



Actuator Failures and Sensor Bias Compensation by Combination of Model Reference Adaptive Control & Kalman Filter Theory

Hossein Rahmani Amoli^{1*}, Karim Salahshoor²

¹Department of Electrical Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran.

²Department of Automation and Instrumentation Engineering, Petroleum University of Technology, Ahvaz, Iran.

Received: 22-Apr-2019, Revised: 08-May-2019, Accepted: 14-May-2019.

Abstract

The actuator failure and sensor bias pose very important challenges in the aerospace industry. Why adaptive control is a good way to deal with these problems? The paper addresses the problem of actuator failures and sensor bias compensation using the combination of a model reference adaptive control (MRAC) approach and Kalman filter (KF) for state tracking objective. To comply this condition, an enhanced MRAC method is introduced for state tracking based on state feedback configuration and a number of adaptation laws have been formulated to maintain the desired system performance. The Kalman filter is used to estimate the states despite sensor bias, providing excellent and reliable estimates. Simulation results demonstrate the effectiveness of the presented method to achieve good state tracking performances in spite of the presence of actuator failures and sensor bias.

Keywords: Actuator failures, sensor bias, MRAC controller, Kalman Filter.

1. INTRODUCTION

One of the important applications of adaptive control is in actuator failures and sensor bias compensation which has many applications including aircraft, flight control systems and so on.

An adaptive control system is defined as a

feedback control system intelligent enough to adjust its characteristics in a changing environment so as to operate in an optimum manner according to some specified criterion [8]. By using MRAC, systems can eventually achieve stability, closed-loop signals boundedness and desired system performance. MRAC method can be used for state tracking or output tracking, related to a

*Corresponding Author's Email:

st_h_rahmani@azad.ac.ir

reference model, by using the matching parameters from the point of view of the stability and control ability [5].

Actuator fault can be unknown, in other words, the type and value of fault and the fact that which actuator failing is occurring are uncertain. For example, some unknown inputs may be stuck at some unknown fixed values at unknown time instants. Adaptive control of systems with actuator failures intends to compensate for uncertain failures with adaptive tuning of controller parameters based on system response errors to achieve desired system performance.

Unknown biases can appear during operation in sensors such as rate gyros, accelerometers, altimeter and etc. It should be noted that using standard MRAC laws cannot achieve closed-loop stability in the presence of sensor bias in state feedback [11]. Sensor bias estimation and compensation are important challenges in the past researches. For example, in Patre and Joshi (2011), a method is provided for sensor bias estimation as a part of the adaptive control law. In Bevely and David M (2004) [1], Vemuri.A.T (2001) [18], Healey.A.J et al. (1998) [4] bias estimation for multisensory systems has been investigated. A multivariable MRAC scheme with sensor uncertainty compensation has been studied in [3]. Adaptive detection of sensor uncertainties and failures was presented in [16]. Output feedback MIMO MRAC schemes with sensor uncertainty compensation was presented in [12]. In addition, both actuator failures and sensor bias compensation were studied in [6]. In the latest research, Ge Song and Gang Tao in 2017 [14] designed new adaptive controller structure that has the capability of ensuring

plant-model matching with ability of compensating all possible uncertain sensor failures at the same time. It can also guarantee asymptotic output tracking and closed-loop signal boundedness in the presence of the various uncertainties. Furthermore, Zehui Mao and Gang Tao [10] presented an adaptive failure compensation scheme with the adaptive controller for healthy system, which are developed to deal with the unknown parameters in the plant and traction system actuator failures [14].

In this paper, KF is used for sensor bias estimation as the optimal observer. KF can estimate the system states and outputs based on measurement noise and random inputs [2]. As a result, an unbiased estimation is achieved that is crucial for enhanced MRAC tracking control. The enhanced MRAC control can stabilize the closed loop system provided that the unbiased estimation is in hand.

The organization of the paper is as follows; in section 2 the system architecture and bias estimation are presented, section 3 describes the enhanced MRAC control and section 4 shows the result of simulation.

2. SYSTEM ARCHITECTURE AND BIAS ESTIMATION

Consider the linear plant dynamics as:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y &= x(t) + \beta\end{aligned}\quad (1)$$

where $A \in R^{n \times n}$ is the system matrix, $B \in R^{n \times m}$ is a known input matrix, $x(t) \in R^n$ is

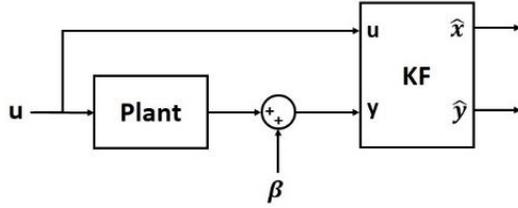


Fig. 1. KF Performance.

the system states, $u(t) \in R^m$ is the control input and $y(t) \in R^n$ is the available state measurement with an unknown constant bias $\beta \in R^n$.

2.1. Sensor Bias Compensation

To achieve the state tracking and sensor bias estimation, firstly, states and output estimation is performed by KF, which results an unbiased estimation.

The KF is an optimal observer. An observer is used to calculate state variables that are not accessible from the output [9]. Due to the presence of sensor bias, the state variable cannot be measured. Actually, KF is a dynamical system whose inputs are the plant inputs and outputs. The system states and outputs may be accompanied by process or measurement noises respectively. KF outputs are estimates of the state variables and the output without noise [2], [7]. When state variables are not measurable for state feedback and when the plant output has bias, KF can be used. Figure 1 shows the KF filter inputs and outputs.

To compensate sensor bias, 'corrected' state $\bar{x}(t) \in R^n$ is defined as:

$$\bar{x} = y - \hat{\beta} \quad (2)$$

Therefore, we can define corrected-state equation as:

$$\dot{\bar{x}} = A\bar{x} + Bu \quad (3)$$

$$\bar{y} = C\bar{x}$$

Considering (3), the state-space equation for above KF dynamical system is:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x}(t) + Bu(t) + L(\bar{y} - \hat{y}) \\ \hat{y} &= C\hat{x} \end{aligned} \quad (4)$$

where $\hat{x} \in R^n$ and L should be selected so that the estimation error converges to zero asymptotically. Subtracting equation (4) from (3) gives:

$$\begin{aligned} (\dot{\bar{x}} - \dot{\hat{x}}) &= A(\bar{x} - \hat{x}) - L(\bar{y} - \hat{y}) \\ (\bar{y} - \hat{y}) &= C(\bar{x} - \hat{x}) \end{aligned} \quad (5)$$

and substituting the output equation into the state equation results:

$$(\dot{\bar{x}} - \dot{\hat{x}}) = (A - LC)(\bar{x} - \hat{x}) \quad (6)$$

Letting $\tilde{x} = \bar{x} - \hat{x}$, therefore

$$(\dot{\tilde{x}}) = (A - LC)\tilde{x} \quad (7)$$

If all eigenvalues of $(A - LC)$ are negative, then the estimated state vector error, \tilde{x} , will decay to zero.

For the values of L to yield a desired characteristic equation and good response in equation (7), the equation:

$$\det [\lambda I - (A - LC)] = 0 \quad (8)$$

is solved for L , given desired eigenvalues.

3. ADAPTIVE CONTROLLER STRUCTURE

The adaptive controller guarantees boundedness and state tracking in the presence of actuator failures and unknown sensor bias. The type of actuator failures is modeled as:

$$u_j(t) = \bar{u}_j, t \geq t_j, j \in \{1, 2, \dots, m\} \quad (9)$$

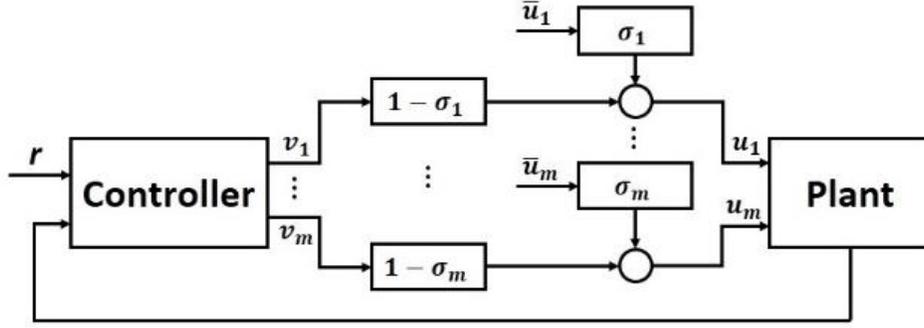


Fig. 2. Control system with actuator failures [15].

where the constant value \bar{u}_j and the failure time instant t_j are unknown. For example, we can mention an aircraft control surface (such as the rudder or an aileron) is stuck at some unknown fixed value. As a basic assumption, the remaining actuators (controls) can still achieve a desired control objective for any value up to $m - 1$ actuator failures.

In the presence of actuator failures, the input vector $u(t)$ can be described by:

$$u(t) = v(t) + \sigma(\bar{u} - v(t)) \quad (10)$$

where

$$\begin{aligned} \bar{u} &= [\bar{u}_1, \bar{u}_2, \dots, \bar{u}_m]^T \\ \sigma &= \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_m\} \\ \sigma_i &= 1 \text{ if the } i\text{th actuator fails,} \\ &\text{i. e., } u_i = \bar{u}_i \\ \sigma_i &= 0 \text{ otherwise.} \end{aligned} \quad (11)$$

σ is a diagonal matrix (failure pattern matrix) whose entries are piecewise constant signals that take on the values of zero or one as shown in Fig. 2. The actuator failures are uncertain in value, pattern and time of occurrence. The objective is to design an adaptive feedback control law using the available measurement $y(t)$ with unknown

bias β that closed-loop signal boundedness is ensured and the system states $x(t)$ track the states of a reference model described by:

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \quad (12)$$

where $x_m \in R^n$ is the state of reference model, $A_m \in R^{n \times n}$, $B_m \in R^{n \times m_r}$, and $r(t) \in R^{m_r}$ ($1 \leq m_r \leq m$) (this paper considers $m_r = 1$) is a bounded reference input used in the system operation (e.g., pilot input in the case of aircraft).

For every failure pattern the following conditions are assumed in a way that there are gains $K_1 \in R^{n \times m}$, $K_2 \in R^m$, and $K_3 \in R^m$ which satisfy following equation [6]:

$$\begin{aligned} A + B(I - \sigma) K_1^T &= A_M \\ B(I - \sigma) K_2 &= B_M \\ B\sigma\bar{u} &= -B(I - \sigma)(K_1^T \beta + K_3). \end{aligned} \quad (13)$$

The reference model must have been designed using optimal and robust control methods such as LQR, H_2 , or H_∞ . For the adaptive control scheme, only A_M and B_M should be known. Because A_M is a Hurwitz matrix, there are positive definite matrices $P = P^T, Q = Q^T \in R^{n \times n}$ in a way that the following Lyapunov inequality holds:

$$PA_M + A_M^T P \leq -Q. \quad (14)$$

Using the KF theory, we have:

$$\hat{\beta} = y - \hat{y} \quad (15)$$

where $\hat{\beta}$ is an estimate of the unknown sensor bias β . From equation (2), we have:

$$\bar{x} = x + \beta - \hat{\beta} = x + \tilde{\beta} \quad (16)$$

that $\tilde{\beta} = \beta - \hat{\beta}$ is the sensor bias estimation error. An adaptive control law is defined as:

$$v = \hat{K}_1^T y + \hat{K}_2 r + \hat{K}_3 \quad (17)$$

where $\hat{K}_1(t) \in R^{n \times m}$, $\hat{K}_2(t) \in R^{m \times m_r}$ and $\hat{K}_3(t) \in R^m$ are the adaptive gains. Therefore, the closed-loop corrected-state equation is:

$$\begin{aligned} \dot{\hat{x}} &= Ax + B(I - \sigma)(\hat{K}_1^T y + \hat{K}_2 r \\ &\quad + \hat{K}_3) + \dot{\tilde{\beta}} + B\sigma\bar{u} \\ &= Ax + B(I - \sigma)(K_1^T y + K_2 r \\ &\quad + K_3) \\ &\quad + B(I - \sigma)(\tilde{K}_1^T y \\ &\quad + \tilde{K}_2 r + \tilde{K}_3) + \dot{\tilde{\beta}} \\ &\quad + B\sigma\bar{u} \\ &= (A + B(I - \sigma)K_1^T)x \\ &\quad + B(I - \sigma)(\tilde{K}_1^T y \\ &\quad + \tilde{K}_2 r + \tilde{K}_3) \\ &\quad + B(I - \sigma)K_2 r \\ &\quad + B(I - \sigma)K_1^T \beta \\ &\quad + B(I - \sigma)K_3 + \dot{\tilde{\beta}} \\ &\quad + B\sigma\bar{u} \end{aligned} \quad (18)$$

where $\tilde{K}_1 = \hat{K}_1 - K_1$, $\tilde{K}_2 = \hat{K}_2 - K_2$, and $\tilde{K}_3 = \hat{K}_3 - K_3$. Using (13) in (18), we have:

$$\begin{aligned} \dot{\hat{x}} &= A_m \bar{x} + B_m r + B(I - \\ &\quad \sigma)(\tilde{K}_1^T y + \tilde{K}_2 r + \tilde{K}_3) - A_m \tilde{\beta} + \\ &\quad \dot{\tilde{\beta}}. \end{aligned} \quad (19)$$

Define a measurable auxiliary error signal $\hat{e}(t) \in R^n$ as:

$$\hat{e} = \bar{x} - x_m. \quad (20)$$

Therefore, from (16), we have:

$$\hat{e} = x - x_m + \tilde{\beta} = e + \tilde{\beta}. \quad (21)$$

where $e = x - x_m$ denotes the state tracking error. Differentiating (21) with respect to time, the closed-loop auxiliary error system can be expressed as:

$$\dot{\hat{e}} = \dot{x} - \dot{x}_m. \quad (22)$$

Substituting (19) and (12) into (22) yields:

$$\begin{aligned} \dot{\hat{e}} &= A_m \hat{e} + B(I - \sigma)(\tilde{K}_1^T y + \tilde{K}_2 r + \\ &\quad \tilde{K}_3) - A_m \tilde{\beta} + \dot{\tilde{\beta}}. \end{aligned} \quad (23)$$

Theorem 1: By using the KF theory, the bias estimation error converges to zero asymptotically.

Proof 1: from equation (16), we have:

$$\tilde{\beta} = \bar{x} - x \quad (24)$$

Differentiating the above equation and using (3) and (1), the following equation is obtained:

$$\begin{aligned} \dot{\tilde{\beta}} &= \dot{\bar{x}} - \dot{x} = A\bar{x} + Bu - Ax - \\ &\quad Bu = A(\bar{x} - x). \end{aligned} \quad (25)$$

Therefore

$$\dot{\tilde{\beta}} = A\tilde{\beta} \quad (26)$$

Given that A is a known Hurwitz matrix, $\tilde{\beta}$ converges to zero asymptotically.

Theorem 2: For the system given by (1), (12); the adaptive controller (10), and the gain adaptation laws:

$$\begin{aligned}\dot{\hat{K}}_1 &= -\eta_1 y \hat{e}^T P B \\ \dot{\hat{K}}_2 &= -\eta_2 B^T P \hat{e} r^T \\ \dot{\hat{K}}_3 &= -\eta_3 B^T P \hat{e}\end{aligned}\quad (27)$$

where $\eta_1 \in R^{n \times n}$, $\eta_2 \in R^{m \times m}$, $\eta_3 \in R^{m \times m}$ are constant symmetric positive definite matrices and P was defined in (14) guarantee that all the closed-loop signals including adaptive gains are bounded and the tracking error is bounded accordingly.

Proof 2: Define

$$\begin{aligned}V &= \hat{e}^T P \hat{e} + \sum_i^n \tilde{K}_{1i}^T \eta_1^{-1} \tilde{K}_{1i} + \\ &\sum_i^{m_r} \tilde{K}_{2i}^T \eta_2^{-1} \tilde{K}_{2i} + \tilde{K}_3^T \eta_3^{-1} \tilde{K}_3 + \\ &\tilde{\beta}^T P_A \tilde{\beta}\end{aligned}\quad (28)$$

where the subscript i denotes the i^{th} column of \tilde{K}_1 , \tilde{K}_2 ; and $P_A = P_A^T \in R^{n \times n}$ is a positive definite solution to the lyapunov inequality:

$$P_A A + A^T P_A \leq -Q_A \quad (29)$$

For some positive definite matrix $Q_A = Q_A^T \in R^{n \times n}$. Differentiating (28) with respect to time, using (14), (23), (26), (29), and properties of the matrix trace, the following expression is obtained:

$$\begin{aligned}\dot{V} &\leq -\hat{e}^T Q \hat{e} + 2Tr \left[\tilde{K}_1^T \left(y \hat{e}^T P B + \right. \right. \\ &\left. \left. \eta_1^{-1} \dot{\hat{K}}_1 \right) \right] + 2Tr \left[\tilde{K}_2^T \left(B^T P \hat{e} r^T + \right. \right. \\ &\left. \left. \eta_2^{-1} \dot{\hat{K}}_2 \right) \right] + 2\tilde{K}_3^T \left(B^T P \hat{e} + \eta_3^{-1} \dot{\hat{K}}_3 \right) - \\ &2\hat{e}^T P (A_m - A) \tilde{\beta} - \tilde{\beta}^T Q_A \tilde{\beta}\end{aligned}\quad (30)$$

Using the gain update laws (27) in (30), we get:

$$\dot{V} \leq -\hat{e}^T Q \hat{e} - 2\hat{e}^T P (A_m - A) \tilde{\beta} - \tilde{\beta}^T Q_A \tilde{\beta}. \quad (31)$$

By defining $\psi \in R^{2n}$ as:

$$\psi = [\hat{e}^T \quad \tilde{\beta}^T]^T \quad (32)$$

and $\delta \in R^{2n \times 2n}$ as:

$$\delta = \begin{bmatrix} Q & P(A_m - A) \\ (A_m - A)^T P & Q_A \end{bmatrix} \quad (33)$$

we have:

$$\dot{V} \leq -\psi^T \delta \psi \quad (34)$$

The determinant of δ is:

$$\det(\delta) = \det(Q) \cdot \det(Q_A - (A_m - A)^T P Q^{-1} P (A_m - A)) \quad (35)$$

Since Q is positive definite, so δ is positive definite if:

$$Q_A - (A_m - A)^T P Q^{-1} P (A_m - A) > 0. \quad (36)$$

Accordingly, Q_A should be chosen in a way that the relation (36) is satisfied. Therefore, $\dot{V} \leq 0$ and $V(t)$ is bounded for all T , $\hat{e}(t)$, $\hat{\beta}(t)$, $y(t)$, \hat{K}_1 , \hat{K}_2 , \hat{K}_3 are all bounded too and $\hat{e}(t) \in L^2$. For the closed-loop signal boundedness, we have $\dot{\hat{e}}(t) \in L^\infty$. Therefore, $\lim_{t \rightarrow \infty} \hat{e}(t) = 0$.

4. SIMULATION RESULTS

Simulation studies are performed on a fourth-order longitudinal dynamics model of a large transport aircraft in a wings-level cruise condition with known nominal trim conditions presented in [6]. The linear time-invariant plant is described by (1) and the state variables are as follows: pitch rate $q(t)$ (deg/s), true airspeed $v(t)$ (m/s),

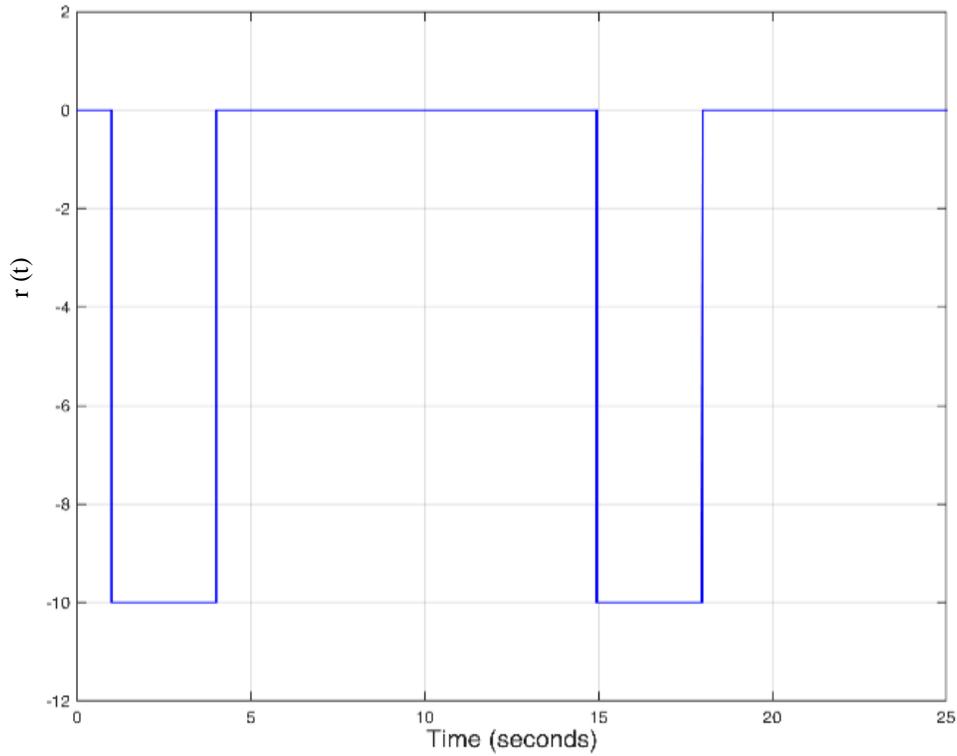


Fig. 3. Reference command.

angle of attack $\alpha(t)(deg)$, pitch angle $\theta(t)(deg)$, and $\{i.e., x = [q \ v \ \alpha \ \theta]^T\}$. The system matrices are:

$$A = \begin{bmatrix} -0.6803 & -0.0115 & -1.0490 & 0 \\ -0.0026 & -0.0062 & -0.0815 & -0.1709 \\ 1.0050 & -0.0344 & -0.5717 & 0 \\ 1.0000 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -44.5192 & -44.5192 \\ 0 & 0 \\ -11.4027 & -11.4027 \\ 0 & 0 \end{bmatrix}$$

The actuators are two identical elevators (i.e., two pairs of elevators operating symmetrically): $u_{e1}(t)$ and $u_{e2}(t)(deg)$, and we will assume that the second elevator gets stuck at $t = 15.1$ s. We consider an

unknown constant bias in the state measurement which is arbitrarily chosen as: $\beta = [5 \ 2 \ -1 \ 10]^T$ for $t > 10$ s. The reference model is chosen in a way that the closed-loop system has $A_m = A + BK_1^T$ and $B_m = BK_2$, where K_1 is the LQR gain designed to minimize a quadratic performance function. All control and adaptation parameters (η_i) are chosen by trial and error. The plant and reference model states, tracking errors, bias estimates by KF, bias estimates error and adaptive control input are shown in figures 4, 5, 6,7 and 8 respectively. The figures indicate that in the presence sensor bias and actuator failure, state tracking and the closed-loop signal boundedness or bounded tracking error can be proved.

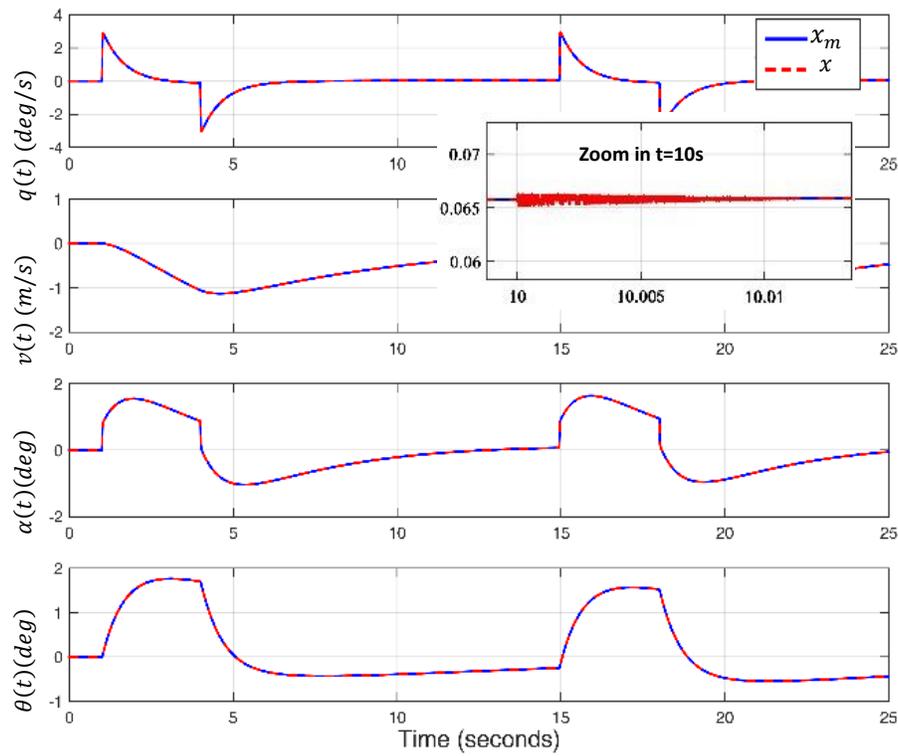


Fig. 4. Plant and reference model states in the presence of actuator failures and sensor bias.

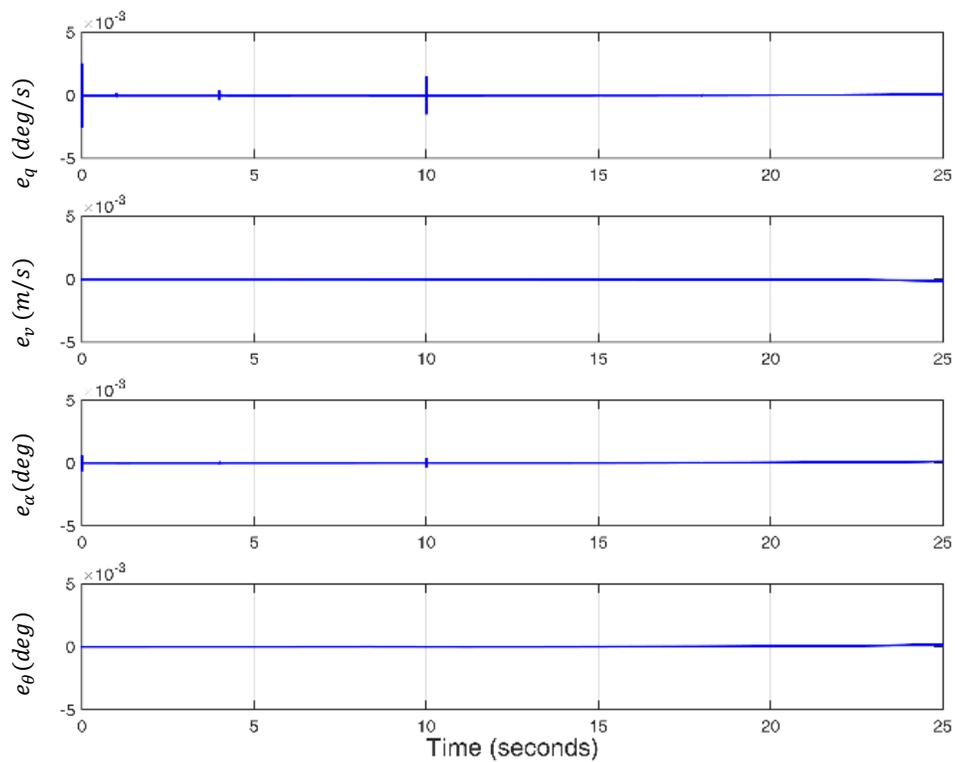


Fig. 5. State tracking error.

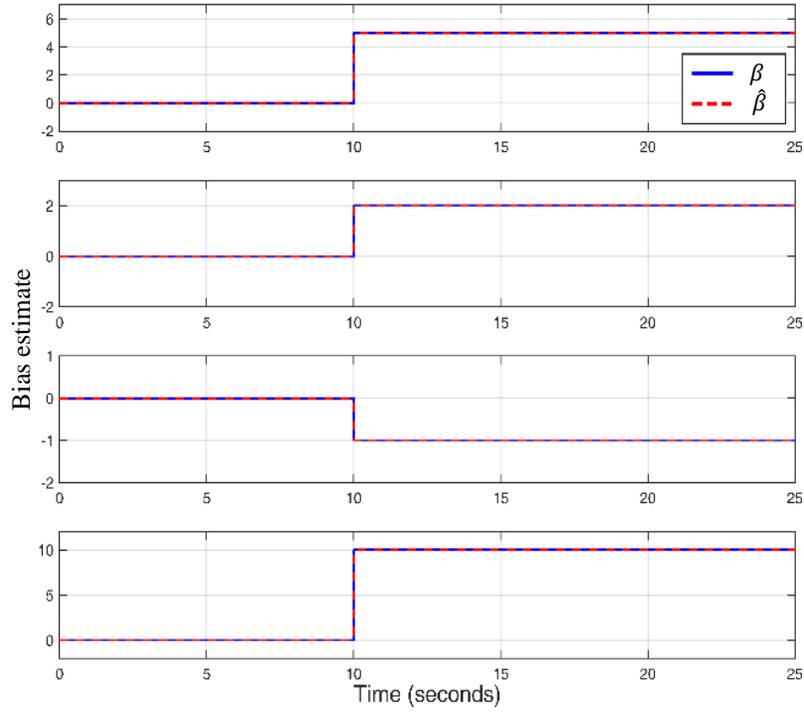


Fig. 6. Bias estimates by KF.

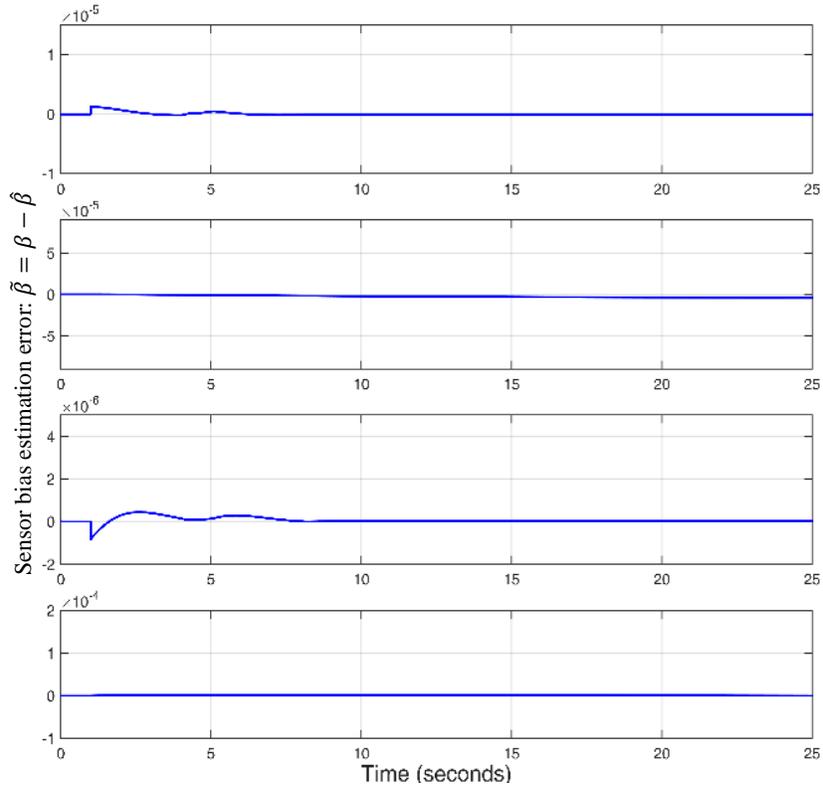


Fig. 7. Bias estimates error.

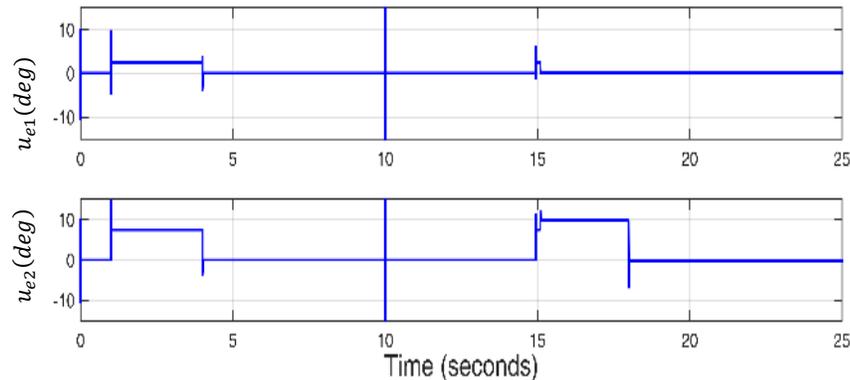


Fig. 8. Adaptive control input.

5. CONCLUSION

The state feedback for state tracking by an enhanced MRAC method is presented based on the state feedback for state tracking objective in the presence of actuator failure and sensor bias. A KF was developed to obtain an accurate estimate of the sensor bias. Simulation results prove that the system state $x(t)$ can effectively track the desired state trajectory based on a reference model.

REFERENCES

- [1] Bevlly David M. (2004). GPS: A Low Cost Velocity Sensor for Correcting Inertial Sensor Errors on Ground Vehicles. *Journal of Dynamic Systems, Measurement, and Control*, 126(2) (pp. 255-264).
- [2] Grover BR, P.Y.C. Hwang. (1997). *Introduction to Random Signals and Applied Kalman Filtering*. 3rd edition, John Wiley and Sons.
- [3] Guo Jiaxing and Gang Tao. (2009). A Multivariable MRAC Scheme with Sensor Uncertainty Compensation. *Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference Shanghai, P.R. China* (pp. 6632-6637).
- [4] Healey. A. J, An. E. P, and Marco. D. B. (1998). Online compensation of heading sensor bias for low cost AUVs. *Workshop on Autonomous Underwater Vehicles* (pp. 35-42).
- [5] Ioannou PA, B. Fidan. (2006). *Adaptive Control Tutorial*. SIAM California.
- [6] Joshi SM and Patre. P. (2014). Direct Model Reference Adaptive Control with Actuator Failures and Sensor Bias. *Journal of Guidance, Control, and Dynamics*, 37(1) (pp. 312-316).
- [7] Kalman, R.E and Bucy, R.S. (1961). New Results in Linear Filtering and Prediction Theory," *J. Basic Eng, Trans. ASME, Ser. D*, vol. 83 (pp. 95-108).
- [8] Narendra KS, AM. Annaswamy. (1989). *Stable Adaptive Systems*. Prentice Hall New Jersey.
- [9] Norman S Nise. (1992). *Control Systems Engineering*. The Benjamin/Cummings Publishing Company, INC.

- [10] Mao Zehui, Gang Tao. (2017). Adaptive Compensation of Traction System Actuator Failures for High-Speed Trains. *IEEE Transactions on intelligent transportation systems*.
- [11] Patre P , Suresh M. Joshi. (2011). Accommodating Sensor Bias in MRAC for State Tracking. In *AIAA Guidance, Navigation, and Control Conference*, Portland.
- [12] Shanshan Li, Gang Tao. (2010). Output Feedback MIMO MRAC Schemes with Sensor Uncertainty Compensation. In *American Control Conference Marriott Waterfront, Baltimore, USA* (pp. 3229-3234).
- [13] Sun J, P. A. Ioannou. (1996). *Robust Adaptive Control*. PTR Prentice-Hall Michigan.
- [14] Song Ge, Gang Tao. (2017). An Adaptive State Feedback Control Scheme with Sensor Failure Compensation. In *11th Asian Control Conference (ASCC) Gold Coast Convention Centre, Australia*.
- [15] Tao G, Chen SH, Tang XD, Joshi SM. (2004). *Adaptive Control of Systems with Actuator Failures*. Springer Berlin.
- [16] Tao G, Jason O. Burkholder. (2009). Adaptive Detection of Sensor Uncertainties and Failures. *AIAA Guidance, Navigation, and Control Conference*, Chicago, Illinois.
- [17] Tao G, Suresh M. Joshi, and Xiaoli Ma. (2001). Adaptive State Feedback and Tracking Control of Systems with Actuator Failures. *IEEE Trans on Automatic Control*, 46(1) (pp. 78-95).
- [18] Vemuri. A.T. (2001). Sensor bias fault diagnosis in a class of nonlinear systems. *IEEE Transactions on Automatic Control*, 46(6) (pp. 949 - 954).
- [19] Zollitsch AW, Florian Holzapfel, and Anuradha M. Annaswamy. (2015). Application of Adaptive Control with Closed-Loop Reference Models to a Model Aircraft with Actuator Dynamics and Input Uncertainty. In *American Control Conference, Chicago, USA* (pp. 3848-3853).