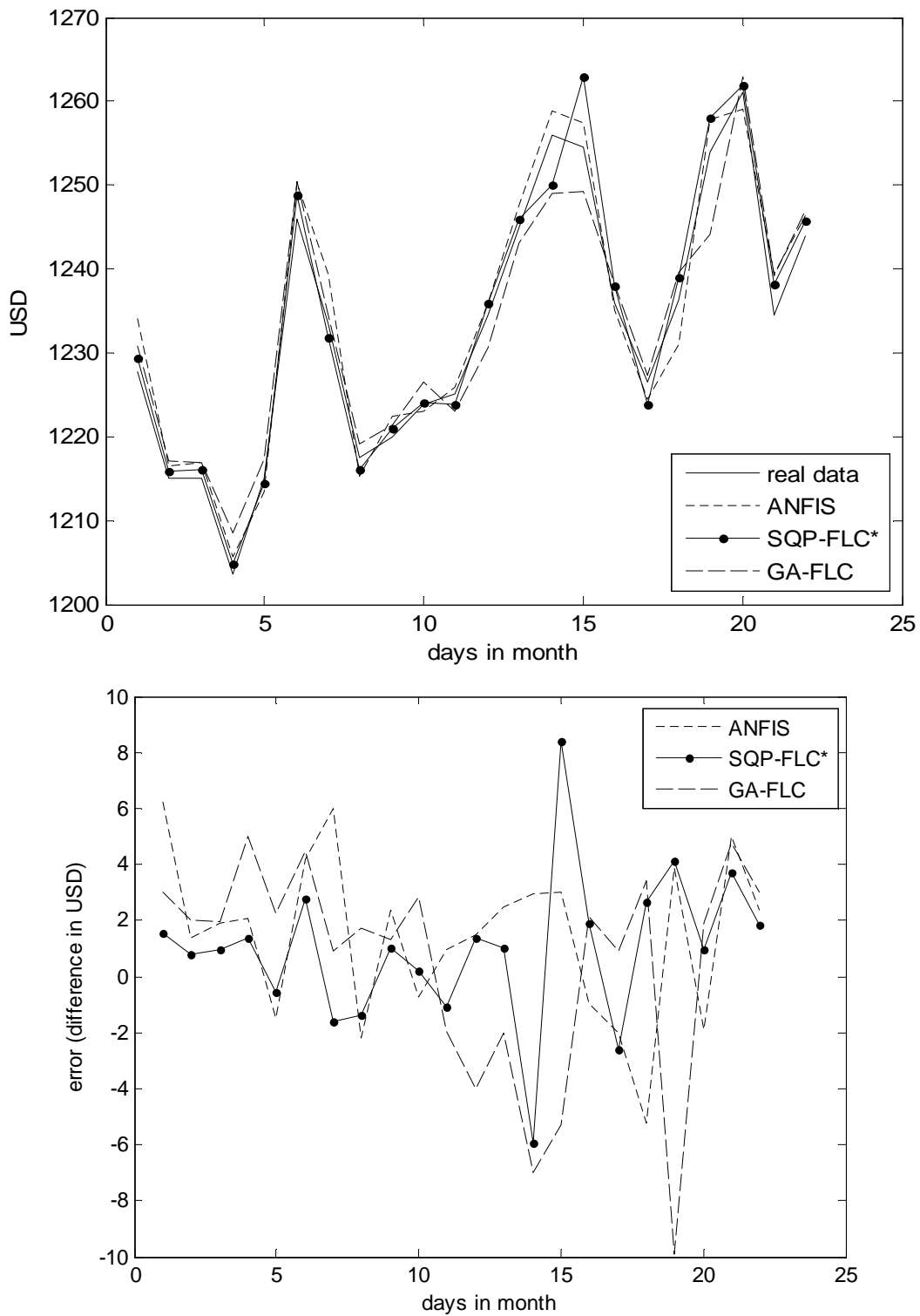
Figure 10, Predicted daily gold price of June 2010 compared with true data (above) and the relative error percentage (below) for 7-input fuzzy model

Prediction results are shown comparably pleasing. Comparing results reveal that prediction performance for data of June that is most correlated with the evaluation time series, data of February and March, is more precise. More importantly, the 7-input fuzzy scheme performs much better rather than those of 3-inout and 5-input ones. This fact reveals that considering higher dynamics of the chaotic time series into the model can do its best to have a more exact approximation of the true model or series. In section 2, this fact was discussed that delay or lag of 3, 5, and 7 are more proper for modeling the chaotic time series of gold price. As a matter of fact, simulation results justify that the best lag number for reconstructing time series is 7 that explained in section 2.

The convergence of the first and second elements (mean and variance of the first fuzzy Gaussian membership function of the first input) of optimized design parameter, vector x, can be seen in figure 11 for 7-input fuzzy model. The SQP optimization applied for fuzzy auto-tuning then guarantees the convergence of the proposed SQP-FLC.

Figure 11.a   Figure 11.b
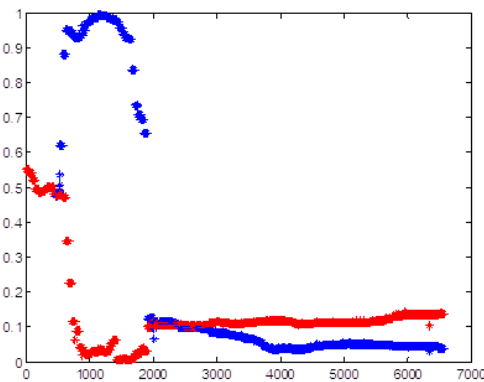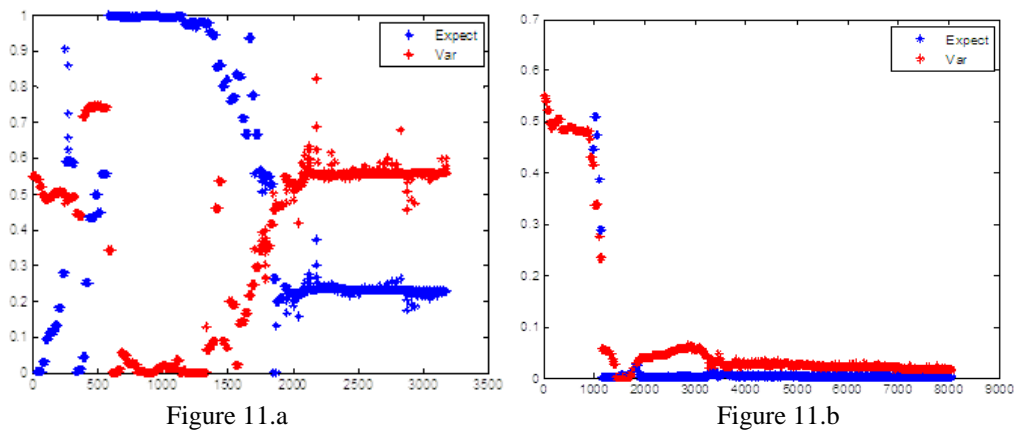

Figure 11.c

Figure 11, Mean and variance of the first optimized Gaussian membership function of the first fuzzy input. (a) 3-input fuzzy model, (b) 5-input fuzzy model, (c) 7-input fuzzy model

Finally, the norm values computed in each case are represented in table 2, 3 for verification of the prediction performance of August and June. The values justify that

considering higher dynamics in modeling leads to much more exact approximation of a time series where the best performance is procured once the lag of 7 is selected.

The various error measures as verification criteria show great performance of the proposed fuzzy prediction structure. However, the performance is much better for prediction of June daily price in comparison with August due to higher correlation coefficient of August relative to February and March (the two evaluation data set for fuzzy model). Results are compared with conventional widely used structures, ANFIS and GA-FLC for different cases. It is obviously observed that SQP-FLC has shown better prediction performance in all the cases except for 3-lagged scheme. This happened because the genetic algorithm for auto-tuning fuzzy structure has a stochastic nature. It may has a best solution to an optimization problem. While, in many cases the results are not satisfied and the prediction performance is even less accurate than ANFIS. Conclusively, proposed SQP-FLC has a trusted performance in each case that is comparably better and more accurate.

TABLE 2, Comparison of August prediction error (MAE, MAD, MSE) for 3,5,7-lagged structures

Prediction Error	MAE	MAD	MSE
Anfis (3-lagged)	6.7725	0.0056	82.7935
SQP-FLC (3-lagged)	4.0955	0.0034	44.4950
GA-FLC (3-lagged)	4.0318	0.0033	36.3550
Anfis (5-lagged)	5.8423	0.0048	59.7129
SQP-FLC (5-lagged)	3.0518	0.0025	22.0685
GA-FLC (5-lagged)	5.8805	0.0049	243.5917
Anfis (7-lagged)	4.8764	0.0040	39.2350
SQP-FLC (7-lagged)	2.5732	0.0021	17.3639
GA-FLC (7-lagged)	4.0429	0.0034	70.2912

TABLE 3, Comparison of June prediction error (MAE, MAD, MSE) for 3,5,7-lagged structures

Prediction Error	MAE	MAD	MSE
Anfis (3-lagged)	5.1956	0.0042	39.7078
SQP-FLC (3-lagged)	4.2283	0.0034	26.9369
GA-FLC (3-lagged)	6.4243	0.0052	66.0937
Anfis (5-lagged)	4.4709	0.0036	31.4357
SQP-FLC (5-lagged)	3.4525	0.0028	18.4372
GA-FLC (5-lagged)	4.8425	0.0039	33.2610
Anfis (7-lagged)	2.6509	0.0021	9.7700
SQP-FLC (7-lagged)	2.0861	0.0017	7.9192
GA-FLC (7-lagged)	3.1179	0.0025	14.4266

5. Conclusion and further discussion

In this paper, a fuzzy modeling approach denoted SQP-FLC is presented for prediction of chaotic time series. Considering chaotic measure of a time series, SQP-FLC shows great ability to obtain an acceptable approximation of the time series. The proposed structure is applied to gold price prediction. According to correlation analysis, two months of February and March are chosen for evaluation of the optimized fuzzy model and daily gold price of June and August are predicted. In order to determine the order of fuzzy model, it is tested with 3, 5, and 7 inputs in each stage where the most proper in chaotic analysis of the time series. The simulation results justify that the

optimized fuzzy model can represent significant prediction power in comparison with two most used methods in the literature. However, in the case of considering more previous data samples, i.e., taking into account of higher dynamics, the fuzzy model works much better where the best performance obtained for 7-lagged structure. Computed verification criteria show the more accurate trusted prediction performance of the proposed SQP-FLC in almost every case.

6. References

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