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Observer-Based Control for a Modified Duffing System Using Twisting Algorithm

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Abstract

This paper presents a new observer-based control scheme for a class of nonlinear systems. In the proposed method, nonlinear observer and twisting algorithm controller are employed to realize a sensor-less control strategy for complex systems which makes use of non measurable process information instead of installing as many sensors as possible. Due to lack of availability of the complex system states, controlling of them will be faced with undesirable performance. This deficiency can be solved with adding observer in the control strategy. In order to estimate unavailable states, an adaptive neural network observer is considered in the present article. This observer is tuned online and no exact information of the nonlinear function in the observed system is required. Also, to realize control purpose, 2nd order sliding mode controller called twisting algorithm is located in the close loop structure. This control strategy is implemented on the modified Duffing chaotic system and the simulation results confirm the capability of this method.

Keywords: *twisting algorithm, adaptive neural network observer, Modified Duffing system, chaos*

1. Introduction

Control of nonlinear system with complex dynamic has attracted extensive interest from different fields of science and engineering. One of the most popular of them is chaotic system. The main characteristic of chaotic system is that the reaction of the initial condition affects extensively the response. Due to unpredictable behavior of chaotic state especially in physical systems, chaos control is an important task. Various controllers based on both feedback and without feedback are proposed to control this phenomenon [1-2], such as controller with adaptive structure [3-4] or using back-stepping technique [5]. The most common performance of these different controls method is that the internal state variable are assumed to be available to construct the control forces; however, due to complexity of systems, limited state information may be available and only the process output can be measured. Under this circumstance, an observer with desirable performance in nonlinear system must be employed to estimate unavailable states. In this letter, a new observer-based approach is presented to control a class of chaotic systems. To control unpredictable behavior of chaotic system, a 2nd order sliding mode controller is employed. This controller is based on a kind of robust control method that is suitable for controlling uncertainty of systems [6, 7]. In first order sliding mode, the sliding variable is chosen such that it has relative degree of one with respect to the control. The first derivative of the sliding variable $(\dot{s})$ keeps the system trajectory in the sliding set $s=0$. As expected the relative degree restriction of the first order sliding mode causes analysts to improve this method. Therefore, the higher sliding mode methods were presented to control some higher-relative degree systems [8, 9]. Among them a 2nd order sliding mode is more popular which includes twisting algorithm, super twisting algorithm, drift algorithm and suboptimal algorithm [10-13]. Second order sliding mode reduce a chattering phenomenon by second derivative of sliding variable $(\ddot{s})$ to keep the system trajectory in the sliding mode set $\dot{s}=s=0$. In this study, we will discuss about twisting algorithm.

To estimate the unavailable states of system, a nonlinear observer is considered. Since the 1960's, many papers have been developed to overcome the problem of state estimation. Therefore, several nonlinear state estimation schemes have been presented to improve the accuracy of designing the control systems, such as extended Kalman filter and unscented Kalman filter. The literatures about these observers are extensive so that we refer the reader to [14-17]. A bunch of the observers that have been proposed is adaptive observer. The first nonlinear adaptive observer was presented in [18]. According to different circumstances, several observers were proposed in [19-21]. In this study, we use the adaptive neural network observer in which the neural network weights are tuned online [22].

The neural network can estimate any nonlinear system. The training process of neural network is implemented online and offline, but online weight adjusting is so useful and can be implemented in experimental projects. Online training has been used in nonlinear modeling, estimating and identification.

The rest of paper is organized as follows:

In section 2, the performance of twisting algorithm is presented. In section 3, the characteristic of proposed observer is rendered. In the next section, the Modified Duffing chaotic system is given. The simulation results are demonstrated in section 5 and in the last section, the conclusion of this paper is presented.

2. Twisting Algorithm

This algorithm is a systematic algorithm to apply for dynamic with relative degree two. The dynamic is represented in the following state space format:

$$\dot{x}=Ax(t)+Bu(t), \qquad y=Cx, \qquad e=r-y \tag{1}$$

where $x\in R^{n}$, $u\in R^{1}$, r are state, control effort and the command signal respectively. The output is denoted by y and e stands for the error signal. The control signal u, of the twisting algorithm is presented as follows [23]:

$$u(t)=c_1.\text{sgn}(e)+c_2\,\text{sgn}(\dot{e}) \tag{2}$$

Where c_1,c_2 are positive coefficients. The block diagram of twisting algorithm is as follows:

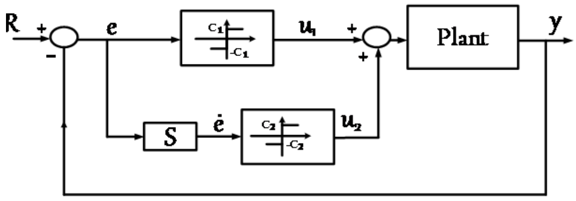

Figure 1. Block diagram of twisting algorithm

3. Adaptive Neural Network Observer

Consider the following single input-single output system assuming that pair of (A,C) is observable i.e. of a canonical observer form [22]:

$$\begin{aligned} \dot{x} &= Ax + b[f(x) + g(x)u + d(t)] \\ y &= C^T x \end{aligned} \tag{3}$$

That $x \in R^n, y \in R, u \in R, b \in R^n$ and $d(t)$ is the unknown disturbance with known upper bound and $f, g : R^n \to R$ unknown smooth function.

The linear system is defined as an observer canonical form if A and C are given as follows:

$$\dot{x} = Ax$$
$$y = C^T x$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & & & \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 0 \\ . \\ . \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

However there is no restriction on the input matrix coefficient i.e. b. The observer dynamic as a replica of the system states but includes a correction term which is considered as follows:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + b[\hat{f}(\hat{x}) + \hat{g}(\hat{x})u - v(t)] + k[y - C^T \hat{x}] \\ \hat{y} &= C^T x \end{aligned} \tag{5}$$

That $\hat{x}$ denotes the estimates of state x and $K = [k_1\, k_2 \,.........\, k_n]^T$ is the observer gain chosen where $(A - KC^T)$ is strictly Hurwitz. $v(t)$ is robustifying term to control disturbance.

The neural network equation that used in this observer is as follows:

$$
\begin{aligned}
f(x) &= W_f^T \sigma_f(x) + \varepsilon_f \\
g(x) &= W_g^T \sigma_g(x) + \varepsilon_g
\end{aligned} \tag{6}
$$

Where consist of two layers, the weight of first layer will be $V = I$ but the second layer weight must be tuned.

One of the advantages of neural network that is used in this paper is that there is no need to data for training. In fact, in [22]; an equation is proposed for training the network using the equations correspondent to the stability of system and provides the value of network weight for system at each moment. Training of this neural network for two nonlinear terms is achieved from the following differential equation:

$$
\begin{aligned}
\dot{\hat{W}}_f &= F_f\, \hat{\sigma}_f\, \tilde{y} - k_f F_f\, |\tilde{y}| \hat{W}_f \\
\dot{\hat{W}}_g &= F_g\, \hat{\sigma}_g\, \tilde{y}\, u - k_g F_g\, |\tilde{y}| \hat{W}_g
\end{aligned} \tag{7}
$$

Where σ denotes the activation function and $\tilde{y} = y - \hat{y}$.Activation function is applied in the neural networks in different ways. In this observer The activation function for first layer is chosen sigmoid and for second layer is chosen purline.

4. Modified Duffing system

The modified Duffing system is derived from Metamorphic shape-changing Underwater autonomous vehicle (MUV). The MUV can imitate the swimming of amoebae, hence the noise generated by propulsion system is reduced the performance of the vehicle. Numerous experiments denote that the equation (8) can represent actual MUV system with desirable approximation [24].

$$
\ddot{x} + (a + b\cos wt)\,\dot{x} + p\,x + q\,x^3 + \Delta f(t, x, \dot{x}, u) = u \tag{8}
$$

Where Δf denotes the un-modeled parts and u denotes the control action. Obviously, the corresponding nominal system of equation (8) can be described as follows [24]:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -p\,x_1 - q\,x^3{}_1 - (a + b\cos(\omega t))\,x_2 + d(t)
\end{aligned} \tag{9}
$$

Where $d(t)$ stands for the disturbance. The system will be of chaotic $p = -1$, $q = 1$, $b = -1, \omega = 1, a = -0.001$ where can be seen is the phase portrait in Figure 2.

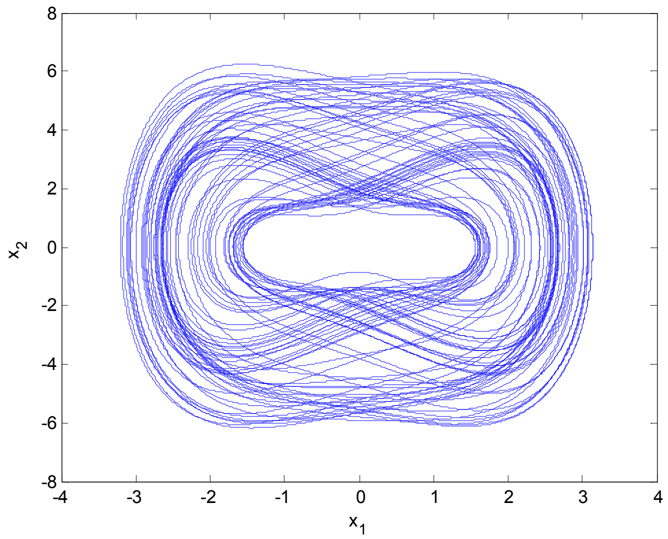

Figure 2. Chaotic behavior of modified Duffing system in 250 sec.

In Figure 1 the chaotic trajectory of this system is rendered. Due to destructive behavior of chaotic systems, controlling of modified Duffing system can be very important in MUV.

5. Simulation result

In this case it is assumed that just state x_1 can be measured. This means another state x_2 must be estimated in a closed loop observer structure.

According to the observer in (5), the overall closed loop control block diagram using the lower
order estimator is as follows:

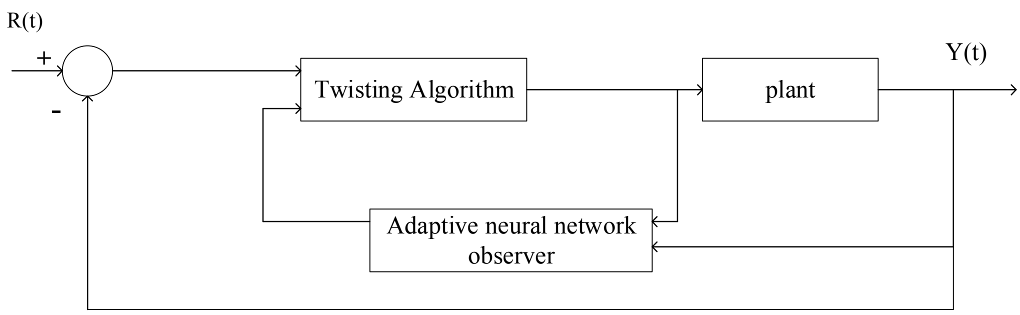

Figure 3. Block diagram of proposed control scheme

The error is defined on x_1 surface to design the twisting algorithm controller. But real states i.e. x_2 are replaced with the estimated ones i.e. $\hat{x}_2$. Therefore the error for the zero tracking tasks ($r \equiv 0$) can be written as follows:

$$e = r - y \Rightarrow e = -y = -x_1 \tag{10}$$

So the control signal u, of the twisting algorithm is defined as follows [23]:

$$u(t) = c_1.\mathrm{sgn}(e) + c_2\,\mathrm{sgn}(\dot{e}) = c_1.\mathrm{sgn}(x_1) + c_2\,\mathrm{sgn}(\hat{x}_2) \qquad (11)$$

According to (11) the twisting algorithm controller is designed and applied on the chaotic system. The simulation results are shown in following figures. The trajectories of states are depicted in Figure 4 & 5.

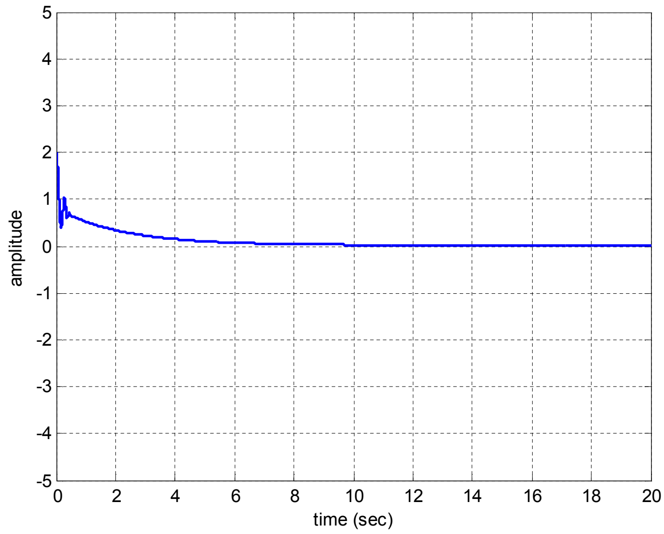

Figure 4. State x_1 after applying controller input

The convergence of first state is acceptable and the simulation section will e followed for second state.

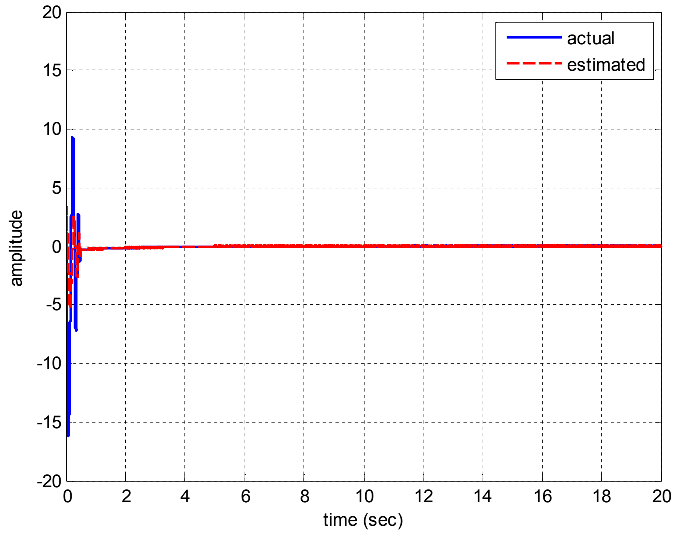

Figure 5. State x_2 after applying controller input and its estimation

Although most of nonlinearity of the chaotic system arises in the second state dynamic the control strategy could effective cope with to make the system state to converge to zero. Indeed the observer proposes a good estimation of the state after one second with a small estimation error. The estimation error of state x_2 is presented as follows:

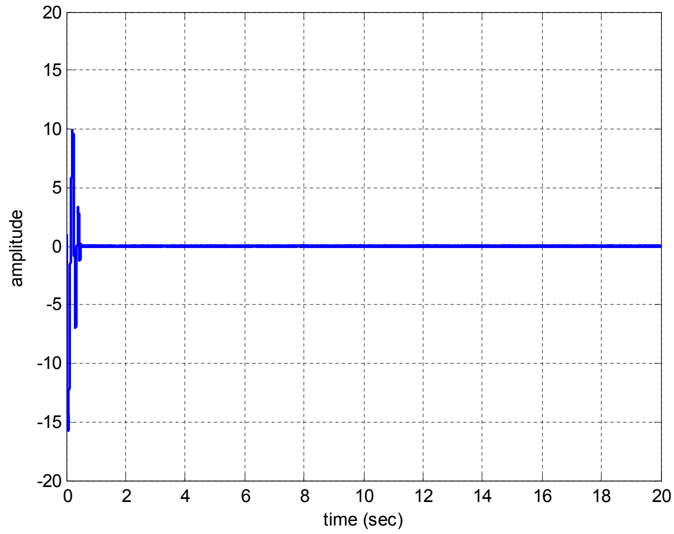

Figure 6. Estimation error of x_2

A phase portrait of dynamic (9) using the twisting algorithm controller is depicted in Figure 7.

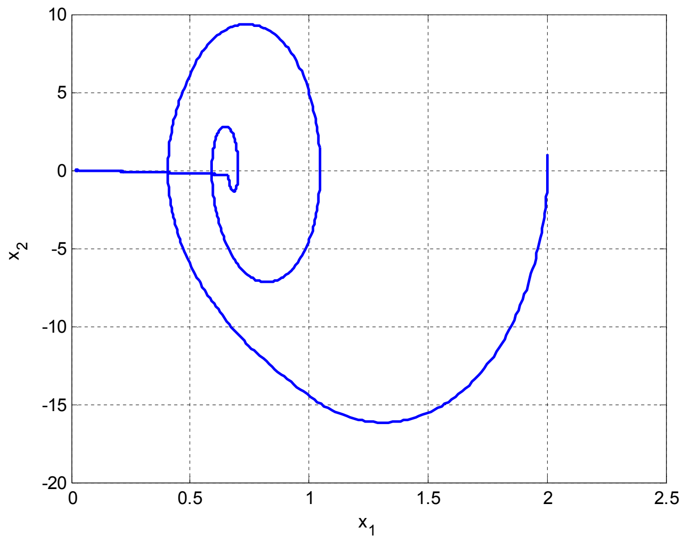

Figure 7. Modified Duffing behavior after applying twisting algorithm controller input

The control effort of proposed method is demonstrated in Figure 8.the trajectory of control effort is desirable and can be implemented in practical case.

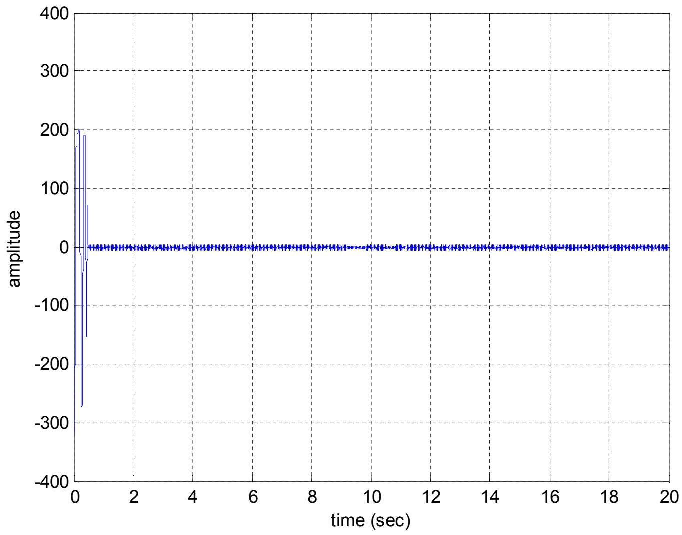

Figure 8. Control effort

6. Conclusion

In this paper, a novel control scheme has been presented for a class of nonlinear systems. In the proposed method, a new combination of the twisting algorithm and adaptive observer has been utilized to control nonlinear system in the presence of disturbances and uncertainty. Due to online performance of adaptive neural network observer, this strategy can be implemented on the practical case. This sensor-less method has been implemented on the modified Duffing chaotic system and the trajectory of the states after applying the control effort of twisting algorithm, endorses the accuracy of this method.

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