Effect of asymmetric quantum dot rings in electron transport through a quantum wire

ABSTRACT

The electronic conductance at zero temperature through a quantum wire with side-connected asymmetric quantum ring (as a scatter system) is theoretically studied using the non-interacting Hamiltonian Anderson tunneling method. In this paper we concentrate on the configuration of the quantum dot rings. We show that the asymmetric structure of QD-scatter system strongly influences the amplitude and spectrum of electron transport characteristics through a quantum wire. We find that the unbalanced number of quantum dots in two rings rather increases the anti-resonant in quantum wire conductance than balances the number of quantum dots rings. Also changing the distance between quantum dot rings influences the amplitude of resonant peaks in the QW conductance spectrum. The proposed asymmetric quantum ring scatter system idea in this paper opens a new insight on the designing of nanostructure quantum wire for a given electrical conductance.

Keywords: Scatter system; Nano scale; Asymmetric quantum dots; Quantum wire; Quantum ring; Electron transport.

INTRODUCTION

Advances in nano-sized system technology have allowed the study of electron transport and especially the conductance through the quantum nano-scale systems like quantum wires (QW) and arrays of quantum dots (QD) in a very controllable way that are very interesting from nano-electronics applications point of view [1-2]. Quantum dots are very promising systems due to their physical properties as well as their potential applications in electronic devices. These structures are small semiconductors or metal structures in which electrons are confined in all spatial dimensions. As a consequence, discreteness in the energy and charge develops. For this reason QD’s are often referred to as artificial atoms. Due to the coupling of the continuum states an even-odd parity effect occurs in the conductance when the Fermi energy is localized at the center of the energy band occurs [3-9].
For this reason the Quantum effects in the rings connected to terminals Quantum dot wire structure, and multi-chain nano-rings can serve as promising prototypes for nano-scale devices.

The systems such as nano-wires [10], uniform QD array [11] asymmetric quantum dot chains [12], and nano-ring [13-14] connected to a quantum wire act as a scatter system for electron transmission through the QW and have a major effect on electronic conductance of nano-device. For a uniform nano rings with M sites, it is shown that the transmission characteristic has M anti-resonances and M-1 resonance, M mini-gaps and M-1 allowed mini-bands [12]. Also for two asymmetric quantum dot chains with M and N quantum dots in every chain we show that the number of forbidden mini-bands is equal to N+M.

The theoretical electron transport properties through a quantum wire (QW) system with side-attachment asymmetric nano-rings is also studied in this paper. The effect of number of dots in the balanced and unbalanced nano-rings is considered and the distance between nano-rings on the electronic conductance is also evaluated. It is shown that the asymmetric scattered QD-rings system makes us to control the spectrum and amplitude of electron transport characteristic through the quantum wire. In section II, the proper model for computation of the conductance value through QW with asymmetric quantum ring using a non interacting Hamiltonian Anderson tunneling model is presented. In Section III the Simulation results are shown and finally, the paper is concluded with a brief discussion.

Figure 1 shows a form of asymmetric quantum dot-rings as a scatter system for manipulation of electron transport through a quantum wire. This type of asymmetry may contain different number of quantum dots in each ring along with a gap between them. The QW system contains QD-ring attachment where there are N and M QDs in each ring. By assuming the equilibrium and letting the distance between two rings to be t, the using the Hamiltonian Anderson tunneling model [15], we can write,

\[ H = H_{QW} + H_{QW-QR} + H_{QR} \]  

Where

\[ H_{QW} = \frac{\theta}{2} \sum_{i=-\infty}^{+\infty} \hat{a}_i^\dagger \hat{a}_{i+1} + h.c \]  

\[ H_{QR} = \epsilon_0 \sum_{r=1}^{R} \hat{b}_r^\dagger \hat{b}_r + V_c \sum_{r=1}^{R} \hat{b}_r^\dagger \hat{b}_{r+1} + h.c + V_e e^{i\phi} \hat{b}_r^\dagger \hat{b}_r + h.c. + \]  

\[ + \epsilon_0 \sum_{p=1}^{P} \hat{s}_r^\dagger \hat{s}_p + V_c \sum_{p=1}^{P} \hat{s}_r^\dagger \hat{s}_{p+1} + h.c + V_e e^{i\phi} \hat{s}_r^\dagger \hat{s}_p + h.c. \]  

\[ H_{QW-QR} = V_0 (\hat{b}_r^\dagger \hat{a}_1 + h.c + \hat{s}_r^\dagger \hat{a}_p + h.c) \]  

In this model, \( H_{QW} \) describes the dynamics of the QW, \( \theta \) being the hopping between neighbor sites of the QW, and \( \alpha_2 (\alpha_i) \) creates (annihilates) an electron in the jth site. \( H_{QR} \) is the Hamiltonian of the asymmetric QD-ring \( \hat{b}_r^\dagger (\hat{b}_r) \) and \( \hat{s}_r^\dagger (\hat{s}_p) \) are the creation operators of an electron in the r and p quantum dots of the first and second QD-rings respectively. The magnetic flux is measured in terms of the elemental quantum flux \( \phi = \phi_0/\phi_0 \) and \( \phi_0 = h/e \) [13]. \( \epsilon_0 \) is the corresponding single level energy and \( V \) is the tunneling coupling between sites in the quantum dots in every ring assumed all equal. \( H_{QW-QR} \) is the coupling of the QW with the asymmetric 2-rings. However \( H_{QW-QR} \) may be changed for different configuration of QD-scatter system (is explained in the following in detail).

![Fig. 1. Side-coupled asymmetric quantum dot-rings as a scatter system attached to a perfect quantum wire.](image-url)
The amplitudes $d_{1}^{k}$ and $c_{1}^{k}$ are the probability amplitude to find the electron in the site 1 of the QW (in the first and second ring respectively) all in the state k where can be calculated as below:

$$d_{1}^{k} / D_{R} = \frac{V_{0}^{2} a_{1}^{k}}{\varepsilon - \varepsilon_{2} - \varepsilon_{0}}$$ \hspace{1cm} (2.a)$$

$$c_{1}^{k} / D_{P} = \frac{V_{0}^{2} a_{1}^{k}}{\varepsilon - \varepsilon_{2} - \varepsilon_{0}}$$ \hspace{1cm} (2.b)$$

Where $D_{R}$=det $(\varepsilon I-H_{R})$ depend on the number of R and P respectively [12].

$$W_{1} / D_{R} = \varepsilon - \varepsilon_{0} - \frac{V_{0}^{2}}{\varepsilon - \varepsilon_{0}}$$ \hspace{1cm} (3.a)$$

$$W_{2} / D_{P} = \varepsilon - \varepsilon_{0} - \frac{V_{0}^{2}}{\varepsilon - \varepsilon_{0}}$$ \hspace{1cm} (3.b)$$

Which $W_{1}$ and $W_{2}$ , depends on the number of dots in every ring. For the calculation of transmission equation, it is assumed that the electrons are described by a plane wave incident from the far left of QW with unity amplitude and a reflection amplitude $r$ and at the far right of QW by transmission amplitude [10].

$$a_{i}^{k} = e^{ikd} + re^{-ikd} \hspace{1cm} \text{i}(1)$$

$$a_{i}^{k} = t e^{ikd} \hspace{1cm} \text{i}(2)$$

Finally transmission amplitude can be calculated by inserting $a_{i}^{k}$ into corresponding equations which leads to an inhomogeneous system of linear equations for $t$, as follows:

$$t = \frac{j \sin kd}{j \sin kd + X}$$ \hspace{1cm} (5)

That value of X will change for different structures of quantum dot rings.

**EXPERIMENTAL**

**Computational Methods**

- **The effect of number of dots in transmission characteristics**
  In this part we would like to consider the effect of asymmetric as a function of number of dots in each one of the QD-rings. To do that lets start with the balance case that is $R=P$ as is shown in Figure 1. By considering the number of QD in the first and second rings being R and P respectively, we obtain the following relationship for X:

$$\beta = 2 \hspace{1cm} (6.a)$$

$$X = \frac{2\mu}{W} - \frac{\mu^{2} e^{ikd}}{W^{2}} \hspace{1cm} (6.b)$$

For the case where $R\neq P$, if $R$ be the number of Quantum dots in the first ring and P be the number of Quantum dots in the second ring, the relationship for the transmission parameter $X$ is given by:

$$\beta = 2 \hspace{1cm} (7.a)$$

$$X = \frac{\mu}{W_{1}} + \frac{\mu^{2} e^{ikd}}{W_{2} W_{1}} \hspace{1cm} (7.b)$$

- **The distance Effect between rings**
  We now consider the effect of distance (gap) between two QD-rings. Assume that the first QD ring is connected to the i site in QW, and the second QD is connected to the (g+p)th ring, then by assuming two different values for P, the corresponding transmission coefficients are computed. That is for $t=1, \beta=3, X=-\alpha$
\[ \beta = 3 \]  

\[ X = \frac{\mu}{W_1} + \frac{\mu}{W_2} - \frac{\mu^2 e^{i\phi}}{W_1W_2} - \frac{\mu^2 e^{i\phi}}{W_1W_2} \]  

(8.a)

\[ G(\varepsilon) = \frac{2e^2}{h} T(\varepsilon) \]  

(9)

\[ \text{and } t=2, p=2, \beta=4, X=-\alpha \]

\[ \beta = 4 \]  

(10.a)

\[ X = \frac{\mu}{W_1} + \frac{\mu}{W_2} - \frac{\mu^2 e^{i\phi}}{W_1W_2} - \frac{\mu^2 e^{i\phi}}{W_1W_2} - \frac{\mu^2 e^{i\phi}}{W_1W_2} \]  

(10.b)

**RESULTS AND DISCUSSION**

In this section, we present the simulation results of electric conductance through quantum wire asymmetric QD-ring structure as a scatter system. In Figure 2, the electron transmission through QW/ Quantum dot ring structure for two cases that is balanced and unbalanced are shown. The dimensionless conductance is plotted in terms of the Fermi energy \( (\varepsilon / \mu) \). For the case where \( R=P \), the number of forbidden mini-bands are exactly the same as Quantum dot rings. The results indicates the existence of broad mini-band and less anti-resonant in the Quantum ring transmission structure transmission. As a result increasing the number of Quantum dots also increases the number of anti-resonance, implying developments of alternating forbidden mini-bands in the range \([-V_c; V_c]\). Also for the balance case anti-resonant is observed for \( \varepsilon=0 \). Conversely for the unbalanced case, electric conductance contains rich spectral properties.

The dimensionless conductance for both cases that is for \( P=1 \) and \( P=2 \) as a function of the Fermi energy is shown in Figure 3. In Figure 3a, the number of quantum dot in the first and second ring are \( R=8 \) and \( P=9 \) respectively. It is shown that the number of antiresonant is independent of the value of gap between two rings. As is shown in this figure, the amplitude of the conductance is different for two different values of distance between two successive QD-rings and it is clear is that amplitude is larger for \( t=1 \) than \( t=2 \). We think that the amplitude of conductance is larger for \( t=1 \) because of coherence constructive-interference in the ballistic channel occurs for the electron stationary states in this case. It should be attended that the location of ant-resonance points aren’t change for two cases. If we consider the number of dots in two-rings same (\( R=P=3 \), the number of anti-resonances will be equal with the number of dots and we have observed three anti-resonances in the conductance spectrum (Figure 3b).

![Figure 2. Dimensionless conductance versus Fermi energy with t=0 a) balance case b) unbalance case](image-url)
value we can change the broad of minigaps. This structure is completely different from chains arrays.

Fig. 3. Dimensionless conductance versus Fermi energy a) P=8, R=9 and b) R=P=4, with Vc =\mu, \varepsilon_0=0 & \phi=0.3

Fig. 4. Compare of electron conductance through QW attached QD-chains with QD-rings for 5 and quantum dots in every array.

CONCLUSIONS

It is shown that for different QD-Rings structures electron transfer characteristic along QW have different amplitudes. As the number of dots in the QD-ring structure increases the number of the mini-band in the transfer characteristic also increases. That is for equal number of QDs in each ring, the mini-band bandwidth also increases [15].

REFERENCES


