Optimization
Iranian Journal of Optimization
Volume 11, Issue 2, 2019, 277-284
Research Paper
Online version is available on: www.ijo.iaurasht.ac.ir

Islamic Azad University
Rasht Branch
ISSN: 2588-5723
E-ISSN:2008-5427

On Solving Possibilistic Multi- Objective De Novo linear Programming

Hamiden Abd El-Wahed Khalifa*

Department of Operations Research, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt

Received: 19 January 2019
Accepted: 28 May 2019

Keywords:
De Novo programming
Multi-objective
Linear programming
Possibilistic variables
δ- possibily efficient solution
Trade-offs

Abstract

Multi-objective De Novo linear programming (MODNLP) is problem for designing optimal system by reshaping the feasible set (Fiala, 2011). This paper deals with MODNLP having possibilistic objective functions coefficients. The problem is considered by inserting possibilistic data in the objective functions coefficients. The solution of the problem is defined and established under the using of efficient and necessary condition. Also, the relation between possibilistic levels corresponding to the solution is constructed. A solution procedure for solving the problem is proposed. A numerical example is given for illustration.

* **Correspondence E_mail**: hamidenkhalifa@cu.edu.eg

INTRODUCTION

In general, there is no single optimal solution in multi- criteria problems, but rather a set of non- inferior (Pareto optimal) solutions from which the decision maker must select the most preferred or best compromise solution as the one to implement. One of the difficulties which occur in the application of mathematical programming is that the coefficients in the formulation are not constants but fluctuating and uncertain. Sakawa and Yano, 1989 introduced the concept of Pareto optimality of fuzzy parametric program. Khalifa, 2018 proposed an approach for solving fuzzy MOLP problems. Kiruthiga and Loganathan, 2015 reduced the Fuzzy MOLP problem to the corresponding ordinary one using the ranking function and hence solved it using the fuzzy programming technique. Hamadameen, 2018 proposed a technique for solving fuzzy MOLP problem in which the objective functions coefficients are triangular fuzzy numbers. Under uncertainty, vague, and imprecise of data, Garg, 2018 suggested an alternative approach for solving multi- objective reliability optimization problem.

De Novo programming defined by Zeleny and Raff, 1980 emphasizes optimal design of the original problem instead of just optimizing a sub problem where the constraints are fixed and given. This approach is much more flexible than the usual multi- objective linear programming (MOLP) (Li and Lee, 1990). Zelany 1980 created an optimal level model for DNP with resource and budget. The concept of the optimal systems design was first applied by Zeleny 1990. Trade-offs are of inadequately properties designed system and through the designing better one can be eliminated. Fiala, 2011 proposed approaches for solving multi- objective De Novo linear programming, also introduced possible extensions, methodological and real applications. Umarusman and Turkmen, 2013 have built the optimum production setting through the de novo programming with the global criterion method. Tabucanon (1988) has shown that the de novo programming formulation deal with the best mixture of input specified as well as the best mixture of the output. Luhandjula, 1986, and 1987 deals with the problem by incorporating possibilistic data into single and MOLP framework. Li and Lee (1990) utilized a two phases for multi- criteria DNP that yields fuzzy solution, and also studied fuzzy multiple criteria De Novo programming based on fuzzy set possibility concept. Zhang et al., 2009؛ Chen and Tzeng, 2009؛ Huang et al., 2006؛ Chen and Hsieh, 2006؛ and Zeleny, 2010 studied De Novo programming. Khalifa (2018) studied a multi- criteria de novo linear programming problem with fuzzy parameters. Zhuang and Hocine (2017) used the Meta –goal-programming approach for solving multi-criteria de Novo programming problems. Eren (2017) discussed the selection of projects according to the evaluating criteria of support mechanisms considered by Regional Development Agencies (RDAs) through the procedure provided by a practical solution methodology, which is an integrating of fuzzy parametric programming and fuzzy linear programming. Umarksman (2019) applied Lexicographic goal programming for solutions of a multiobjective De Novo programming problem with positive ideal solutions and the same problem solved with global criterion method , and hence compared the results. Babic et al., (2018) showed an example of how to use De Novo programming instead of linear programming in real business situations.

In this paper, the MODNLP having possibilistic objective functions coefficients is studied. The solution of the problem is defined and established based on necessary and sufficient conditions, and the relation between possibilistic levels corresponding to the solution is constructed.

The remainder of the paper is as: In section 2; some preliminaries needed in the paper are presented. In section 3, a Poss MODNLP problem is formulated. In section 4, the δ- possibly efficient solution of the Poss MODNLP problem is characterized. In section 5, a numerical example is given for illustration. Finally some concluding remarks are reported in section6.

PRELIMINARIES

In order discuss our problem conveniently, some necessary results on possibilistic variables and its δ-cut are viewed (Luhandujla, 1987; Hussein, 1998).

Definition1. (Hussein, 1998). A possibilistic variable X on V is a variable characterized by a possibility distribution ζ_x (i.e., a rule which associated to each v∈V, a value ζ_x (v) indicates the degree of compatibility of the variable X with the realization v∈V.

If V is a Cartesian product of V_1 , V_2 ,…, V_n, then ζ_x (v_1 , v_2 , …, v_n)is an n- ary possibility distribution, i.e., $\zeta_x(v)=(\zeta_{x1}(v_1), \zeta_{x2}(v_2), \dots, \zeta_{xn}(v_n))$

Definition2. The δ-cut of a possibilistic variable X is

$(X)_\delta=\{v\in V: \zeta_x(v)\geq\delta\}$.

Definition3. A possibility distribution ζ_x on V is said to be convex if

$$\zeta_x(\alpha v^1+(1-\alpha) v^2)\geq \min(\zeta_x(v^1), \zeta_x(v^2)); \quad \forall v^1,v^2\in V; \alpha\in[0, 1].$$

Definition4. The support of a possibilistic variable *X* is

$$\mathrm{Supp}(X)=\left\{v\in V: \sup_{u\in M_\chi(x)}(\zeta_x(v)>0;\ \forall\chi>0)\right\},$$

where $M_\chi(v)=\{u\in V: \|u-v\|<\chi\}$.

PROBLEM STATEMENT AND SOLUTION CONCEPTS

A typical multi objective De Novo linear programming (MODNLP) which is designing optimal system by reshaping the feasible set (Fiala, 2011) is formulated with possibilistic data in the objective functions coefficients as (Poss MODNLP)

$$\max \tilde{Z}_k=\sum_{i=1}^{n}\tilde{c}_i^k\, y_i=\tilde{c}^k y,\ k=1,2,\ \dots,s \tag{1}$$

Subject to
A y-b≤0,
p b≤B,
y≥0.

Where, $b\in\mathbb{R}^m$ is unknown resources restrictions, $p\in\mathbb{R}^m$ is resource prices, $c\in\mathbb{R}^{k(1\times n)}$, are possibilistic variables on ℝ characterized by possibily distribution ζ_{ri}^{k},) $A\in\mathbb{R}^{m\times n}$,and Bis the given total available budget. It is assumed that all possibility distributions involved in Poss MODNLP are convex ones with bounded and closed supports.

Remark1: It is noted that Poss referred to possibility.

It follows from Poss MODNLP problem that:

$$pAy\leq pb\leq B. \tag{2}$$

By defining the unit cost $pA=w\in\mathbb{R}^n$. So, Poss MODNLP problem can be rewritten as

$$\max \tilde{Z}_k\left(x,\tilde{c}^k\right)=\tilde{c}^k y=\sum_{i=1}^{n}\tilde{c}_i{}^k y_i,\ k=1,2\dots,s \tag{3}$$

Subject to
$y\in Y=\{y\in\mathbb{R}^n: wy\leq B,\ y\geq 0\}$.

Definition5 (δ-Possibly efficient solution). A point y^*∈Y is said to be δ- possibly efficient solution to the (Poss MODNLP)1 problem if there is no y∈Y such that:

$$\mathrm{Poss}\begin{pmatrix} Z_1(y^*,\tilde{c}^1)\leq Z_1(y,\tilde{c}^1), Z_2(y^*,\tilde{c}^2)\leq Z_2(y,\tilde{c}^2),\dots, Z_{k-1}(y^*,\tilde{c}^{k-1})\leq Z_{k-1}(y,\tilde{c}^{k-1})\\ Z_k(y^*,\tilde{c}^k)\leq Z_k(y,\tilde{c}^k), Z_{k+1}(y^*,\tilde{c}^{k+1})\leq Z_{k+1}(y,\tilde{c}^{k+1}),\ \dots, Z_s(y^*,\tilde{c}^s)\leq Z_s(y,\tilde{c}^s)\end{pmatrix}\geq\delta \tag{4}$$

On account of the extension principle,

$$\text{Poss}\begin{pmatrix} Z_1(y^*,\tilde{c}^1) \le Z_1(y,\tilde{c}^1), Z_2(y^*,\tilde{c}^2) \le Z_2(y,\tilde{c}^2), \dots, Z_{k-1}(y^*,\tilde{c}^{k-1}) \le Z_{k-1}(y,\tilde{c}^{k-1}) \\ Z_k(y^*,\tilde{c}^k) \le Z_k(y,\tilde{c}^k), Z_{k+1}(y^*,\tilde{c}^{k+1}) \le Z_{k+1}(y,\tilde{c}^{k+1}), \dots, Z_s(y^*,\tilde{c}^s) \le Z_s(y,\tilde{c}^s) \end{pmatrix}$$
$$= \sup_{(c^1,c^2,\dots,c^s)\in D} \max\left(\zeta_{\tau^1}(\tilde{c}^1), \zeta_{\tau^2}(\tilde{c}^2), \dots, \zeta_{\tau^{k-1}}(\tilde{c}^{k-1}), \zeta_{\tau^k}(\tilde{c}^k), \zeta_{\tau^{k+1}}(\tilde{c}^{k+1}), \dots, \zeta_{\tau^s}(\tilde{c}^s)\right) \quad (5)$$

where,

$$D = \begin{Bmatrix} (c^1, c^2, \dots, c^s) \in \mathbb{R}^{s(1\times n)}: Z_1(y^*,\tilde{c}^1) \le Z_1(y,\tilde{c}^1), Z_2(y^*,\tilde{c}^2) \le Z_2(y,\tilde{c}^2) \\ , \dots, Z_{k-1}(y^*,\tilde{c}^{k-1}) \le Z_{k-1}(y,\tilde{c}^{k-1}), Z_k(y^*,\tilde{c}^k) \le Z_k(y,\tilde{c}^k), \\ Z_{k+1}(y^*,\tilde{c}^{k+1}) \le Z_{k+1}(y,\tilde{c}^{k+1}), \dots, Z_s(y^*,\tilde{c}^s) \le Z_s(y,\tilde{c}^s) \end{Bmatrix}, \quad (6)$$

and $\zeta_{\tau k}$ (k=1,2,...,s), and m×n- ary possibility distributions.

CHARACTERIZATION OF δ- POSSIBLY EFFICIENT SOLUTION FOR PROBLEM

Our aim is to characterize the δ-possibly efficient solution for the problem (3), so let us apply the following δ-parametric MODNLP problem, i.e., (δ-PMODNLP)

(δ-PMODNLP) $\max Z_k (x, c^k) = c^k y = \sum_{i=1}^{n} c_i^k y_i, i=1,2,...,s$ (7)
Subject to
$y \in Y, c_i^k \in (\tilde{c}_i^k)_\delta$.

Where $(\tilde{c}_i^k)_\delta$ are the δ -cut of the possibilistic variables $\tilde{c}_i^k$. By the convexity assumption,

Where $(\tilde{c}_i^k)_\delta$ are the δ -cut of the possibilistic variables $\tilde{c}_i^k$. By the convexity assumption, $\zeta_{\tau ik} (\tilde{c}_i^k)_\delta$, i=1,2,...,n;k=1,2,...,s are real intervals that are denoted by are real intervals that are denoted by $[c_i^k (\delta)^L, c_i^k (\delta)^U]$. Let ξ_δ^k be the set of 1×n matrices $c^k=(c_i^k)$, with $c_i^k \in [c_i^k (\delta)^L, c_i^k (\delta)^U]$,k=1,2,...,s. It clear that δ-PMODNLP may be rewritten as

$\max Z_k (x, c^k)$,k=1,2,…,s (8)
Subject to
$y \in Y$, and $c^k \in \xi_\delta^k$,k=1,2,...,s.

Definition6. (δ-Parametric efficient solution). A point $y^* \in Y$ is said to be an δ- parametric efficient solution to the problem (8) if and only if there are no $y \in Y$, and $c^k \in \zeta_\delta^k$ such that $Z_k (y^*, c^k) \le Z_k (y, c^k)$; ∀k and strict inequality holds for at least one k

Theorem1. A point $y^{\wedge *} \in Y$ is an δ- possibly efficient solution to the problem (3) if and only if $y^* \in Y$ is an δ- parametric efficient solution for problem (8).

Proof. Necessity: Let $y^* \in Y$ be an δ- possibly efficient solution to the problem (3) and is not $y^* \in Y$ an δ- parametric efficient solution for problem (8). Then there are $y^1 \in Y$ and $d^k \in \zeta_\delta^k$,k=1,2,...,s such that

$Z_t (y^*, d^t) \le Z\_t (y^1, d^t)$; ∀ t∈{1,2,...,s} and k∈{1,2,...,s} such that

$Z_k (y^*, d^t) \le Z\_k (y^1, d^t)$. As $d^k \in \zeta_\delta^k$, we have

$$\text{Poss}\begin{pmatrix} Z_1(y^*,\tilde{c}^1) \le Z_1(y,\tilde{c}^1), Z_2(y^*,\tilde{c}^2) \le Z_2(y,\tilde{c}^2), \dots, \\ Z_{k-1}(y^*,\tilde{c}^{k-1}) \le Z_{k-1}(y,\tilde{c}^{k-1}), Z_k(y^*,\tilde{c}^k) \le Z_k(y,\tilde{c}^k) \\ , Z_{k+1}(y^*,\tilde{c}^{k+1}) \le Z_{k+1}(y,\tilde{c}^{k+1}), \dots, Z_s(y^*,\tilde{c}^s) \le Z_s(y,\tilde{c}^s) \end{pmatrix} \ge \delta \quad (9)$$

Contradiction to the assumption that $y^* \in Y$ is a δ- possibly efficient solution to the problem (3).

Sufficiency: Let $y^* \in Y$ be an δ- parametric efficient solution to the problem (8) and $y^* \in Y$ is not an δ-possibily efficient solution for problem (3). Then there are $y^2 \in Y$ and k∈{1,2,...,s} such that

$$\text{Poss}\begin{pmatrix} Z_1(y^*,\tilde{c}^1) \le Z_1(y^2,\tilde{c}^1), Z_2(y^*,\tilde{c}^2) \le Z_2(y^2,\tilde{c}^2), \dots, \\ Z_{k-1}(y^*,\tilde{c}^{k-1}) \le Z_{k-1}(y^2,\tilde{c}^{k-1}), Z_k(y^*,\tilde{c}^k) \le Z_k(y^2,\tilde{c}^k) \\ , Z_{k+1}(y^*,\tilde{c}^{k+1}) \le Z_{k+1}(y^2,\tilde{c}^{k+1}), \dots, Z_s(y^*,\tilde{c}^s) \le Z_s(y^2,\tilde{c}^s) \end{pmatrix} \ge \delta$$

i.e.,

$$\sup_{(c^1,c^2,\dots,c^s)\in\bar{D}} \max\begin{pmatrix} \zeta_{\tau^1}(\tilde{c}^1), \zeta_{\tau^2}(\tilde{c}^2), \dots, \zeta_{\tau^{k-1}}(\tilde{c}^{k-1}), \zeta_{\tau^k}(\tilde{c}^k) \\ , \zeta_{\tau^{k+1}}(\tilde{c}^{k+1}), \dots, \zeta_{\tau^s}(\tilde{c}^s) \end{pmatrix} \ge \delta \tag{10}$$

Where

$$\bar{D} = \left\{\begin{matrix} (c^1, c^2, \dots, c^s) \in \mathbb{R}^{s(1\times n)}: Z_1(y^*,\tilde{c}^1) \le Z_1(y^2,\tilde{c}^1), Z_2(y^*,\tilde{c}^2) \le Z_2(y^2,\tilde{c}^2) \\ , \dots, Z_{k-1}(y^*,\tilde{c}^{k-1}) \le Z_{k-1}(y^2,\tilde{c}^{k-1}), Z_k(y^*,\tilde{c}^k) \le Z_k(y^2,\tilde{c}^k), \\ Z_{k+1}(y^*,\tilde{c}^{k+1}) \le Z_{k+1}(y^2,\tilde{c}^{k+1}), \dots, Z_s(y^*,\tilde{c}^s) \le Z_s(y^2,\tilde{c}^s) \end{matrix}\right\}.$$

For this supremum exists, there is $(r^1, r^2,\dots,r^s)\in \bar{D}$ with $\max(\zeta_{\tau^1}(r^1),\zeta_{\tau^2}(r^2),\dots,\zeta_{\tau^s}(r^s)) < \delta$ then

$$\sup_{(r^1, r^2,\dots,r^s)\in\bar{D}} \max(\zeta_{\tau^1}(r^1), \zeta_{\tau^2}(r^2), \dots, \zeta_{\tau^s}(r^s)) < \delta.$$

Contradiction (6). Thus there is $(r1, r^2,\dots,r^s)\in \bar{D}$ satisfying

$$\max(\zeta_{\tau^1}(r^1), \zeta_{\tau^2}(r^2),\dots,\zeta_{\tau^s}(r^s))\ge, \tag{11}$$

i.e.,

$$r^k\in \xi_\delta{}^k, k=1,2,\dots,s \tag{12}$$

It follows from (9) , and (11) that contradiction the efficiency y^* for the problem (8).

By solving the problem (8) which is the continuous knapsack problems, we have

$$y_i^k = \begin{cases} 0, & k \ne i \\ \frac{B}{w_i}, & i = i_k \end{cases}$$ be the solution with the corresponding ideal objectives Z_k^* for the ideal system with respect to B, where

$$i_k \in \left\{ i \in \mathbb{R}^n : \max_i \frac{c_i{}^k}{w_i} \right\}$$

The Meta- optimum problem corresponding to the problem (8) is formulated as

$$\min F=w\,y \tag{13}$$

Subject to

$c^k y \ge Z_k^*$,

$y\ge0$, $c^k\in\zeta_\delta{}^k$, k=1, 2, …, s

Using GAMS software the solution of problem (13) is: y^*;$B^*=w\,y^*$, and $b^*=A\,y^*$.

To provide an effective and fast tool for the efficient optimal redesign of large- scale linear systems; let us introduce the optimum- path ratio (Shi, 1995) as

$$g_1=B/B^*, B\le B^*$$

The optimal design of the system for the budget B: $y=g\_1\ {}_y{}^*$,$b=g_1 b^*$,$Z=g_1 Z^*$.

It is noted that if k<n, synthetic solutions can be obtained by solving the problem individually, Where, Shi,1995 defined the synthetic optimal solution as $y^{**}=(y_{i1}{}^1, y_{i2}{}^2,\dots,y_{ik}{}^k,0,\dots,0)$, where $y_{i\theta}{}^\theta$ is the optimal solution of problem(8). There are six types of optimum- path ratios (Shi, 1995):

$$g_1 = \frac{B}{B^*}, g_2 = \frac{B}{B^{**}}, g_3 = \frac{B^*}{B^{**}}, g_4 = \frac{\sum_i \lambda_i B_i^k}{B},$$

$$g_5 = \frac{\sum_i \lambda_i B_i^k}{B^*}, g_6 = \frac{\sum_i \lambda_i B_i^k}{B^{**}}$$

There is possible to establish different optimal system design as option for the DM.

NUMERICAL EXAMPLE

Consider the following Poss MOLP problem

$$\max \tilde{Z}_k = \sum_{i=1}^{2} \tilde{c}_i^{\,k}\, y_i, k = 1, 2 \qquad (14)$$

Subject to
$3x_1+4x_2\leq 40$,
$x_1+3x_2\leq 30$,
$x,x_2\geq 0$.

The Poss MODNLP problem can be formulated as

$$\max \tilde{Z}_k = \sum_{i=1}^{2} \tilde{c}_i^{\,k}\, y_i, k = 1, 2 \qquad (15)$$

Subject to
$1.9\, x_1+3.2\, x_2 \leq 42$,
$x, x_2\geq 0$.

With the price p=$(0.5, 0.4), and the budget level B=$ 35.

The possibilistic variables $\tilde{c}_i^1$ and $\tilde{c}_i^2$ is characterized by a possibility distributions $\zeta_{\tau_i^1}(.)$, and$\zeta_{\tau_i^2}(.)$,,
respectively. The supports of $\tilde{c}_i^1$ and $\tilde{c}_i^2$ are[1, 3], and [1, 5], and may be defined as:

$$Supp\left(\tilde{c}_1^{\,1}\right) = 1+\delta, \qquad \zeta_{\tau_1^1}(1) = \zeta_{\tau_1^1}(2) = 0$$
$$Supp\left(\tilde{c}_2^{\,1}\right) = 2-\delta, \qquad \zeta_{\tau_2^1}(1) = \zeta_{\tau_2^1}(2) = 0$$
$$Supp\left(\tilde{c}_1^{\,2}\right) = 1+2\delta, \qquad \zeta_{\tau_1^2}(1) = \zeta_{\tau_1^2}(3) = 0$$
$$Supp\left(\tilde{c}_2^{\,2}\right) = 4+\delta, \qquad \zeta_{\tau_2^2}(4) = \zeta_{\tau_2^2}(5) = 0$$

At $\delta=0$, the problem corresponding to problem (5) is

$$\begin{aligned}\max Z_1 &= x_1 + 2x_2\\ \max Z_2 &= 1x_1 + 5x_2\end{aligned} \qquad (16)$$

Subject to
$1.9x_1 + 3.2x_2 \leq 42$,
$x_1, x_2 \geq 0$

Solving the problem (16) individually with respect to the constraints of problem (14), we get:

$Z_1^*. = 20$, $Z_2^* = 40$ Thus

$$\min F = 1.9\, y_1 + 3.2 y_2 \qquad (17)$$

Subject to
The solution is:

$$y_1^* = 0,\ y_2^* = 13.125, B^* = v\, y^* = (1.9 \quad 3.2)\begin{pmatrix}0\\13.125\end{pmatrix} = 42,$$

$$b^* = A\, y^* = \begin{pmatrix}3 & 4\\1 & 3\end{pmatrix}\begin{pmatrix}0\\13.125\end{pmatrix} = \begin{pmatrix}52.50\\39.375\end{pmatrix}.$$

Hence,

$$r_1 = \frac{B}{B^*} = \frac{35}{42} = 0.8333$$

Thus, the possibilistic optimal design of the system for the budget B is:

$$y = r_1\, y^* = 0.8333(0 \quad 13.125) = (0 \quad 10.9375),$$

$$b = r_1\, b^* = 0.8333\begin{pmatrix}52.50\\39.375\end{pmatrix} = \begin{pmatrix}43.74825\\32.8112\end{pmatrix},$$

$$Z = r_1\, Z^* = \begin{pmatrix}21.875\\43.75\end{pmatrix}.$$

CONCLUDING REMARKS

In this paper, possibilistic MODNLP has been studied. The solution of the problem has been defined and established under the using of efficient and necessary condition. In addition, the relation between possibilistic levels corresponding to the solution was constructed. A solution procedure for solving the problem has proposed. In addition, we deduce that the optimal system should of free tradeoff, and the De Novo programming is one of the methodology used in the optimal system design.

ACKNOWLEDGMENTS

The author would like to thank the referees for their suggestive valuable and helpful comments that have led to an improved version of the paper.

REFERENCES

Babic, Z., Veza, I., Balic, A., & Crnjac, M. (2018). Application of De Novo Programming Approach for Optimizing the Business Process. *World Academy of Science, Engineering and Technology, International Journal of Industrial and Systems Engineering*, *12*(5), 519-524.

Chen, J. K., & Tzeng, G. H. (2009). Perspective strategic alliances and resource allocation in supply chain systems through the De Novo programming approach. *International journal of sustainable strategic management*, *1*(3), 320-339.

Eren, M. (2017). A two-phase procedure for a multi-objective programming problem with fuzzy coefficients based on group decision-making for project selection. *Croatian Operational Research Review*, *8*(1), 317-333.

Fiala, P. (2011). Multiobjective De Novo Linear Programming. *Acta universitatis palackianae olomucensis. Facultas rerum naturalium. Mathematica*, *50*(2), 29-36**.**

Garg, H. (2018). Analysis of an industrial system under uncertain environment by using different types of fuzzy numbers. *International journal of system assurance engineering and management*, *9*(2), 525-538.

Hamadameen, O. A. (2018). A noval technique for solving multi- objective linear programming problems. *Aro- The Scientific Journal of Koya University*, 5(2), 1-8.

Huang, J. J., Tzeng, G. H., & Ong, C. S. (2006). Choosing best alliance partners and allocating optimal alliance resources using the fuzzy multi-objective dummy programming model. *Journal of the operational research society*, *57*(10), 1216-1223.

Hussein, M. L. (1998). Complete solutions of multiple objective transportation problems with possibilistic coefficients. *Fuzzy Sets and Systems*, *93*(3), 293-299.

Khalifa, H. A. (2018). On Solutions of Fuzzy Multi-Objective Programming Problems through Weighted Coefficients in Two-Phase Approach. *J. Appl. Res. Ind. Eng. Vol*, *5*(2), 110-1218.

Khalifa, H. A. (2018). On solving fully fuzzy multi-criteria De Novo programming via fuzzy goal programming approach. *Journal of Applied Research on Industrial Engineering*, *5*(3), 239-252.

Kiruthiga, M., & Loganathan, C. (2015). Fuzzy multi-objective linear programming problem using membership function. *International journal of science, engineering, and technology, applied sciences*, *5*(8), 1171-1178.

Li, R. J., & Lee, E. S. (1990). Fuzzy approaches to multicriteria De Novo programs. *Journal of mathematical analysis and applications*, *153*(1), 97-111.

Li, R. J., & Lee, E. S. (1990). Multi-criteria de novo programming with fuzzy parameters.*Computers & Mathematics with Applications*, *19*(5), 13-20.

Luhandjula, M. K. (1986). On possibilistic linear programming. *Fuzzy Sets and Systems*, *18*(1), 15-30.

Luhandjula, M. K. (1987). Multiple objective programming problems with possibilistic coefficients. *Fuzzy sets and systems*, *21*(2), 135-145.

Sakawa, M., & Yano, H. (1989). Interactive decision making for multiobjective nonlinear programming problems with fuzzy parameters. *Fuzzy sets and Systems*, *29*(3), 315-326.

Shi, Y. (1995). Studies on optimum-path ratios in multicriteria De Novo programming problems. *Computers & Mathematics with Applications*, *29*(5), 43-50

Tabucanon, M. T. (1988). *Multiple criteria decision making in industry* (Vol. 8). Elsevier Science Ltd.

Umarksman, O.U.N. (2019). Using global criterion method to define priorities in Lexicographic goal programming and an application for optimal system design. MANAS *Journal of Social Studies*, 8(1/1): 333- 348.

Umarusman, N., & Turkmen, A. (2013). Building optimum production settings using de novo programming with global criterion method. *International journal of computer applications*, *82*(18).

Zeleny, M., & Raff, S. (1980). Mathematical programming with multiple objectives. *Compuf. Ops Res*, *7*(1-2)

Zeleny, M. (1986). Optimal system design with multiple criteria: De Novo programming approach. *Engineering costs and production economics*, *10*(1), 89-94.

Zeleny, M. (1990). Optimizing given systems vs. designing optimal systems: The De Novo programming approach. *International journal of general systems*, *17*(4), 295-307.

Zeleny, M. (2010). Multiobjective optimization, systems design and De Novo programming. In *Handbook of Multicriteria Analysis* (pp. 243-262). Springer, Berlin, Heidelberg.

Zhang, Y. M., Huang, G. H., & Zhang, X. D. (2009). Inexact de Novo programming for water resources systems planning. *European journal of operational research*, *199*(2), 531-541.

Zhuang, Z. Y., & Hocine, A. (2018). Meta goal programing approach for solving multi-criteria de Novo programing problem. *European journal of operational research*, *265*(1), 228-238.