Fault-tolerant adder design in quantum-dot cellular automata

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Abstract
Quantum-dot cellular automata (QCA) are an emerging technology and a possible alternative for faster speed, smaller size, and low power consumption than semiconductor transistor based technologies. Previously, adder designs based on conventional designs were examined for implementation with QCA technology. This paper utilizes the QCA characteristics to design a fault-tolerant adder that is more powerful in terms of implementing robust digital functions. By considering two-dimensional arrays of QCA cells, fault properties of such block adder can be analyzed in terms of misalignment, missing and dislocation cells. In order to verify the functionality of the proposed device, some physical proofs are provided. The results confirm our claims and its usefulness in designing digital circuits.

Keywords: Adder; Design and modeling; Fault-tolerant circuits; Nanoelectronic circuits; Quantum-dot cellular automata.

INTRODUCTION
Quantum-dot cellular automata (QCA) have been recognized as one of the new technologies that may replace semiconductor transistor based technologies at the nano-scale level. The principle of QCA was first proposed by Lent and Tougaw [1]. The superior features of QCA over current CMOS VLSI devices along with the feasibility of designing logic gates, circuits, and massively parallel architectures indicate the potential of QCA as a promising novel computing paradigm. In the sense that it would potentially allow the implementation of massively parallel computing architectures which could outperform the current CMOS VLSI counterparts in every performance aspect, that is, integration density, power consumption, and speed, while also enabling new applications by overcoming inherent limitations of VLSI technology [2].

There are, however, several obstacles for a practical realization of QCA and exploiting full potential of this new technology. Here, it suffices to mention the following issues:

The first major obstacle is the realization of QCA hardware capable of performing in room temperature. Current semiconductor technologies that are being considered for the QCA implementation would operate only in cryogenic temperatures due to the large size of the cells. This, in turn, has motivated the investigation of molecular realization of QCA. The smaller size of molecules means that Coulomb energies are much larger, so room temperature operation is possible. In fact, there are indications that realization of QCA-based molecular devices capable of functioning in the current commercial regime is possible. It should be mentioned that the focus of our work is on electronic realization of QCA devices as opposed to magnetic realization. It has been demonstrated that magnetic quantum dots, despite their large size, can operate at room temperature.

The second obstacle is the means by which input state is fixed and the output state is measured. Obviously, the issue of connecting the nano-world to the micro-world is one that is germane to all type of nano-devices.

The third issue is the required precision in the assembly and tolerance to fabrication defect. In fact, it is widely believed that QCA devices and circuits will highly sensitive to imprecision in their...
assembly. Here again it seems that molecular implementation provides an additional advantage by allowing the use of various self-assembly techniques. However, there are still questions as to whether molecular self-assembly techniques would give sufficient control over cell positioning.

In this note, we will not address the first and second item. We will focus on an approach to overcome some of the issues related to the third item. Our approach is based upon considering two-dimensional arrays of QCA cells. Assuming a certain amount of blocks in the assembly of the QCA cells, it is still possible to design circuits that perform the desired functions despite their faulty assembly. This is the direction that we have been pursuing for designing fault tolerant QCA adders. Two fundamental units of QCA based design are majority and inverter gates; hence, efficient construction of QCA circuits using majority and inverter has attracted a lot of attention [3-18].

A single-bit full-adder can be implemented by using only majority and inverter gates [11-12, 19]. As full-adder is the principle element of the arithmetic systems, its performance directly affects the performance of the entire system. Hence, efficiently constructing a full-adder in QCA is of great importance [9-13, 20-22].

Fault-tolerant design of QCA logic circuits is absolutely necessary for characterization of defective behavior of QCA circuits. In recent years the fault tolerance properties of QCA circuits has been demonstrated by many researchers [4-8, 23-28].

QCA background

Quantum cellular automata are new device architecture, which is proper for the nanometer scale. The basic computational element in QCA is a quantum cell. A quantum cell can be viewed as a set of four charge dots, positioned at the corners of a square. The cell contains two extra mobile electrons, which are allowed to tunnel between neighboring sites [19, 30]. The electrons are forced to the corner positioned by columbic repulsion. The two possible polarization states represent logic “0” and “1” as shown in Fig. 1(a) [19, 30].

As shown in Fig. 1(b), an ordinary QCA majority gate requires only five QCA cells; three inputs labeled A, B and C, the device cell and the output. The logic function of majority gate is:

\[ M(A, B, C) = AB + AC + BC \]  \hspace{1cm} (1)

Alternatively, as shown in Fig. 1(c), each single-bit full-adder can be implemented with only inverters and majority gates. The device has three inputs: two operands \( a \), \( b \), and the previous carry result. The two outputs are the sum \( \text{Sum} \) and the carry \( \text{Carry} \). Full-adder cells can be easily chained together to produce a multi-bit adder [22]. And in Fig. 1(d) a QCA inverter is shown.

Advantages and Difficulties of QCA-based design

QCA offers several distinct advantages over traditional technologies: (1) This schema inherently allows for very small feature size and thus high computational density. (2) Because current does not flow through QCA-based circuits, these designs can operate at very low power levels. This low power cost is vital to achieving the device densities. (3) QCA design support massively parallel computational architectures, which can allow for more efficient information processing.

Many obstacles must be overcome before QCA-based circuits are available as a viable technology: (1) quantum cells must be small, on the order of 18nm, to be efficient. Currently the technology does not exist to reliably manufacture quantum cells of this size and assemble them into particular structures. Fortunately much time and effort is being spent on these scale related issues. (2) As with any technology on this scale, it is difficult to create interfaces between the computational circuits and I/O devices such as monitors and keyboards that would allow the user to interact with the computer. Also this limitation is face by other technologies. (3) QCA structures exhibit propagation delays. This delay can be attributed to the finite amount of time that it takes for the electrons in a cell to tunnel to their new position [4]. (4) In addition to robustness capabilities of any future QCA device, another difficulty in its practical implementation is patterning a circuit. That is, if a simple gate is used within a QCA circuit then a high degree
of accuracy is needed for proper alignment of cells. With today’s technology, it is very hard to assemble a specific pattern, let alone making it precise. This issue should also be considered in the context of another problem associated with the manufacturing of massive arithmetic circuits. It is believed that QCA architectures could eventually be implemented with self-assembled molecules, although there are no candidates as yet and there are questions to whether molecular assemblies would give enough control over cell positioning [2, 4]. This suggested that, while great QCA array with a very large number of cells can be implemented, the exact position of cells would be hard to control. In other words, practical implementation QCA array would represent a high degree of fault in cell positioning.

This has motivated us to investigate the design of QCA devices from a different perspective. In fact, instead of analyzing the behavior of a single cell, we have analyzed the behavior of two-dimensional arrays of cells for designing fault-tolerant QCA device [6-8].

Faults and Fault tolerance

Three major categories of faults can occur during the assembly of a QCA circuit. First, faults may occur when quantum cells are shifted from their intended locations which are called “misalignment” cells. Sometimes misalignment cells have no effect on functionality of a QCA circuit, and also sometimes they can cause a circuit to have an unexpected output. A second type of faults occurs when the quantum cell itself is “missing” resulting in the cell becoming defective and it would have no influence on its neighbors and it can cause a circuit to cease functioning well. A third type of faults occurs when quantum cells are rotated relative to the other cells in the array which is called “dislocation” cells. Also, in this case, the circuit may cease to function.

Fig. 2(a) shows misalignment cell in a full-adder. Obviously, due to symmetric, the direction of the cell movement is not important and it may cause the design does not function as a full-adder. In Fig. 2(b), a missing cell in full-adder is presented that the design may cease to function. As shown in Fig. 2(c), the dislocation cell with 45° rotation angle, also can cause a full-adder to have unexpected output. Based on the researches which have been performed to date, some fault-tolerant QCA circuits have been with faults. In next section, we have attempted to make a novel fault-tolerant full-adder using physical relation; in such a manner that it can continue to operate correctly in the event of the mentioned faults.
EXPERIMENTAL

Computational Methods

Fault-tolerant full-adder and fault-tolerant adder

The basis of functioning of the full-adder can be easily understood by considering Coulomb interaction among four neighboring QCA cells. However, this also suggested that the correct function of such a device would strongly depend on the precision and geometry of its implantation. In order to assess the impact of the precision and geometry, we have studied and validated various configurations for implementation of the full-adder. This validation is performed by some physical relations using kink energy.

The layout of the proposed fault-tolerant full-adder is shown in Fig. 3. In this new structure, a fault-tolerant full-adder can be implemented only with fault-tolerant majority and inverter gates. In this scheme we have three inputs labeled a, b and c and two output cells are shown by Sum and Carry. In addition, three block 9th middle cells labeled Block1, Block2 and Block3, and seven block 4th middle cells labeled 1, 2, 3, 4, 5, 6 and 7. Polarization of input cells is fixed and middle cells and output cells are free to change. The rest of cells are considered as wire.

As already mentioned, the property of all blocks is depending on number and position of device cells in each block. These remarkable collective of QCA cells are important from robustness point of view and they may also alleviate some of the problems related to patterning circuits.

The proposed design is based on majority and inverter blocks that allow several paths of information travel between inputs and outputs. This design allows some faults to be cancelled out by cells in blocks that are in correct state. Using the layout of fault-tolerant full-adder, we design a fault-tolerant four-bit QCA adder as shown in Fig. 4. Regarding the physical proofs, assume that all cells are similar and the length of each one is \(a=18 \text{ nm}\) and there is a space of \(x=2 \text{ nm}\) between each two neighbors. In all figures, rectangles show a QCA cell and the circles inside show the electrons within that cell. It should be noted that in order to achieve more stability, electrons of QCA cell are arranged in such a manner that reaches minimum kink energy (the difference in electrostatic energy between the two polarization states). The kink energy between two electron charges is calculated using Eq. (2a). In this equation, \(U\) is kink energy, \(k\) is fixed colon, \(q_i\) and \(q_j\) are electric charges and \(r\) is the distance between two electric charges.

By putting the values of \(k\) and \(q\), we obtain Eq. (2b). \(U_i\) is the summation of kink energies that is calculated from Eq. (3) [31].

\[
U = \frac{k q_i q_j}{r}
\]  
\[
k q_j = 9 \times 10^7 \times 1.6 \times 10^{-19} = 23.04 \times 10^{-20} = A = \text{cte}
\]  
\[
U_i = \sum_{i=1}^{2} U_i
\]
Fig. 3: Layout of proposed fault-tolerant full-adder.

Fig. 4: Layout of proposed fault-tolerant four-bit adder.
RESULTS AND DISCUSSIONS

Physical Proof

As the proposed design has 55 different middle cells, we should check all the faults that may occur in middle cells to verify the correctness of this scheme. Here, only one of the faults (missing cell 5 in Block3) is proved and the others can be proved as well. The assumed value of input cells are \( a=b=0 \) and \( c=1 \).

First, we calculate the kink energy existing between each electron \( e_{x}, e_{y} \), and \( e_{y} \) with electrons “x” and “y” in (a) and (b) states using (2a) and (2b) equations. For example, \( U_{i} \) is the kink energy existing between electrons \( e_{i} \) and \( x \) (or y). Also, \( r \) is the distance between two electron charges. Then we calculate the total kink energy (\( U_{r} \)) in both states using Eq. (3). The comparison of total kink energies in both (a) and (b) states shows that which state (a or b) is more stable. We consider the state that has the lower kink energy level as the more suitable one.

By considering the value of input cells we can gain the value of input cells for Block3 \( (a,b,c) \) and \( c=0 \).

As the proof method is similar for all cells and their values and also due to lack of space, only the first part of this proof is stated and the rest of relations are omitted (Fig. 5).

Fig. 5(a) (electron x)

\[
U_{i} = \frac{A}{r_{i}} = \frac{23.04 \times 10^{-29}}{40 \times 10^{-9}} \approx 0.58 \times 10^{-20} \text{ (J)}
\]

Fig. 5(b) (electron y)

\[
U_{i} = \frac{A}{r_{i}} = \frac{23.04 \times 10^{-29}}{60.73 \times 10^{-9}} \approx 0.38 \times 10^{-20} \text{ (J)}
\]

With comparison of the achieved results, the electrons in cell 8 are positioned in state (a) which is more stable and has lower kink energy. It is worth mentioning that in all cells \( U_{i} \) is the kink energy in +1 polarization and \( U_{i} \) is the kink energy in -1 polarization.

By considering middle cells in Block3 (Fig. 3) the results are summarized in Table 1.

The following observation can be made from Table 1:

1) In all cases, proposed schemes with single defective cell function as the majority function or majority like function (majority function with one complemented variables).
2) Defective cell occurring on non-coherent with input cells (cells 1, 3, 7 and 9) does not change the
logic function of Block3, thus confirming the non-defect tolerant design of a single-bit full-adder.

3) Whenever cell 8 is defective, the polarization level experiences a drop (about ±0.1), but it also acts as majority gate. In all blocks, the total average of maximum polarization level decreased by increasing the number of defective cells.

4) Since the schemes Block1, Block2 and Block3 are similar to each other as well as schemes 1, 2, 3, 4, 5, 6 and 7, one of the significant specifications of the proposed design is that it can tolerate multi-faults. For instance, if one of the mentioned faults in “Faults and Fault tolerance” section simultaneously occurs in Block1 and Block3 or 1, 2 and 3 schemes, the proposed structure will still perform proper operation; that can be proved by physical relations.

Considering the above computing, we can infer that the proposed structure for implementing a fault-tolerant full-adder is correct and resulted in a correct state for the output cell when some faults occur.

Our study demonstrated the potential of this new approach to the design of fault-tolerant QCA arithmetic circuits. These results indicate the superior fault tolerance properties of QCA arrays in terms of misalignment, dislocation and missing cells. The next question is whether such proposed design can be implemented. As the first step toward this end, the parallel blocks are distinguishing by the fact that different clocks drive them in a pipeline fashion. Note that the whole edge of the first array is used as the input to the second blocks. In fact, a given QCA circuit can be divided into a set of smaller sub-arrays by assigning different clocks to each sub-array. In such a circuit, each sub-array can perform a logic function similar to or different from other sub-arrays and the output of each sub-array is used as input to other sub-arrays.

Fig. 5: a) The one value in cell 8, b) The zero value in cell 8.
CONCLUSION

A novel fault-tolerant full-adder for quantum-dot cellular automata is presented in this paper. High performance logic component can be achieved by utilizing this fault-tolerant full-adder. Some physical proofs have verified the functionality of the presented structure. The proposed design demonstrates significantly more robust than the standard full-adder and adder to single or multiple faults in misalignment, missing and dislocation cells. Improving the robustness of the full-adder leads to efficient designing of many fault-tolerant arithmetic circuits.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interests regarding the publication of this manuscript.

REFERENCES


Table 1: Single defective cell in proposed scheme.

<table>
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<th>Defective Cell No.</th>
<th>Out</th>
<th>Defective Cell No.</th>
<th>Out</th>
</tr>
</thead>
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<td>M(a1, b1, c1)</td>
<td>1</td>
<td>M(a1, b1, c1)</td>
</tr>
<tr>
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<td>M(a1, b1, c1)</td>
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<td>M(a1, b1, c1)</td>
</tr>
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<td>M(a1, b1, c1)</td>
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<td>8</td>
<td>M(a1, b1, c1)</td>
<td>9</td>
<td>M(a1, b1, c1)</td>
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Thesis in Wichita State University.


