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A Dual-Objective Nonlinear Model for Network Design with NSGA Algorithm

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Abstract

This study concerns the development of a nonlinear programming model capable of solving an adapted version of a single-objective nonlinear problem. The original problem was adapted via the inclusion of an additional constraint and term in the objective function. The resultant aim is twofold: to optimize a three-level supply chain so as to decrease objective costs (such as shortage periods) while simultaneously increasing customer service levels. Demand is random and the inventory control system continuous. Lost sales due to urgent demand are assumed. After evaluating the formulated mathematical model, a meta-heuristic algorithm is developed capable of determining the number of open distribution centers and allocating retailers to these centers. Experiments to evaluate the proposed method's performance are conducted on small to medium-sized problems. Results are compared against those of e-constraint and None Dominated Sorting Genetic Algorithms (NSGA2) (whose parameters are adjusted using the Taguchi method). Final results indicate the superiority of the proposed meta-heuristic in comparison to other, competing approaches.

Keywords: *Nonlinear Programming, Dual-Objective Function, Taguchi Method, Meta-Heuristic Algorithm*

1. Introduction

Today's highly competitive marketplace requires strategic supply chain programming capable of not only satisfying customers, but also decreasing inventory costs [1].

Therefore, to improve supply chain efficiency and management, network design should focus on determining the number and location of distribution centers in addition to the allocation of retailers to such centers.

Other factors such as daily fluctuation in customer demand and the consequent bullwhip effect in the supply chain may be remedied by designing a network which simultaneously decrease costs and increase customer service levels by appropriately managing distribution center shortages [2].

The principal behind our approach is taken from a platelet distribution generation system. The optimal allocation of retailers to distribution centers, and therefore three-level supply chain efficiency, is achieved given demand urgency, product cost and the number and locations of distribution centers. Demand is considered random, given its variable nature, while shortage is considered in the form of lost sales. Several other

classic cost measures and service requirements are also found within the objective function [3].
Since the shortage cost of each inventory unit cannot be obtained by referring to the cost documents and calculation of the shortage cost in the cost objective function, the accuracy of the formulated model can be more emphasized by applying the number of shortages and shortage situations**.** The location of suppliers and retailers is predetermined and suppliers have unlimited capacity. The demand is also random with normal distribution. To gain a more scientifically sound model and decrease the role of estimation, the model is formulated and maximized in a separate objective function as a service level objective. The number of shortages and the shortage situations are considered at a service level, with more attention given to customer satisfaction so as to increase model accuracy.
Another problem characteristic is that allocating demands to farther distribution centers is also possible. According to the importance of increasing service level in addition to decreasing the costs, a dual objective programming model was presented for allocation by factory managers [4].
More practical dual objective inventory models compared to single objective models. Another contrast between single- and dual-objective functions is that there may exist multiple optimal solutions in the latter case.
This means the cardinality of Pareto optimal set is greater than 1 and hence the NSGA2 evolutionary and meta-heuristic method which presented non-dominated optimal solutions as a Pareto set is recommended. The NSGA2 algorithm has the possibility to solve nonlinear multi-objective models by considering both quality and solution order and therefore appears an appropriate solving method for this model [5].
Finally, the proposed model is solved using e-constraint method which is a classic method to solve multi-objective problems and the results of both methods are compared. In fact, the e-constraint method is an exact solution method but since it is impossible to make the model linear, results are trapped in a local solution.
This paper is organized as follows. Section 2 discuss the related work. In Section 3, author shave proposed a method for the development of a nonlinear programming model, Section 4 obtain simulation and comparison results with other papers. In Section 5, conclusion is presented.

2. Literature Review

Syed Abdul Rahman Khan et al. (2018) test the effect of three factors of green supply chain practices on conformational performance in the perspective of Pakistani FMCG organization. A sample of 191 organizations was gathered from the FMCG industry. Three independent variables including green transportation, green distribution and green purchasing evaluate the GSC practices. By using predatory agent analysis and linear multiple regression, the findings show that except green purchasing rest of the two variables (green transportation and green distribution) have been found positive and important to estimate organizational performance. Therefore, the manager’s firms should adopt green practices in their distribution and transportation operations to enhance overall conformation performance [6].
Saeed Fazayeli et al. addresses a location-routing problem on multimodal transportation network. The introduced problem follows four objectives simultaneously which form main contribution of the paper; determining multimodal

routes between supplier and distribution centers, locating mode changing facilities, locating distribution centers, and determining product delivery tours from the distribution centers to retailers. An integer linear programming is presented for the problem, and a genetic algorithm with a new chromosome structure proposed to solve the problem. Proposed chromosome structure consists of two different parts for multimodal transportation and location-routing parts of the model [7].

Shuangyan Li et al. develop an optimization model to integrate facility location and inventory control for a three-level distribution network consisting of a supplier, multiple distribution centers (DCs), and multiple retailers. The integrated model addressed in this study simultaneously determines three types of decisions: (1) facility location (optimal number, location, and size of DCs); (2) allocation (assignment of suppliers to located DCs and retailers to located DCs, and corresponding optimal transport mode choices); and (3) inventory control decisions on order quantities, reorder points, and amount of safety stock at each retailer and opened DC. A mixed-integer programming model is presented, which considers the carbon emission taxes, multiple transport modes, stochastic demand, and replenishment lead time. The goal is to minimize the total cost, which covers the fixed costs of logistics facilities, inventory, transportation, and CO_2 emission tax charges [8].

Amiri designed an integer programming model concerning a single-period, two-level distribution network for supply chain management. The problem includes factory and distribution center locations and these facilities' capacity. Decisions concerning the transport of products from factory to warehouse and, ultimately, warehouse to customer are elements of this problem. Constraints included were those related to single-product and demand uncertainty [9].

A supply chain problem where the number and location of distribution centers are determined and customers exhibit random demand was considered by Maxshen and Kim. A certain amount of safety stock is maintained within each distribution center to satisfy a certain customer service level. An algorithm based on Lagrange Relaxation was employed for this problem where facility capacity is considered constant [10].

Tsu demonstrated how traditional multi-objective optimization techniques attempt to solve these problems through ranking objectives and optimizing an objective by considering others as constraints often results in no satisfactory answers. Furthermore, in cases where the objective space in non-convex, the authors found the Pareto optimal solutions, therefore indicating how it is better to use evolutionary algorithms to solve such problems [11].

Maiti developed an NSGA2 algorithm capable of solving the multi-objective multi-level inventory problem, with the results evaluated using classic methods such as ideal programming [12].

Sajadi and Poor stated that by removing the single-product and the single transportation state constraints imposed by Amiri a multi-product, two-level model with the possibility of multi-mode transportations which selects the best transportation method was possible. The problem was formulated using mixed integer programming and solved with a heuristic algorithm based on Lagrange Relaxation [13].

Cardonain 2014 evaluated a two-level product distribution network including multi-objective factories of distribution centers and a set of potential warehouses. In this paper, the uncertainty of demand The uncertainty concerning warehouse demand and decision-making in relation to allocation of transportation mode is investigated during this paper. Network design is performed while satisfying two decision-maker objectives: service quality and two-level supply network affordability. According to previously published studies, the single-objective model of decreased cost has been evaluated with the conclusion being the necessity of adding a service level objective in addition to that of decreased costs. Table 1 shows important literature review [14].

Razvani attempt to use improve Stud GA to find optimal weights for multi-layer Perceptron neural network. Stud GA is improved from genetic algorithms that perform information sharing in a particular way. In this study, chaotic system will be used to improve Stud GA. The comparison of proposed method with Imperialist Competitive Algorithm, Quad Countries Algorithm, Stud GA, Cuckoo Optimization Algorithm and Chaotic Cuckoo Optimization Algorithm on tested data set (Wine, Abalone, Iris, WDBC, PIMA and Glass) with determined parameters, as mentioned the proposed method has a better performance [15].

Falaht pisheh intend to find the location of the underwater sensor nodes by introducing a new method based on the Cuckoo Optimization Algorithm (COA). She will compare the proposed method with the related methods in terms of the localization error rate and the number of nodes discovered. The results of the comparisons show that the proposed method can greatly reduce the error rate of the localization of the sensor nodes. [16]

Nodehi proposes election of cluster heads in the Ad-hoc networks. Although different methods have been successfully proposed by researchers to tackle this problem, nearly all of them have the deficiency of providing a single combination of head clusters as the solution. On the contrary, in the proposed method, using a Multi-Objective Genetic Algorithm, a set of near optimum solutions is provided. In this method, energy consumption, numbers of cluster heads, coverage and degree difference are considered as objectives. Numerical results reveal that the proposed algorithm can find better solutions when compared to conventional methods in this area namely, weighted clustering algorithm (WCA), comprehensive learning particle swarm optimization (CLPSO) and multi objective particle swarm optimization(MOPSO) [17].

Table 1. literature review

Subject	Ref	Problem	Contribution
The impact of green supply chain practices in business performance: Evidence from Pakistani FMCG firms	[6]	Optimization of green transportation, green distribution and green purchasing	using approbatory agent analysis and linear multiple regression
A model for distribution centers location-routing problem on a multimodal transportation network with a meta-heuristic solving approach	[7]	location-routing problem on multimodal transportation network	An integer linear programming and a genetic algorithm with a new chromosome
Joint Optimization of Distribution Network Design and Two-Echelon Inventory Control with Stochastic Demand and CO_2 Emission Tax Charges, PLOS ONE	[8]	integrate facility location and inventory control for a three-level distribution network consisting of a supplier, multiple distribution centers	A mixed-integer programming model is to minimize the total cost, which covers the fixed costs
Designing a distribution network in a supply chain system: Formulation and efficient solution procedure	[9]	transport of products from factory to warehouse and, ultimately, warehouse to customer are elements	integer programming model concerning a single-period, two-level distribution network

Incorporating inventory and routing costs in strategic location models,	[10]	number and location of distribution centers are determined and customers exhibit random	An algorithm based on Lagrange Relaxation was employed
Evolutionary pareto optimizers for continuous review stochastic inventory systems	[11]	traditional multi-objective optimization techniques attempt to solve these problems through ranking objectives and optimizing an objective	Pareto optimal solutions, therefore indicating how it is better to use evolutionary algorithms
Utilization of multi-objective genetic algorithm forone-item multi-level inventory distribution system	[12]	the multi-objective multi-level inventory problem	developed an NSGA2 algorithm using classic methods such as ideal programming.
Two-echelon,multicommodity supply chain network design with mode selection, lead-times and inventory costs	[13]	by removing the single-product and the single transportation state constraints imposed by Amiri (2006) a multi-product, two-level model with the possibility of multi-mode transportations	using mixed integer programming and solved with a heuristic algorithm based on Lagrange Relaxation.
Metaheuristic procedure for a bi-objective supply chain design problem with uncertainty	[14]	the uncertainty ofdemand The uncertainty concerning warehouse demand and decision-making in relation to allocation of transportation mode is investigated during this paper.	a two-levelproductdistribution network including multi-objective factories of distribution centers and a set of potential warehouses
Multi-layer Perceptron Neural Network Training Based on Improved of Stud GA	[15]	Finding optimal weights for multi-layer Perceptron neural network. Stud GA	Chaotic system will be used to improve Stud GA.Stud GA is improved from genetic algorithms that perform information sharing in a particular way.
Localization of Underwater Wireless Sensor Network Nodes Using Cuckoo Optimization Algorithm	[16]	Finding the location of the nodes	Introducing a new method based on the Cuckoo Optimization Algorithm (COA)
Determining Cluster-Heads in Mobile Ad-Hoc Networks Using Multi-Objective Evolutionary based Algorithm	[17]	Finding the location of the cluster heads	*Using a Multi-Objective Genetic Algorithm. energy consumption, number of cluster heads, coverage and degree difference are considered as objectives*

3. Mathematical model

The presented problem, as detailed, concerns an examination of a three-level supply chain which includes multiple suppliers, distribution centers and retailers. Transportation is performed indirectly among the components, with the optimal determination of the number of distribution centers and their allocation with regard to retailers being addressed.

3.1. Assumptions

The assumptions of the proposed model are as following:

- There is a factory whose role is that of supplier.
- Supplier capacity and the number of distribution centers are unlimited (if capacity is unlimited, the factory can be also multi-product).
- Demand is random in retailers with normal distribution.
- Demand in any distribution center is a function of the demand of the retailers allocated to it.
- Retailers are only allocated to active distribution centers.
- Single sourcing implies all demand from each retailer should be satisfied by a single distribution center.
- Inventory is not held by the retailers due to the urgency of demand.
- Distribution centers with an inventory model (Q, r) encountered the service type1.
- Shortage is considered in the form of lost sales.
- Reallocating retailers to farther distribution centers is possible.
- Supplier and retailer locations are predetermined and suppliers have unlimited capacity.
- The amount of orders assigned to each distribution center equals the total demand of those retailers allocated to it.

Since customers demand is imported to the each retailer with the normal distribution and on a daily basis with a daily average of $\mu_i$andvariance of ${\sigma_i}^2$and also considering that the inventory is not maintained by retailers safety stock in retailers is zero. The amount of orders assigned to each distribution center (DC) is equal to the demand of those retailers allocated to that center. Urgency and random demand requires inventory to be monitored continuously using FOS**.** When inventory levels drop below a predefined order point (r), q replacement products are ordered.

It should be noted that while the quantity of goods ordered remains constant, the time interval between such orders is variable. Assuming the shortage with the certain probability (service type 1) in the inventory model (Q,r), the possibility of encountering a shortage is a constant value and since the demand function is normally distributed with certain mean and variance, r can be calculated from the reversed cumulative distribution function.

The manufacturer (factory) also delivers the product to distribution centers. It has an average of $\sum \mu_i$ and variance of $\sum {\sigma_i}^2$ at the delivery time (fixed period). DCs consider urgency before estimating the demand for each of their allocated retailers.

As shown in the above model, retailersare potentially allocated to the farther distribution centers because of the highly competitive market, shortages are considered in the form of lost sales. Also, considering the inventory model (Q, r) andservicetype1 for distribution centers, the probability of shortages is fixed. The study network is illustrated in Figure 1.

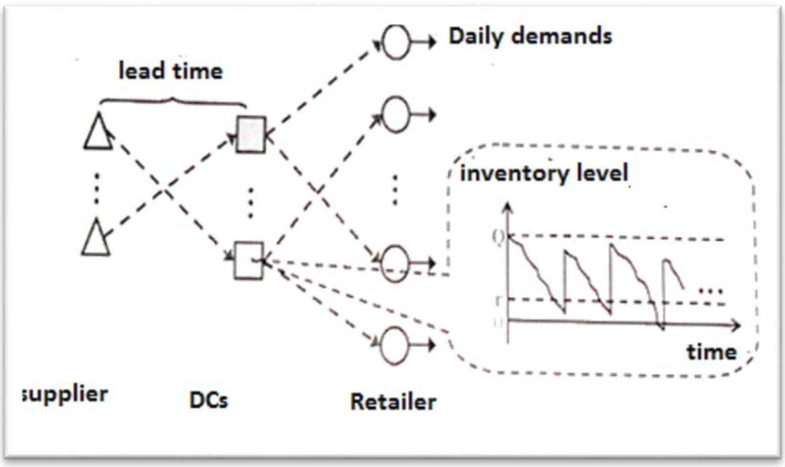

Figure1. Three-level supply chain network with indirect transportation and continuous review system [3]

3.2. Decision variables and model parameters

For the mathematical formulation, the following parameters and decision variables are introduced

I: Set of retailers

J: Set of potential locations for the establishment of a distribution center

f_j: Fixed cost for establishment of each distribution center j

d_{ij}: Transportation cost of each product unit between retailer and potential distribution center j

X_j:1 if *j* is selected as the distribution center, 0 otherwise.

Y_{ij}:1 if retailer *i* is assigned to distribution center *j*, 0 otherwise

r_j: Re-order point for the distribution center j is achieved according to service type 1 and shortage occurrence with a certain probability.

β:Weightof the transportation cost

x: Number of working days

θ: Weight of warehousing costs

h: holding cost of each product unit per year

α:Shortage probability.

L: Time delay for product delivery from supplier to distribution center.

F_j:Fixed cost of ordering in each distribution center

a_j:Variable cost of transportation

g_j: Fixed cost of transportation

γ: Maximum amount of demand that each distribution center can supply

μ_i: Mean or expectation of demand in each retailer

σ_i: Standard deviation of demand in each retailer

3.3. Modeling

According to the assumptions and mentioned parameters, the mathematical model of this study defined as follows

Minimize

$\sum_{j\in J} f_j X_j + \left[\beta \sum_{j\epsilon J} \sum_{i\epsilon I} x d_{ij} \mu_i Y_{ij}\right] +$

$$\left[\sum_{j\epsilon J}\sqrt{2\theta h\left(F_j+\beta g_j\right)\sum_{i\epsilon I} x d_{ij}\mu_i Y_{ij}}+\beta\sum_{j\epsilon J} a_j\sum_{i\epsilon I} x\,\mu_i Y_{ij}\right]$$
$$+\theta h z_\alpha\sum_{j\epsilon J}\sqrt{\sum_{i\epsilon I} L\,\sigma_i^2 Y_{ij}} \qquad (1)$$

Minimize

$\sum_j \frac{D_j}{Q_j}\, p(D_{lj}>r_j)X_j$(2)

S. t

$$\sum_i \frac{D_j}{Q_j}\left[\frac{\sigma_{ij}}{\sqrt{2\pi}} e^{-\left(\frac{r_j-\mu_{ij}}{\sigma_{ij}}\right)^2 \Big/ 2}+\left(\mu_{ij}-r_j\right)p\left(D_{lj}>r_j\right)\right] Y_{ij} \le \gamma\sum_i x\mu_{ij}Y_{\,ij}\ \forall j\in j \qquad (3)$$

$\sum_{j\in J} Y_{ij}=1\,.\forall\, i\in I$ (4)

$Y_{ij}\le X_j,\forall i\in I\,.\forall j\in J$ (5)

$X_j\in[0.1]$(6)

$\forall j\in J$ (7)

$Y_{ij}\in[0.1]\forall i\in I\,.\forall j\in J$ (8)

In the above model, by adding the equations 7 and 8 as well as decision variable r_j to the Deb 's model. It has developed sophisticated penalty functions specific to the problem at hand and the search algorithm used for optimization and this model is used to solve a real problem in this paper.

In equation 1, the total system cost is determined by summing the following:

- Fixed costs related to establishing and adjusting DC
- Cost of product delivery in the area
- Total inventory costs, including
  - fixed cost of order in distribution center
  - Product transportation costs from supplier to distribution centers
  - holding cost of product inventory in distribution center
  - Cost of safety stock inventory

Equation 2 maximizes service level by reducing the number of expected annual shortage periods in distribution centers. Constraint 3 ensures that there are lost sales during shortage periods. Consequently, the average number of annual shortages cannot be greater than the maximum supply demand of each distribution center.

Equations 4 states that each retailer is assigned to exactly one DC.

and Equation 5 states that as long as the distribution center j is not established, retailer cannot be allocated to it. Equations 6 and 7 present the binary decision variables.

3.4. Model validation via e-constraint

It is a valid model that, despite the inaccuracy, provides a significant prediction of the system. For this purpose, e-constraint method is used to evaluate system performance by transforming the objective function to the constraint. Steps to perform this method for two-objective programming are as follows:

- One of the objective functions is selected as the main
- the solution and optimal values and each single objective is achieved for the selected objective function
- Interval between two optimal values and subsidiary single objective function divided into the number of pre-specified intervals and an efficiency table is created
- Each time the problem is solved with main objective function and specific epsilon value

3.5. Validation of the model

Jafarnejad's numerical example is employed to enable solving the model via e-constraint. In this example, 5distribution centers and15retailers are considered with values of the prepared parameters detailed in the Appendix. To implement the mentioned method, GAMS22.8 is used on a laptop fitted with, Intel core i5, 6GB, 3.20GHz, with BARON solver and the results detailed in the form of a Pareto set as per Figure 2.

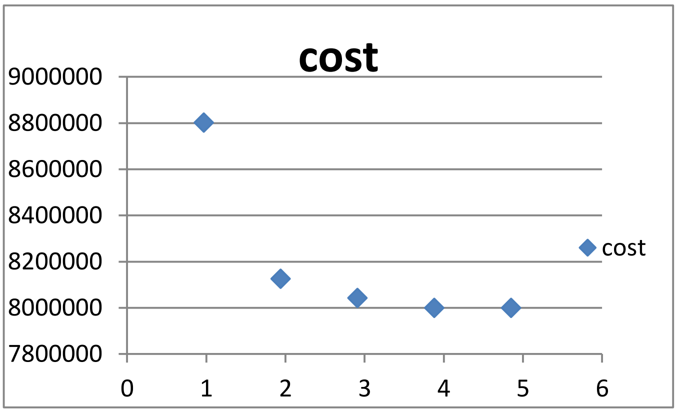

Figure2.Paretoset diagram of e-constraint method

Table 2. Efficiency table of e-constraint method

epsilon	0.97	1.94	2.91	3.88	4.85
cost	8801926.96	8126288.42	8042966.57	7999328.12	79993024.12

Results confirm this method's accuracy. as expected, by distancing the number of shortage periods from the optimal value, the cost function reached to the optimal value, therefore validating the model's performance.

After examining the model's validity, NSGA2 and e-constraint approaches are evaluated to select the most efficient and therefore appropriate algorithm.

4. Proposed meta-heuristic-method

As proven by Jafarnezhad**,** the present problem is NP-hard and there is no possibility of making it entirely linear. In other words, the number of non-linear phrases can be minimized. Thus, the feasible space is non-convex and detailed solving methods are incapable of presenting accurate, optimal solutions.

Therefore, approximate algorithms are proposed. Given that heuristic methods are generally unable to exit local optima, this paper attempts to employ meta-heuristics. Given that the model is both non-linear and dual-objective, it is tested using the NSGA2 algorithm, which is itself tested against e-constraint to access its efficiency. Because the problem is NP hard and exact methods are incapable of solving the problem on a large scale, to demonstrate the efficiency of the proposed algorithm five medium and small problems were generated to enable effective evaluation

Due to the contradiction between the two objective functions of cost reduction and increasing service level, multiple optimal solutions are possible for the mentioned model, meaning that the cardinality of the Pareto optimal set is more than one.

For this reason, among the optimal solutions of the Pareto set, the best solution, considering the criteria of order and quality is necessary. In fact, techniques such as multi-objective optimization, which by often ranking goals or optimizing one goal and considering other goals as constraints, often do not lead to satisfactory answers and cannot be optimized in non-convex spaces. As a result, we adopt the proposed NSGA-II approach to solve this model. Moreover, it does not have the weaknesses of the traditional techniques described above, but It is possible to solve multi-objective nonlinear models considering both the quality and order of the solution and it seems to be a suitable solution for this model

4.1. NSGA2algorithm

NSGA2 begins by producing an initial random population P1 of size Np. Next, the objective functions related to the evaluation model and fitness function are obtained.

. The population is subsequently front organized via non-dominated sorting. To produce children Q_1, first the number of P_1 selection operators, intersection and mutation is applied on the parent population. The objective functions of the produced children are evaluated and their fitness obtained, before an elitism operation is performed with a combination of parents' population P_1 and childrenQ_1, thereby creating population Rt. R_t. The algorithm front organizesR_t in several F_{i} fronts considering the non-dominated sorting. To complete elitism N, the population is selected from the integrated and front organized population. NSGA2 begins from best front F1 to select N population based on non-dominated sorting. If the stop condition is satisfied the final report is presented, if not it returns to the beginning of the child-production cycle and the steps continue until the condition is met. The final population obtained contains 6 rows and 15 columns, with the first five rows showing independent variables and the final the dependents. To generate children, two chromosomes of the parent are chosen via binary selection. One-point method is considered as a cross over operator and the mutation is applied by mutation rate and step mutation. The number of iterations is taken as stopping criterion.

4.2. Determining the parameters

The algorithm's parameters should be properly adjusted so as to engender good quality meta-heuristic solutions. Since evaluating all tests is difficult by increasing the number of algorithm parameters using test designs, some tests were performed to determine independent factors and the best process output level.

4.2.1. Taguchi method

One of the most common statistical methods employed to analyze process output sensitivity is the Taguchi method. This method is used when by performing a part of total required tests to determine the optimal levels of independent factors, the best level for the process output is determined.

Algorithm parameters, assessed in Figure 2, are divided into three levels: low, medium and high. Table 3 shows factors by level.

Table 3: Evaluation of algorithm factors by level.

symbol	parameters	Factor level		
		low	medium	High
maxit	Maximum iteration	50	100	150
npop	Population size	50	100	150
pcrossover	Crossover properties	0.05	0.0.7	0.09
sigma	Algorithm parameters	0.01	0.02	0.05
µ	Algorithm parameters	0.02	0.05	0.10

By performing the Taguchi method using MINITAB software, the diagrams of Figures 4 and 5 were constructed.

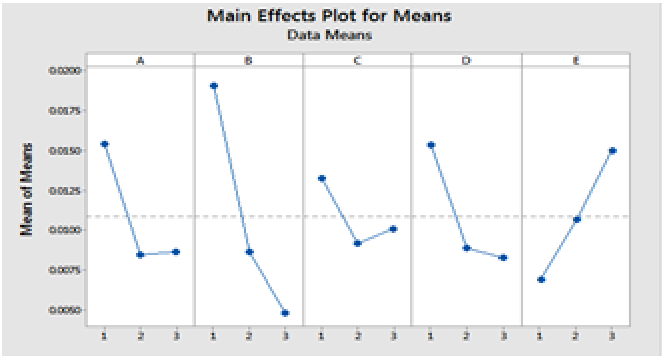

Figure 3. Effects of average

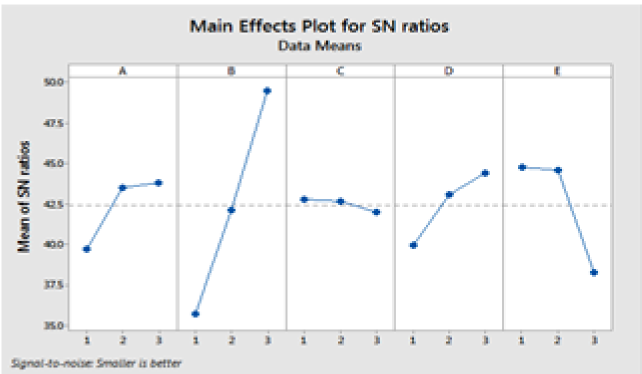

Figure4. Main effectsof signal to noise ratio

Results of both diagrams show that in the best state, each point of the diagram should be positioned in which level. The result of NSGA2 parameters adjustment is as following in table 4.

Table 4. Factors and optimal levels obtained fromthe Taguchi method

symbol	Optimal level
pop	100
maxit	150
pCrossover	0.07
sigma	0.05
µ	0.02

4.3.Computational results

Five small and medium-sized problems are designed to examine the efficiency of the proposed algorithm which employs e-constraint and NSGA2.
The NSGA2 and e-constraint algorithm steps in the designed problems were solved according to the proposed model on a laptop equipped with, Intel core i5, 3.20GHz, and 6GB of memory. Results of the generated problems can be seen in Table 5.

Table 5. Small and medium-sized test results.

		e-constraint			NSGA2			RG	
row	Prob size	Obj1	Obj2	Run time(s)	Obj1	Obj2	Run time(s)	Obj1	Obj2
1	3*4	1.42	1.94	43	1.24	1.94	11	11.94%	0%
2	3*10	4.59	1.94	119	4.72	1.94	16	-2.8%	0%
3	4*12	6.18	2.91	726	5,76	2.91	17	6.8%	0%
4	4*14	5.27	2.91	1124	6.97	2.91	19	6.74%	0%
5	5*15	8.42	2.91	1832	7,59	2.91	22	9.86%	0%
Average		5.61	2.52	768	5,25	2.52	17	6.51%	0%

The problem characteristics are detailed in the first and second columns of Table 5. In the second column, the first number is the number of distribution centers and the second number is the quantity of retailers. In the next three columns, the best time solutions which have obtained for Pareto set, resulted by e-constraint method are shown. The values of NSGA2 method is shown in the next columns and the error caused by NSGA2 method, calculated by the following equation.
To properly evaluate the error level obtained during the best results of each algorithm are compared against each other.
The equation for calculating the error obtained by the proposed approach is detailed below.
$RG = \frac{BR-RA}{BR}$(8)
Where:
RG: (Error of results of proposed algorithm efficiency)
BR: Best results
RA: Result of each algorithm
The results indicate that the error function in the second objective function is 0 and under 6.6% for the first. The time required to solve the problem using e-constraint increases exponentially. By contrast, the computational time required by NSGA2 grows with a gentle slope meaning the average calculation time is, ultimately, negligible when compared against the e-constraint method. Therefore, the NSGA2 method proves the most efficient for solving the present problem.
To accurately evaluate the proposed model's improvement compared to jafarnezhad's study, the presented numerical example including 15 retailers and 5 distribution centers was investigated.
Using R2014aMATLAB software, the two-objective model of the current study is solved with meta-heuristics and the results compared against those when solving the basic model (Excel solver). To ensure the results of the basic solver, the authors re-solved it and achieved the same results. The results concerning the developed model are given in Figure 5 and table 6.

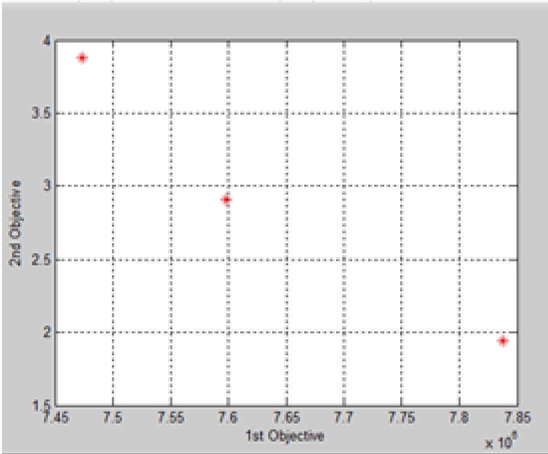

Figure 5. Numerical example are detailed in the using theNSGA2 algorithm

Table 6. Numerical example values using theNSGA2 algorithm

Number of shortage periods	Number of established DCs	established DCs	Cost
1.94	2	1,3	7837528.265
2.91	3	1,3,4	7598281.541
3.88	4	1,3,4,5	7473188.613

the non-dominated solutions of NSGA2 meta-heuristic method are detailed in the Pareto optimal diagram of Figure 5. Solutions consists of 2, 3 and 4 distribution centers should be established in the potential locations. Distribution centers which are acceptable to be established in the Dual objective model with NSGA2 approach, in order of priority, are a combination of $X_5.X_4.X_3.X_1$ and $X_2$was also not proper for establishment as the results of single-objective model in a Jafarnezhad study. According to Pareto optimal diagram, it is also observed that an increase in cost coincides with a decrease in shortage periods. Since the number of shortage periods is proportional to the number of open distribution centers, fewer established distribution centers occur when costs are increased **.**All points are efficient points in the obtained Pareto set, selecting the optimal one is dependent upon the importance coefficient in the objective functions .

5. Conclusions and recommendations for further research

In this study, the problem which is obtained by adding a minimization objective function and a lost sale constraint, is still NP-hard, because the basic problem is NP-hard and certainly the addition of objective function and constraint not only doesn't reduce the complexity of the desired problem, but also makes it more complicated. On the other hand, because of non-linear, two-objective and discrete nature of model, meta-heuristic method such as NSGA2algorithm is suitable to solve the model.

The mathematical model of the problem was presented in accordance with the assumptions and graphical model representation. Jafarnejad (2008)'s study, which includes 15 retailers and 5 distribution centers and is solved using the constraint Epsilon solution method, was employed when validating the introduced model. Results indicate are as expected and indicate how the number of shortage periods decreased coincided with cost increases. The mathematical model was solved by encoding the desired problem in MATLAB. Next, using the Taguchi method, five effectiveness factors are adjusted with respect to three levels. The final algorithm's results, the best Pareto solutions, are then presented for a problem containing 15 retailers and 5 distribution centers.

The results of 5 small and medium-sized problems indicate that the error percentage resulting from meta-heuristics is lower than 6.6% and that the average solution time is far less than those achieved using the Epsilon method. NSGA2 is, therefore, considered the most efficient and appropriate solution method for such problems.

In order to conduct the future research, according to two main weaknesses of e-constraint method, the following cases are suggested.

1. The objective function's amplitude resulting from the efficiency table was not located in the efficient Pareto set due to the multiple optimal solution in objective functions, since the model is dual-objective, there exists the possibility for multiple solutions.

2. Inefficiency of obtained solutions which are known as weak efficient solutions

In order to, manage the weak points in e-constraint method, Lexicography method is suggested develop the model, other objective functions may be defined for the proposed model. An example of which may include considering supplier and distribution center backorders by introducing capacity constraints, limited supplier capacity, or considering other parameters such as order delivery duration or fuzzy demand.

Other meta-heuristic approaches such as MOGA, VEGA and MOPSO may also be employed and the best approach presented in comparison to those obtained during the present study. To develop the model, other objective functions may be defined for the proposed model. An example of which may include considering supplier and distribution center backorders by introducing capacity constraints, limited supplier capacity, or considering other parameters such as order delivery duration or fuzzy demand.

Other meta-heuristic approaches such as MOGA, VEGA and MOPSO may also be employed and the best approach presented in comparison to those obtained during the present study.

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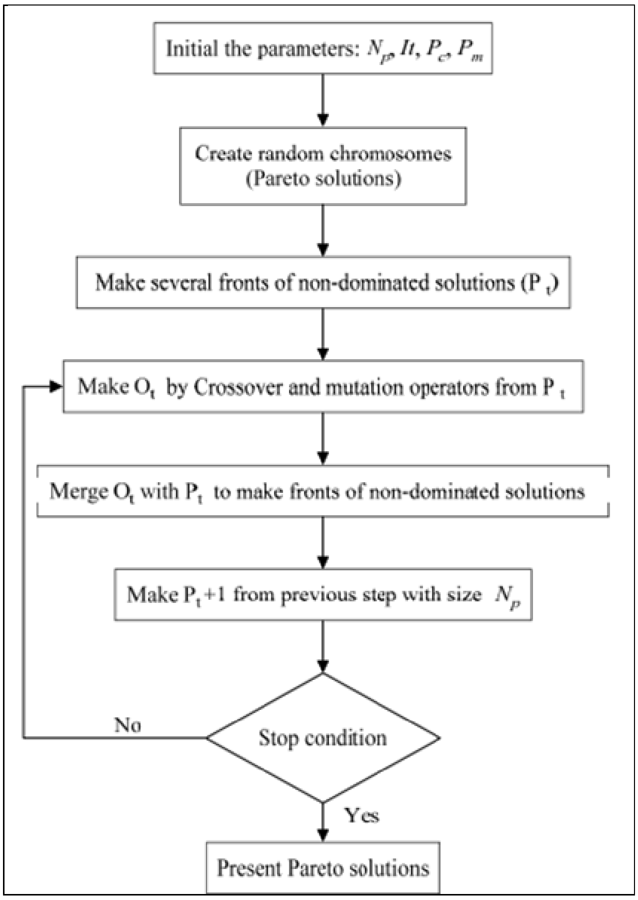
Appendixes

β = ۱
χ = ۲۵۰
θ = ۰٫۰۲
h − ۲
l = ۳
α = ۰٫۹۷

	f_j	F_j	a_j	g_j
1	18000	5000	1	60
2	20000	3000	0.7	50
3	26000	5000	0.8	65
4	25000	2800	0.4	49
5	26000	1900	0.5	56

d_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	18	18	18	9	12	17	13	17	12	10	12	14	18	21	17
2	12	20	20	18	15	15	15	18	14	19	18	18	19	15	18
3	10	12	21	17	13	20	15	20	15	18	19	17	15	18	15
4	10	15	19	15	14	19	14	15	16	17	17	22	17	14	19
5	16	16	17	16	16	15	18	16	21	18	20	20	18	19	17
μ_i	100	120	130	125	134	145	120	130	145	175	180	140	140	160	150
σ^2_i	5	3	2	5	4	6	2	3	2	1	6	5	4	2	3

Appendix A. Numerical example parameters



Appendix B. Flowchart of NSGA2 algorithm

		Retailers(i)																
	y_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	X_j	The number of assigned retailers
DC(j)	1				1	1		1		1	1	1	1				1	7
	2																	
	3	1	1											1		1	1	4
	4								1						1		1	2
	5			1			1											2
	cost																	7473188

		Retailers(i)																
	y_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	X_j	The number of assigned retailers
DC(j)	1			1	1	1	1			1	1	1	1				1	8
	2																	
	3		1											1		1	1	3
	4	1						1	1						1		1	4
	5																	
	cost																	7598281

		Retailers(i)																
	y_{ij}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	X_j	The number of assigned retailers
DC(j)	1			1	1	1	1		1	1	1	1	1	1			1	10
	2																	
	3	1	1					1							1	1	1	5
	4																	
	5																	
	cost																	7837528

Appendix C. Pareto set resulted from the example 15*5 solution using NSGA2 method