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Research Paper

Sensitivity analysis of a caliper formed atomic force microscope cantilever based on a modified couple stress theory

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ABSTRACT: A relationship based on the modified couple stress theory is developed to investigate the flexural sensitivity of an atomic force microscope (AFM) with assembled cantilever probe (ACP). This ACP comprises a horizontal cantilever, two vertical extensions and two tips located at the free ends of the extensions, which forming a caliper. An approximate solution to the flexural vibration problem is obtained using the Rayleigh–Ritz method. The results show that the sensitivities of AFM ACP obtained by the modified couple stress theory are smaller than those evaluated by the classical beam theory at the lower contact stiffness. The results also indicate that the flexural sensitivities of the proposed ACP are strong size dependant when the thickness of the cantilever is close to the material length scale, especially at lower contact stiffness. Furthermore, the greatest flexural modal sensitivity occurs at a small contact stiffness of the system, in which the ratio of the cantilever thickness to the material length scale and the distance between the vertical extensions are also small. In this situation, the distance between the vertical extensions and the clamped end of the cantilever and also the vertical extensions lengths are large. The results reveal that the sensitivity of the right sidewall tip is higher than that of the left one.

Keywords: *Assembled cantilever probe; Atomic force microscope; Modified couple stress theory; Rayleigh–Ritz method; Size dependent.*

INTRODUCTION

The atomic force microscope (AFM), which extends the high resolution capabilities of the scanning tunneling microscope, plays an important role in studying the surface topography of materials and measuring intermolecular forces [1-5]. When a tip scans across a sample surface, it induces a dynamic interaction force between the tip and the surface. The imaging rate and contrast of topographic images are notably influenced by the resonant frequency and the sensitivity of an AFM, respectively. So it is important for researchers to study the resonant frequency and sensitivity of an AFM cantilever [6-9]. Dynamic responses of the AFM cantilever have been investigated by many researches [10-15].

The conventional cantilevers with a sharp tip at the free end have a significant capability of nanoscale surface measurements. Unfortunately, their probe tips never come in close proximity to sidewalls, no matter how sharp and thin the tips are. Therefore, nanoscale surface measurements at sidewalls are urgently demanded. In order to overcome the limitations of conventional AFMs, Dai *et al.* [16] proposed assembled cantilever probes (ACPs) for direct and non-destructive sidewall measurement of nano- and microstructures. Chang *et al.* [17] analyzed the resonant frequency and flexural sensitivity of a form of these AFM ACPs, which consists of a horizontal cantilever and a vertical extension located at its free end. Recently, Kahrobaiyan et al. [18] investigated the resonant frequencies and flexural sensitivities of another form of ACPs proposed by Dai *et al.* [19].

Lacking an internal material length scale parameter, conventional strain-based mechanics theories cannot interpret and predict those micro-structure-dependent size effects when the structural size is in the order of microns and sub-microns [20]. Experimental observations have been indicated that the static and dynamic behavior of the micro-scale structures are size-

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dependent that should be justified by the higher order continuum theories [21, 22]. One of the first higher order continuum theories was couple stress elasticity theory introduced by some researchers such as Mindlin [23], Touplin [24] and Koiter [25] in 1960s, which proposes two higher-order material length scale parameters in addition to the two classical Lame constants. Yang *et al.* [26] modified the couple stress theory to introduce an applicable theory which can capture size dependencies considering only one additional constant other than Lame constants. In the last few years, this theory has been developed and extensively applied to study microbeams and microprobes. Using the modified couple stress theory, Park and Gao [27] investigated the static mechanical properties of an Euler–Bernoulli beam. Kong *et al.* [20] derived the governing equation, initial and boundary conditions of an Euler–Bernoulli beam based on the modified coupled stress theory using the Hamilton principle. Based on the modified couple stress theory, Ma *et al.* [28] developed a microstructure-dependent Timoshenko beam model and studied the bending and vibration problems of a simply supported microbeam. Ke *et al.* [29] investigated the nonlinear free vibration of the size-dependent functionally graded microbeams using the modified couple stress theory. Kahrobaiyan *et al.* [30] studied the resonant frequency and sensitivity of AFM rectangular cantilevers using a modified couple stress theory. Recently, Lee and Chang [31] analyzed the sensitivities of a V-shaped AFM cantilever based on the modified couple stress theory.

In this paper, the flexural vibration sensitivity of a model of an assembled cantilever probe proposed by Die *et al.* [16], consisting of a horizontal cantilever and two vertical extensions, are analyzed utilizing the modified couple stress theory. The sensitivities of this newly proposed AFM probe based on the modified couple stress theory and the classical beam theory are evaluated. In addition, the effects of the sample surface contact stiffness and the geometrical parameters such as the ratio of the vertical extensions lengths to the horizontal cantilever length and the distance between the vertical extensions and also between the vertical extensions and the clamped end of the horizontal cantilever on the flexural sensitivity are investigated.

EXPERIMENTAL

The proposed kind of AFM ACP developed in the present study, comprises a horizontal cantilever, two vertical extensions and two tips located at the free ends of the extensions, which forming a caliper. The geometrical parameters and configuration of this ACP are depicted in Fig. 1. The horizontal cantilever has a uniform cross section thickness *b*, width *a*, which length is *L*. It can be observed that the vertical extensions with the length H and the same cross section are attached to the horizontal cantilever at the distances L_1 and L_2 from the clamped end. Considering the ratio of the extension rigidity to the cantilever rigidity, the deflection of the extension in comparison with the cantilever deflection can be neglected, so it can be assumed that the extension is rigid. So the cantilever of ACP experiences flexural vibration during contact with the sample. As shown in Fig. 1, the ACP interacts with the sample surface at each tips by normal springs K_{n1} and K_{n2} for normal interactions and lateral springs K_{L1} and K_{L2} for lateral interactions. *X* is the coordinate along the center of the cantilever while $W(X,t)$ is the vertical deflection in X-direction at time *t*.

In this study, Rayleigh–Ritz method [32] is utilized in order to obtain the ACP's flexural sensitivities. Considering the couple stress theory, the potential energy for a free-free beam is [20, 31]

$$U = \frac{1}{2}\int_0^L \left(EI + GAl^2\right)\left(\frac{\partial^2 W}{\partial X^2}\right)^2 dx \tag{1}$$

where *E* and *G* are the elastic modulus and the shear modulus, respectively; *l* is the material length scale parameter which indicates the size-dependent behavior of the microcantilever based on the modified coupled stress theory; *A* and *I* are the cross-sectional area and the area moment of inertia of the cantilever, respectively.

Adding the elastic energy of the proposed ACP boundary conditions, gives the following result for the elastic potential energy of the system:

$$U_{max} = \frac{1}{2}\int_0^L \left(EI + \mu Al^2\right)\left(\frac{\partial^2 W(X,t)}{\partial X^2}\right)^2 dx + \frac{1}{2}k_{t1}W^2(L_1,t) + \frac{1}{2}k_{t2}W^2(L_2,t) + \frac{1}{2}k_{n1}\left(H\frac{\partial W(L_1,t)}{\partial X}\right)^2 + \frac{1}{2}k_{n2}\left(H\frac{\partial W(L_2,t)}{\partial X}\right)^2 \tag{2}$$

The harmonic solution to the problem can be expressed as:

$$w(X,t) = Y(X)e^{i\omega t} \tag{3}$$

so the maximum kinetic and potential energies of the proposed ACP are introduced as:

$$T_{\max} = \frac{1}{2}\int_0^L \rho\omega^2 Y^2(X)dX + \frac{1}{2}M_e\omega^2\left(Y^2(L_1) + Y^2(L_2)\right) + \frac{1}{2}J_e\omega^2\left(\left(\frac{dY(L_1)}{dX}\right)^2 + \left(\frac{dY(L_2)}{dX}\right)^2\right) \quad (4)$$

$$U_{\max} = \frac{1}{2}\int_0^L \left(EI - GAl^2\right)\left(\frac{d^2Y}{dX^2}\right)^2 dX + \frac{1}{2}k_{t1}Y^2(L_1) + \frac{1}{2}k_{t2}Y^2(L_2) + \frac{1}{2}k_{n1}\left(H\frac{dY(L_1)}{dX}\right)^2 + \frac{1}{2}k_{n2}\left(H\frac{dY(L_2)}{dX}\right)^2 \quad (5)$$

where $T_{\max}$ and $U_{\max}$ are the maximum flexural kinetic and potential energies respectively. ° represents the cantilever's volume density while µ represents the resonant frequency. Also M_e and J_e are the mass and mass moment of inertia of the vertical extensions, which are defined as [33].

$$M_e = \rho_e H\ ,\ J_e = \frac{1}{3}M_e H^2\ ,\ J_e = \frac{1}{3}\rho_e H^3 \quad (6)$$

where $°_e$ denotes the density of the vertical extensions. The dimensionless variables are defined as:

$$x = \frac{X}{L},\ y = \frac{Y}{L},\ \tilde{L}_1 = \frac{L_1}{L},\ \tilde{L}_2 = \frac{L_2}{L},$$
$$\delta L = \frac{L_2 - L_1}{L} = \tilde{L}_2 - \tilde{L}_1,\ m = \frac{\rho_e}{\rho},\ h = \frac{H}{L}$$
$$\eta = \frac{12G}{E(b/l)^2},\ \beta_{n1} = \frac{k_{n1}L^3}{EI},\ \beta_{n2} = \frac{k_{n2}L^3}{EI},$$
$$\beta_{t1} = \frac{k_{t1}L^3}{EI},\quad \beta_{t2} = \frac{k_{t2}L^3}{EI} \quad (7)$$

Utilizing these dimensionless variables, the normalized form of kinetic and potential energies can be expressed as follows:

$$\tilde{T}_{\max} = \frac{T_{\max}}{\rho L^3\omega^2} = \frac{1}{2}\int_0^1 y^2(x)dx + \frac{1}{2}mh\left(y^2(\tilde{L}_1) + y^2(\tilde{L}_2)\right) + \frac{1}{6}mh^3\left(\left(\frac{dy(\tilde{L}_1)}{dx}\right)^2 + \left(\frac{dy(\tilde{L}_2)}{dx}\right)^2\right) \quad (8)$$

$$\tilde{U}_{\max} = \frac{L}{EI}U_{\max} = \frac{1}{2}\int_0^1(1+\eta)\left(\frac{d^2y}{dx^2}\right)^2 dx + \frac{1}{2}\beta_{t1}y^2(\tilde{L}_1) + \frac{1}{2}\beta_{t2}y^2(\tilde{L}_2) + \frac{1}{2}\beta_{n1}\left(h\frac{dy(\tilde{L}_1)}{dx}\right)^2 + \frac{1}{2}\beta_{n2}\left(h\frac{dy(\tilde{L}_2)}{dx}\right)^2 \quad (9)$$

The dimensionless resonant frequency can be derived as [18]

$$\tilde{\omega} = \frac{\tilde{U}_{\max}}{\tilde{T}_{\max}} \quad (10)$$

In this article, the Rayleigh–Ritz method is used to calculate the natural frequencies.

According to the Rayleigh–Ritz method, it is assumed that

$$y(x) = \sum_{i=1}^{n} a_i u_i(x) \quad (11)$$

where a_i are some constants, and u_i is the admissible function which requires to satisfy the geometric boundary conditions, but need not satisfy the natural boundary conditions [31]. substituting the Eq. (11) into Eqs. (8) and (9) gives

$$\tilde{T}_{\max} = \frac{1}{2}\int_0^1\left(\sum_{i=1}^{n} a_i u_i(x)\right)^2 dx + \frac{1}{2}mh\left(\left(\sum_{i=1}^{n} a_i u_i(\tilde{L}_1)\right)^2 + \left(\sum_{i=1}^{n} a_i u_i(\tilde{L}_2)\right)^2\right) + \frac{1}{6}mh^3\left(\left(\sum_{i=1}^{n} a_i u_i'(\tilde{L}_1)\right)^2 + \left(\sum_{i=1}^{n} a_i u_i'(\tilde{L}_2)\right)^2\right) \quad (12)$$

$$\tilde{U}_{\max} = \frac{1}{2}\int_0^1(1+\eta)\left(\sum_{i=1}^{n} a_i u_i''(x)\right)^2 dx + \frac{1}{2}\beta_{t1}\left(\sum_{i=1}^{n} a_i u_i(\tilde{L}_1)\right)^2 + \frac{1}{2}\beta_{t2}\left(\sum_{i=1}^{n} a_i u_i(\tilde{L}_2)\right)^2 + \frac{1}{2}\beta_{n1}h^2\left(\sum_{i=1}^{n} a_i u_i'(\tilde{L}_1)\right)^2 + \frac{1}{2}\beta_{n2}h^2\left(\sum_{i=1}^{n} a_i u_i'(\tilde{L}_2)\right)^2 \quad (13)$$

where the prime symbol represents differentiation with respect to *x*. Applying the Rayleigh quotient, the dimensionless flexural resonant frequencies can be obtained

$$Ka = \tilde{\omega}^2 Ma \quad (14)$$

where *a* is the eigenvector of expansion coefficients and $\tilde{µ}$ is the normalized frequencies. Also the dimensionless matrices of mass M_{ij} and stiffness K_{ij} can be obtained as

$$\frac{\partial\tilde{T}_{\max}}{\partial a_i} = \sum_{j=0}^{n} M_{ij}a_j \qquad \frac{\partial\tilde{U}_{\max}}{\partial a_i} = \sum_{j=0}^{n} K_{ij}a_j \quad (15)$$

$$M_{ij} = \int_0^1 u_i(x)u_j(x)dx + mh\left[u_i(\tilde{L}_1)u_j(\tilde{L}_1) + u_i(\tilde{L}_2)u_j(\tilde{L}_2)\right] + \frac{1}{3}mh^3\left[u_i'(\tilde{L}_1)u_j'(\tilde{L}_1) + u_i'(\tilde{L}_2)u_j'(\tilde{L}_2)\right]$$

$$K_{ij} = \int_0^1 (1+\eta) u_i''(x) u_j''(x) dx + \beta_{l1} u_i(\tilde{L}_1) u_j(\tilde{L}_1) + \beta_{l2} u_i(\tilde{L}_2) u_j(\tilde{L}_2) + \beta_{n1} h^2 u_i'(\tilde{L}_1) u_j'(\tilde{L}_1) + \beta_{n2} h^2 u_i'(\tilde{L}_2) u_j'(\tilde{L}_2)$$

Eq. (14) is differentiated with respect to β_{li}, by assuming the change in the eigenvector to small changes in stiffness is negligible and the mass matrix, M, does not depend on β_{li}. Then the following equation can be obtained [32].

$$\frac{\partial K}{\partial \beta_{li}} a = 2\omega \frac{\partial \tilde{\omega}}{\partial \beta_{li}} Ma \qquad i = 1,2 \qquad (16)$$

Finally, by applying the normalization condition $a^T Ma=1$, the dimensionless flexural sensitivity, S_i can be obtained as:

$$S_i = \frac{\partial \tilde{\omega}}{\partial \beta_{li}} = \frac{1}{2\tilde{\omega}} a^T \frac{\partial K}{\partial \beta_{li}} a \qquad i = 1,2 \qquad (17)$$

where S_1 and S_2 represents the flexural sensitivity of the tip connected to the left and right sidewall probe, respectively.

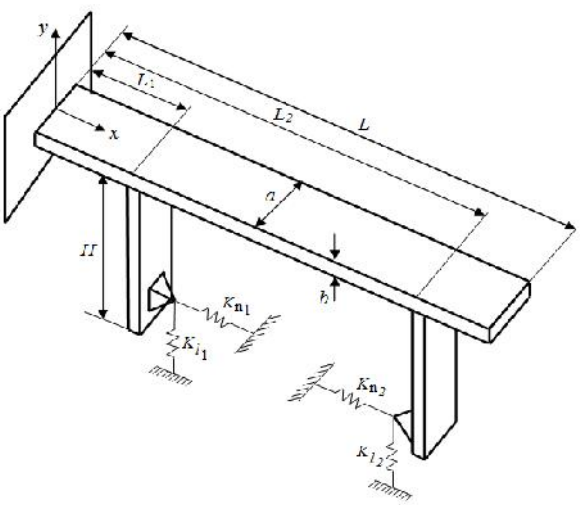

Fig. 1: Schematic diagram of an AFM cantilever microassembled with two vertical extensions and two tips located at the free ends of the vertical extensions.

RESULTS AND DISCUSSION

In this paper, the flexural sensitivity of the vibration modes of an AFM cantilever with two sidewall probes have been analyzed utilizing the Rayleigh–Ritz method. In order to calculate the effects of various geometric parameters on the flexural sensitivities of this kind of ACPs, a ten-term polynomial expansion is used. The default geometric and material parameters which are considered as E =170Gpa, G =64.1Gpa, °=2330 Kg/m^3, $°_e$=3440 Kg/m^3, $\tilde{L}_1 = 0.6$ and ¢$L = 0.4$. The ratio of the cantilever thickness to the internal material length scale parameter, b/l is assumed to be 4. The lateral and normal contact stiffness were assumed as $\beta_{l1} = \beta_{l2}$, $\beta_{n1} = 0.9\beta_{l1}$ and $\beta_{n2} = 0.9\beta_{l2}$. The dimensionless flexural sensitivities of the left and right sidewall tips as a functions of β_{l1} and β_{l2} based on the modified couple stress theory and classical beam theory are compared in Figs. 2 and 3. As it can be seen, the sensitivities of the ACP obtained by the modified couple stress theory are smaller than those predicted by the classical beam theory at the low values of contact stiffness while the situation is different at the large values of contact stiffness. It can also be found that the first mode is the most sensitive mode due to the change in contact stiffness. Comparing Figs. 2 and 3, it can also be inferred that the sensitivity of the second tip is higher than the first one evaluated by either classical beam theory or modified couple stress theory.

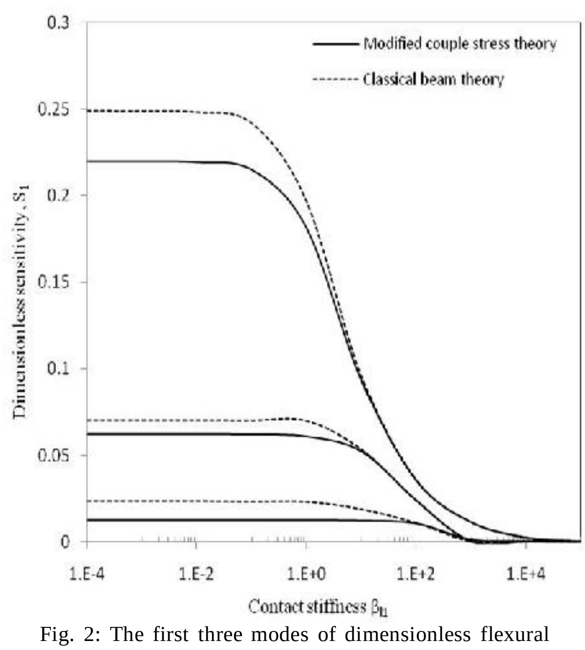

Fig. 2: The first three modes of dimensionless flexural sensitivities of the left sidewall tip based on modified couple stress theory and classical beam theory.

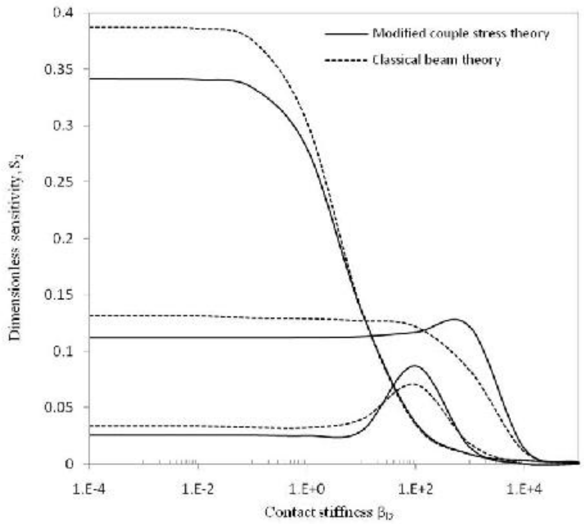

Fig. 3: The first three modes of dimensionless flexural sensitivities of the right sidewall tip based on modified couple stress theory and classical beam theory.

Figs. 4 and 5 illustrate the change in the flexural sensitivity of the left and right sidewall tips, S_1 and S_2, due to the change in the dimensionless distances between the extensions, ¢L and the dimensionless distances of the extensions from the clamped end of the cantilever , $\tilde{L}_1$ and $\tilde{L}_2$, at two different cases when $_{l1} = 10^{-2}$. In the first case, is changed with the change in while is fixed at 0.5. On the other hand, in the second case, the right extension is considered to be fixed at the free end of the cantilever while is changed with the change in. Regarding these two figures, two remarkable results can be inferred. Firstly, an increase in either or can raise the flexural sensitivity, in which the highest value for the sensitivity occurs when is small and the vertical extensions are close to the free end of the cantilever. Secondly, it can be found that a change in does not have a knock on effect on the flexural sensitivity of the first tip while the effect of a change in on the flexural sensitivity of the second tip is considerable.

Figs. 6 and 7 illustrate the change in the resonant frequency of the first and second tips due to the change in the contact stiffness for various values of. The material length scale, *l* make it possible to investigate the size-dependant dynamic behavior. It should be noted that for the case of, the new model will reduce to the classical beam theory model. It can be seen in Figs. 6 and 7 that the sensitivities of both tips display strong size dependant behavior as the ratio of decreases. However, the dimensionless sensitivities of the AFM ACP are approximately size independent for the high values of. It is also should be noted that for the low values of, the effect of material length scale on the sensitivity is considerable when the contact stiffness is also low. In this case, as enlarges, the flexural sensitivity of both sidewall tips enhances.

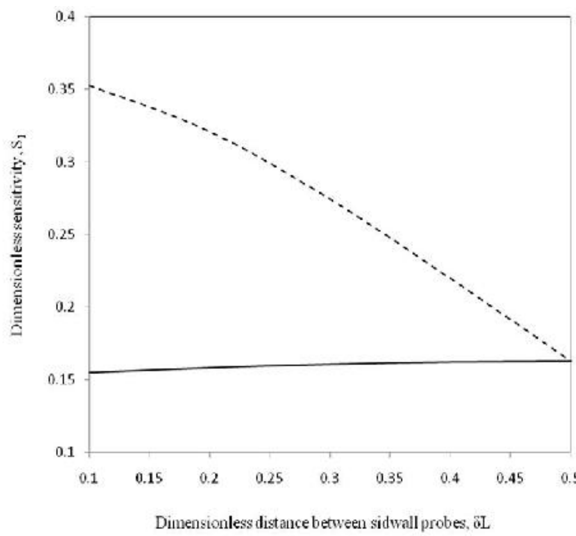

Fig. 4: The dimensionless sensitivity of left sidewall tip as a function of distance between sidewall probe at $_{l1} = 10^{-2}$ for two cases: (1) L_1 is fixed while L_2 is moving across the cantilever; (2) L_2 is fixed while L_1 is moving across the cantilever.

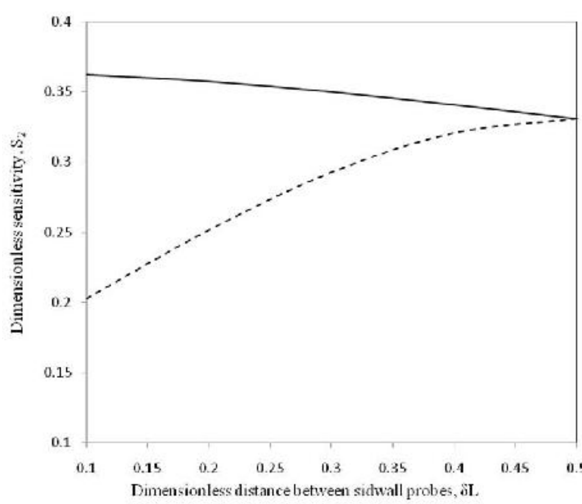

Fig. 5: The dimensionless sensitivity of right sidewall tip as a function of distance between sidewall probe at $_{l2} = 10^{-2}$ for two cases: (1) L_1 is fixed while L_2 is moving across the cantilever; (2) L_2 is fixed while L_1 is moving across the cantilever.

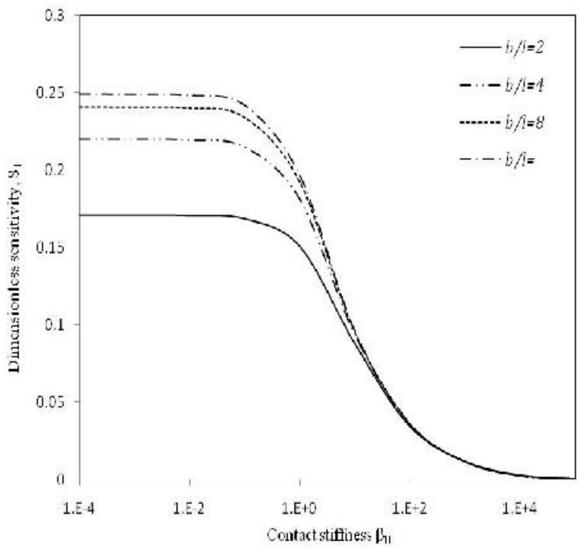

Fig. 6: The dimensionless flexural sensitivity of left sidewall tip, S_1 as a function of contact stiffness, $_{l1}$ at various ratios of microcantilever thickness to internal material length scale parameter *b/l*.

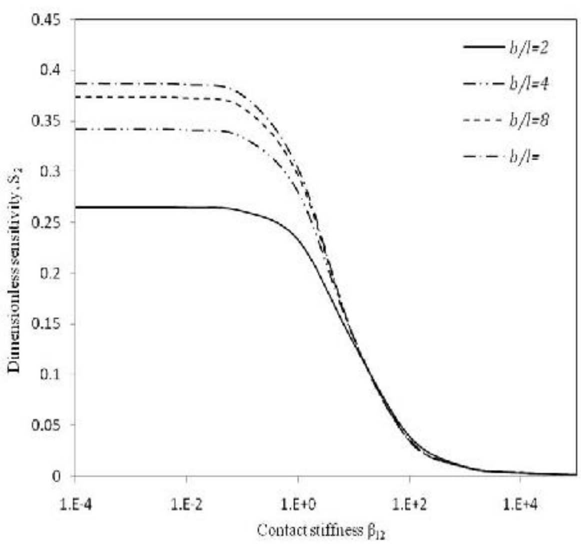

Fig. 7: The dimensionless flexural sensitivity of right sidewall tip, S_2 as a function of contact stiffness, $_{l2}$ at various ratios of microcantilever thickness to internal material length scale parameter *b/l*.

The dimensionless sensitivities of the first and second tips as the functions of and and vertical extension length, *h* are depicted in Figs. 8 and 9. As it can be found, when the contact stiffness is low, the effect of vertical extension length, *h* on the sensitivities is significant. In addition, the sensitivities of the left and right sidewall probes enhance as the vertical extension length increases and accordingly, high accuracy of measurement can be achieved.

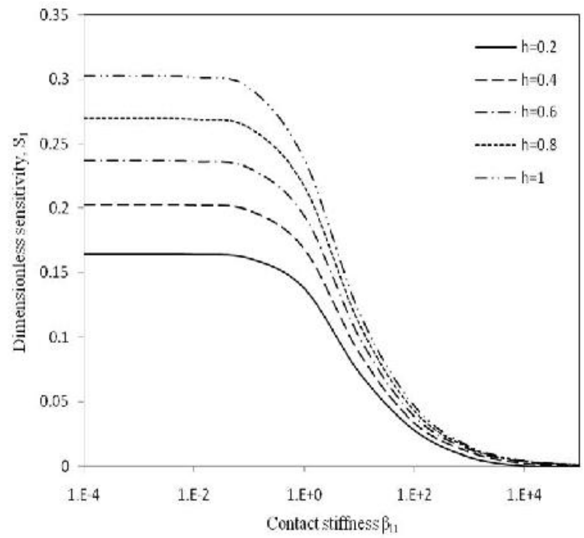

Fig. 8: The dimensionless flexural sensitivity of left sidewall tip, S_1 as a function of contact stiffness, $_{l1}$ at various lengths ratios of vertical extensions to horizontal cantilever.

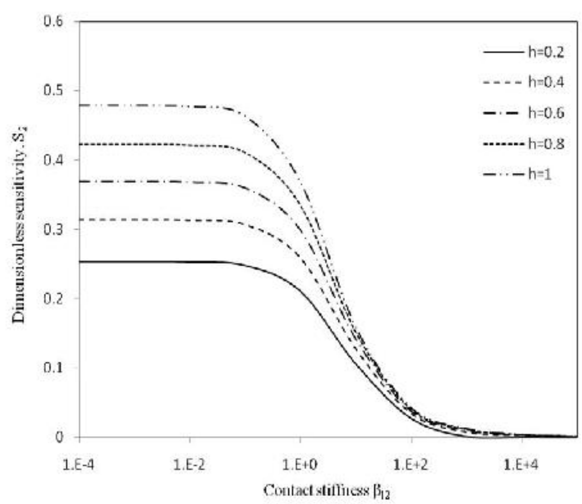

Fig. 9: The dimensionless flexural sensitivity of right sidewall tip, S_2 as a function of contact stiffness, $_{l2}$ at various lengths ratios of vertical extensions to horizontal cantilever.

CONCLUSION

On the basis on the modified couple stress theory, the sensitivities of the flexural vibration modes for an atomic force microscope cantilever with two sidewall probes has been analyzed using the Rayleigh–Ritz method. According to the results, the sensitivities of the proposed ACP at both sidewall probes evaluated by the modified couple stress theory were smaller than those obtained by the classical beam theory especially at lower contact stiffness. The sensitivities of both tips are high when the distance between the extensions is small and the right extension is close to the free end of the cantilever. Hence, the sensitivity of the second tip is always more than that of the first tip basis on either the classical beam theory or the modified couple stress theory. In addition, the effect of the left extension position on the sensitivity of the second tip is considerable. The results also indicate that the difference between the flexural sensitivities of both tips predicted by the modified coupled stress theory and those obtained by the classical beam theory are very significant when the thickness of the AFM microcantilever is close to the internal material length scale parameter. It reveals the size-dependent behavior of the AFM ACP microcantilever. From the results, it can be inferred that an increase in the dimensionless vertical extension length leads to the enhancement of the sensitivities of both tips.

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