Research Article

Exact Equations for The Reliability and Mean Time to Failure Of 1-Out-Of-*N* Cold-Standby System with Imperfect Switching

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Abstract

Standby redundancy is a common and fundamental technique for increasing the reliability and availability of various systems. Cold-standby state is one of the most important strategies that are well used in non-repairable systems and plays an important role in mission-critical systems reliability, such as space exploration and satellite systems. In this paper, closed-form equations are derived using the Markov method to calculate the reliability function and the mean time to failure of a 1-out-of-*n* cold-standby system with non-repairable components under imperfect switching. While it is assumed that the failures of the switch and its associated active components are independent of each other, a constant failure rate is considered for the components and an increasing constant failure rate for the switch as it is used more frequently. In the end, numerical examples are solved for a system with various numbers of components to demonstrate the application of the closed-form equations.

Keywords: Standby redundancy; Cold-standby redundancy; Imperfect switching; Markov method.

1. Introduction

Redundancy is a common and fundamental technique to improve systems reliability and availability. It has been widely used in many industries including railroads, aerospace, automotive, nuclear, and medical to ensure having more reliable systems with higher safety (Dwyer et al., 2012).

Standby redundancy is typically divided into three categories of hot, warm, and cold-standby, among which the cold-standby redundancy due to its wider applications, its simplicity, as well as providing higher reliability and lifetime compared to the other two policies (hot and warm standby) is more applicable. For instance, Sharifi & Yaghoubizadeh (2015), Sharifi et al. (2016), and Kim & Kim (2017) investigated the redundancy allocation problems (RAP), and Ardakan & Rezvan (2018) worked on the use of the cold-standby redundancy strategy.

In the cold-standby state, the redundant components are not under the stress and pressure and their failure rates are very low (often it is assumed that the failure rate of a cold-standby component is zero). Some applications of the cold-standby redundancy can be seen in systems such as carbon recovery used in fertilizer plants, textile manufacturing, satellite, explosion space, electronics, aerospace, and many others, especially electric power systems (Wang et al., 2012; Xing et al., 2012; Amari, 2012 and Jia et al., 2016).

As the *k*-out-of-*n* cold-standby redundant systems are of particular importance, they have been investigated in detail by various authors in terms of reliability and availability. Using the Markov approach, Wang & Loman (2002) studied the availability and reliability of a *k*-out-of-*n* active system with *m*-unit in cold-standby mode. Wang et al. (2012) applied the central limit theorem and investigated the reliability of a *k*-out-of-*n* cold-standby redundant system. Employing the convolution integrals method, Amari (2012) obtained the reliability of the *k*-out-of-*n* standby redundant system in the cold state. Yaghoubi et al. (2019) presented a precise formula to calculate the steady-state availability of a *k*-out-of-*n* cold-standby redundant system.

One of the special and most prevalent cases of the *k*-out-of-*n* cold-standby redundant systems is called the 1-out-of-*n* system that has been extensively discussed in the literature. Coit (2001) obtained a lower bound for the reliability of a 1-out-of-*n* cold-standby subsystem under imperfect switching. Wang et al. (2012) investigated 1-out-of-*n* cold-standby systems using the central limit theorem assuming perfect switching. Zhai et al. (2016) studied the reliability of the 1-out-of-*n* cold-standby systems with imperfect switching using the multi-valued decision diagrams (MDD) method. Moreover, the 1-out-of-*n* cold-standby redundant systems were discussed in optimization problems by many authors. Interested readers are referred to Levitin et al. (2013), Ardakan & Hamadani (2014), and Levitin et al. (2014) for more information.

Typically, the analytical solution for the 1-out-of-*n* cold-standby systems is the integration method. In this method, having the probability density function (PDF) pertaining to the failure times of the components the system reliability can be obtained using the convolution of integrals (Lenz et al., 2011). However, when the failure time of all components of a system follows the

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exponential distribution, another analytical method called Markov can be used.

One of the most important tasks in systems reliability analysis is to obtain an explicit and general formula to determine the reliability of a particular system. There is a common example known as the 1-out-of-n cold-standby redundant system. If the time-to-failure of the system components follows an exponential distribution, a general closed-form is considered for system reliability, when the switch is perfect. This relationship is presented in many textbooks such as Billinton & Allan (1992), Dhillon (2004). However, to the best of the authors' knowledge, there is not such an explicit formula in the literature when the switch is imperfect. Therefore, in this paper, the reliability of the cold-standby redundant systems with the exponentially-distributed time-to-failure of the components is analyzed assuming an imperfect switch.

The structure of the remainder of the paper is as follows. The problem under investigation alongside its assumptions is stated in Section 2. The reliability and mean time to failure of the cold-standby system under an imperfect switching mechanism is obtained in Section 3. An illustration is provided in Section 4. Finally, conclusions are made in Section 5.

2. The Problem and the Assumptions

Figure 1 shows a switching system with an active component connected by the switch and (n-1) cold-standby redundant components. The switch in its mission time is assumed imperfect. Moreover, it is assumed that the failure rate of the switch increases when it is used more frequently.
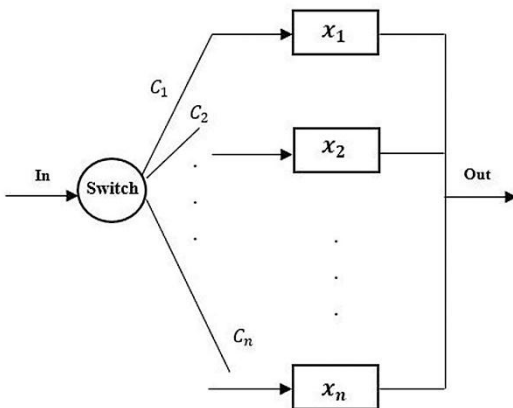

Fig. 1. Configuration of a 1-out-of-n cold-standby redundant system with imperfect switching

The system starts operating with an active component alongside (n-1) cold-standby redundant components in a specific order, e.g., $(x_2, x_3, …, x_n)$ and a switch. The system is initially starting in the active mode with one main component connected by the switch, while the rest of the components are in cold-standby mode. If the active component cannot complete its mission successfully, the

first redundant component becomes active, when it is immediately placed by the switch. If the new active component is not able to complete its mission, the switch instantaneously changes its position with a new active component. This process continues until there is no redundant standby component to be activated. Note that if the switch fails at any time, the system operates as long as the active component does not fail. This means that the system fails to operate at a time when both the switch and the active component fail to operate. It is assumed in this paper that the switch can fail at any time. That is, the failure of the switch is a continuous function of time (case (i), in Coit, 2001). Besides, it is assumed that the switch failure rate increases at each switching step. In other words, a higher failure rate of the components causes the switch to work more frequently and to fail at a higher rate. This means that the switch failure rate is increasing at each step of switching. Figure 2 graphically depicts this assumption.
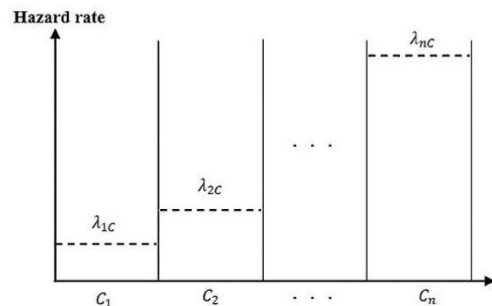

Fig. 2. The failure rate of the switch in each stage of switching

The goal is to find the reliability and the mean time to failure of this system under the following assumptions.

2.1. Assumptions
The assumptions involved are as follows:
- The system is initially operational.
- The components and the switch are binary, that is, they are either working or in a failure condition.
- All the components and the switch are non-repairable and independent of each other.
- The failure rate of all the components in the cold-standby mode is assumed zero.
- The failure behaviors of all the components in the active state and also the switch follows an exponential distribution.

3. Reliability of a 1-out-of-n Cold-Standby Redundant System Under Imperfect Switching

An analytical method to analyze the reliability of a complex system (e.g. standby systems) is based on the Markov method. When the time-to-failure of a component or a system follows an exponential distribution, the Markovian method is used as a prominent and popular method for the reliability analysis (Billinton & Allan, 1992; Dhillon, 2004).

The proposed system has $2n$ states with the state transition diagram shown in Figure 3, where the situation (x_i, x_{iC}) expresses that the ith component and its associated switch are operational and $(x_i, \bar{x}_{iC})$ represents the operational state of the ith component and the failing state of the switch when it replaces the ith component to operate as the active unit.
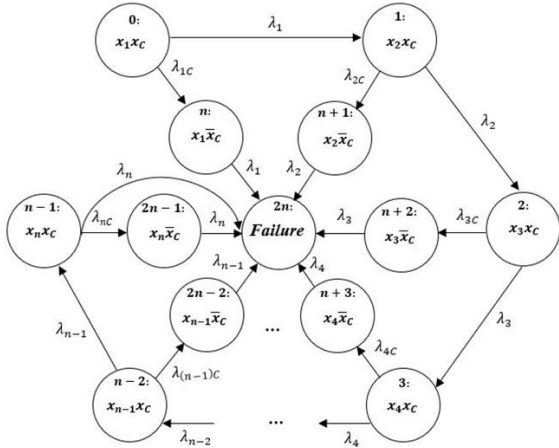

Fig. 3. State transition diagram of Figure 1

$$P_0(s) = \frac{1}{(s + \lambda_1 + \lambda_{1C})} \tag{5}$$

$$P_i(s) = \left(\frac{1}{\lambda_{i+1}}\right) \prod_{j=1}^{i+1} \left(\frac{\lambda_j}{s + \lambda_j + \lambda_{jC}}\right); \ i = 1, 2, \ldots, n-1 \tag{6}$$

$$P_n(s) = \frac{\lambda_{1C}}{(s + \lambda_1)(s + \lambda_1 + \lambda_{1C})} \tag{7}$$

$$P_{n+i}(s) = \left(\frac{\lambda_{(i+1)C}}{s + \lambda_{i+1}}\right)\left(\frac{1}{\lambda_{i+1}}\right) \prod_{j=1}^{i+1} \left(\frac{\lambda_j}{s + \lambda_j + \lambda_{jC}}\right); \ i = 1, 2, \ldots, n-1 \tag{8}$$

In Equations (5)-(8), s and $P_i(s)$ are the Laplace transformation variable and the Laplace transform of P_i, respectively.

$$P_0(t) = e^{-(\lambda_1 + \lambda_{1C})t} \tag{9}$$

$$P_i(t) = \sum_{l=1}^{i+1} \left(\frac{\prod_{j=1}^{i} \lambda_j}{\prod_{j=1, j \neq l}^{i+1}\left((\lambda_j + \lambda_{jC}) - (\lambda_l + \lambda_{lC})\right)}\right) e^{-(\lambda_l + \lambda_{lC})t}; \ i = 1, 2, \ldots, n-1 \tag{10}$$

$$P_n(t) = e^{-\lambda_1 t} - e^{-(\lambda_1 + \lambda_{1C})t} \tag{11}$$

$$P_{n+i}(t) = \sum_{l=1}^{i+1} \left(\frac{\lambda_{(i+1)C}\left(\prod_{j=1}^{i} \lambda_j\right)}{\left(\lambda_{i+1} - (\lambda_l + \lambda_{lC})\right)\left(\prod_{j=1, j \neq l}^{i+1}\left((\lambda_j + \lambda_{jC}) - (\lambda_l + \lambda_{lC})\right)\right)}\right) e^{-(\lambda_l + \lambda_{lC})t}$$
$$+ \prod_{j=1}^{i} \left(\frac{\lambda_j}{\lambda_j + \lambda_{jC} - \lambda_{i+1}}\right) e^{-\lambda_{i+1}t}; \ i = 1, 2, \ldots, n-1 \tag{12}$$

The general differential equations for this system (Figure 3) are derived as follows:

$$P_i(t + \Delta t) = P_i(t)\left(1 - (\lambda_{i+1} + \lambda_{(i+1)C})\Delta t\right) + P_{i-1}(t)\lambda_i \Delta t; \ i = 0, 1, \ldots, n-1 \tag{1}$$

$$P_{n+i}(t + \Delta t) = P_{n+i}(t)(1 - \lambda_{i+1}\Delta t) + P_i(t)\lambda_{(i+1)C}\Delta t; \ i = 0, 1, \ldots, n-1 \tag{2}$$

where λ_0 and $P_{-1}(t)$ are zero in Equation (1).

Defining $\frac{dP_i(t)}{dt} = \lim_{\Delta t \to 0} \frac{P_i(t+\Delta t) - P_i(t)}{\Delta t}$, Equations (1) and (2) can be written as

$$\frac{dP_i(t)}{dt} = -(\lambda_{i+1} + \lambda_{(i+1)C})P_i(t) + \lambda_i P_{i-1}(t); \ i = 0, 1, \ldots, n-1 \tag{3}$$

$$\frac{dP_{n+i}(t)}{dt} = -\lambda_{i+1}P_{n+i}(t) + \lambda_{(i+1)C}P_i(t); \ i = 0, 1, \ldots, n-1 \tag{4}$$

where, $\frac{dP_i(t)}{dt}$ is the derivative of P_i with respect to t.

By the initial conditions $P_i(0) = \begin{cases} 1 & ; \ i = 0 \\ 0 & ; \ i \neq 0 \end{cases}$, and taking the Laplace transform of Equations (3) and (4) and applying the initial conditions, we have:

Finally, applying the inverse Laplace transform on Equations (5)-(8), leads into

Adding Equations (9)-(12) together, the reliability of the cold-standby redundant system with n dissimilar components by imperfect switching is obtained as follows:

$$R_{sys}^n(t) = \left(P_0(t) + \sum_{i=1}^{n-1} P_i(t)\right) + \left(P_n(t) + \sum_{i=1}^{n-1} P_{n+i}(t)\right) = e^{-\lambda_1 t}$$

$$+ \sum_{i=1}^{n-1} \sum_{l=1}^{i+1} \left(\frac{\left((\lambda_{i+1} + \lambda_{(i+1)C}) - (\lambda_l + \lambda_{lC}) \right) \left(\prod_{j=1}^{i} \lambda_j \right)}{(\lambda_{i+1} - (\lambda_l + \lambda_{lC})) \left(\prod_{j=1, j\neq l}^{i+1} \left((\lambda_j + \lambda_{jC}) - (\lambda_l + \lambda_{lC}) \right) \right)} \right) e^{-(\lambda_l + \lambda_{lC})t}$$

$$+ \sum_{i=1}^{n-1} \left(\prod_{j=1}^{i} \frac{\lambda_j}{\lambda_j + \lambda_{jC} - \lambda_{i+1}} \right) e^{-\lambda_{i+1}t} \tag{13}$$

In which, $R_{sys}^n(t)'$ is the reliability of a system with n components under imperfect switching at time t derived by the Markov method.

$$R_{sys}^n(t) = \sum_{i=0}^{n-1} \left(\frac{\lambda^i \lambda_{(i+1)C}}{\prod_{j=1}^{i+1} \lambda_{jC}} \right) e^{-\lambda t} + \sum_{i=1}^{n-1} \sum_{l=1}^{i+1} \left(\frac{\lambda^i (\lambda_{lC} - \lambda_{(i+1)C})}{\lambda_{lC} \left[\prod_{j=1, j\neq l}^{i+1} (\lambda_{jC} - \lambda_{lC}) \right]} \right) e^{-(\lambda + \lambda_{lC})t} \tag{14}$$

Besides, when the failure rate of the switch is the same in all switching steps, i.e., $\lambda_{jC} = \lambda_C$, for $j = 1, \dots, n$, then the system reliability in Equation (13) becomes:

$$R_{sys}^n(t) = e^{-\lambda_1 t} + \sum_{i=1}^{n-1} \sum_{l=1}^{i+1} \left(\frac{(\lambda_{i+1} - \lambda_l)\left(\prod_{j=1}^{i} \lambda_j \right)}{(\lambda_{i+1} - (\lambda_l + \lambda_C)) \left(\prod_{j=1, j\neq l}^{i+1} (\lambda_j - \lambda_l) \right)} \right) e^{-(\lambda_l + \lambda_C)t}$$

$$+ \sum_{i=1}^{n-1} \left(\prod_{j=1}^{i} \frac{\lambda_j}{\lambda_j + \lambda_C - \lambda_{i+1}} \right) e^{-\lambda_{i+1}t} \tag{15}$$

In Equation (13)-(15), it can be easily seen that when λ_{iC}; for all i, is zero, i.e., $\lambda_{iC} = 0$, then the system reliability is

The reliability of the system in Equation (13), when all the components are identical, i.e., $\lambda_j = \lambda; j = 1, \dots, n$, can be obtained as follows:

yielding with a perfect switch for each mentioned equation. For example, in Equation (14) we have

$$\underset{\lambda_{1C} \to 0}{\text{Lim}} \lim_{\lambda_{2C} \to 0} \dots \lim_{\lambda_{nC} \to 0} R_{sys}^n(t)_{\lambda_{1C} \neq \lambda_{2C} \neq \dots \neq \lambda_{nC}} = \lim_{\lambda_{1C} \to 0} \lim_{\lambda_{2C} \to 0} \dots \lim_{\lambda_{nC} \to 0} \left[\sum_{i=0}^{n-1} \left(\frac{\lambda^i \lambda_{(i+1)C}}{\prod_{j=1}^{i+1} \lambda_{jC}} \right) \right.$$

$$\left. - \left(\sum_{i=1}^{n-1} \sum_{l=1}^{i+1} \left(\frac{\lambda^i (\lambda_{(i+1)C} - \lambda_{lC}) e^{-\lambda_{lC}t}}{\lambda_{lC} \left[\prod_{j=1, j\neq l}^{i+1} (\lambda_{jC} - \lambda_{lC}) \right]} \right) \right) \right] e^{-\lambda t}_{\lambda_{1C} \neq \lambda_{2C} \neq \dots \neq \lambda_{nC}} = e^{-\lambda t} \sum_{i=0}^{n-1} \frac{(\lambda t)^i}{i!} \tag{16}$$

The details of the derivation of Equation (16) are given in Appendix 1. Note that Equation (16) is the same as the one obtained for a non-repairable cold-standby system with n-component under perfect switching in Billinton & Allan (1992), Dhillon, (2004), and Wang et al. (2012).

3.1. Mean Time To Failure (MTTF)
The MTTF for the cold-standby 1-out-of-n systems with dissimilar components is obtained by Equation (17). However, when all components are identical, the MTTF is derived by Equation (18), and in case the switch failure rate is constant throughout the time mission, the MTTF is determined according to Equation (19).

$$MTTF = \int_0^\infty R_{sys}^n(t) dt$$

$$= \frac{1}{\lambda_1} + \sum_{i=1}^{n-1} \left(\prod_{j=1}^{i} \frac{\lambda_j}{\lambda_j + \lambda_{jC} - \lambda_{i+1}} \right) \left(\frac{1}{\lambda_{i+1}} \right)$$

$$+ \sum_{i=1}^{n-1} \sum_{l=1}^{i+1} \left(\frac{\left((\lambda_{i+1} + \lambda_{(i+1)C}) - (\lambda_l + \lambda_{lC}) \right) \left(\prod_{j=1}^{i} \lambda_j \right)}{(\lambda_{i+1} - (\lambda_l + \lambda_{lC})) \left(\prod_{j=1, j\neq l}^{i+1} \left((\lambda_j + \lambda_{jC}) - (\lambda_l + \lambda_{lC}) \right) \right)} \right) \left(\frac{1}{\lambda_l + \lambda_{lC}} \right) \tag{17}$$

$$MTTF = \left(\sum_{i=0}^{n-1} \left(\frac{\lambda^{i-1} \lambda_{(i+1)C}}{\prod_{j=1}^{i+1} \lambda_{jC}} \right) \right) - \left(\sum_{i=1}^{n-1} \sum_{l=1}^{i+1} \left(\frac{\lambda^i (\lambda_{(i+1)C} - \lambda_{lC})}{\lambda_{lC} (\lambda + \lambda_{lC}) \left[\prod_{j=1, j\neq l}^{i+1} (\lambda_{jC} - \lambda_{lC}) \right]} \right) \right) \tag{18}$$

$$MTTF = \frac{1}{\lambda_1} + \sum_{i=1}^{n-1}\left(\prod_{j=1}^{i}\frac{\lambda_j}{\lambda_j + \lambda_C - \lambda_{i+1}}\right)\left(\frac{1}{\lambda_{i+1}}\right)$$
$$+ \sum_{i=1}^{n-1}\sum_{l=1}^{i+1}\left(\frac{(\lambda_{i+1} - \lambda_l)(\prod_{j=1}^{i}\lambda_j)}{(\lambda_{i+1} - (\lambda_l + \lambda_C))(\prod_{j=1, j\neq l}^{i+1}(\lambda_j - \lambda_l))}\right)\left(\frac{1}{\lambda_l + \lambda_C}\right) \tag{19}$$

4. A numerical Example

Here, the reliability and the MTTF of a system are determined for a mission time of one year (8,760 h) using Equations (14) and (18) when n=2, 5, 10, and 20. The failure of each active component follows an exponential distribution with a rate of $\lambda = 0.001$ per hour. Besides, it is assumed that the switch fails according to the exponential distribution with an increasing failure rate obtained by $\lambda_{iC} = \frac{i}{n}\lambda$, $i = 1, 2, \dots, n$. The reliability and the MTTF of the system are obtained using the closed-form equations derived in Section 3 when imperfect and perfect switching mechanisms are used. The reliabilities of all of the four abovementioned systems are shown in Figure 4.
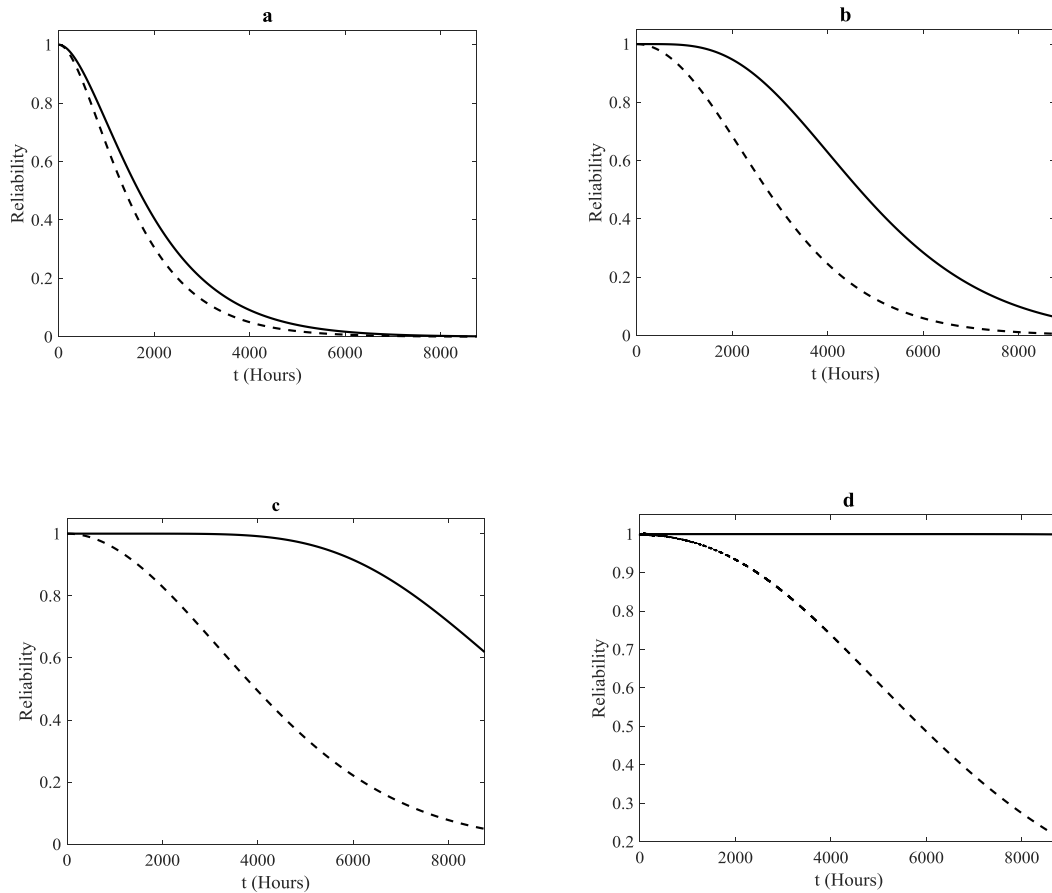

Fig. 4. System reliability with perfect and imperfect switching
a: $n = 2$, b: $n = 5$, c: $n = 10$, d: $n = 20$
Solid line: Perfect switching, Dash line: Imperfect switching

As seen in Figure 4, the reliabilities of all of the four systems when they use a perfect switch are always are higher than the ones with an imperfect switching mechanism, as expected. Besides, when the number of redundant cold-standby components increases, both systems (with perfect and imperfect switching) become more reliable. This is also expected. However, not only one needs to increase the number of redundant components to improve the reliability of a system with an imperfect switching mechanism, but also higher reliable switches are required. Moreover, when there are fewer redundant components in the system, the performances of the systems are closer to each other (see Figures 4-a). Furthermore, as the number of redundant components increases, the difference between the reliabilities of a system with a perfect switch from the one with an imperfect switch becomes larger (see Figures 4-b, 4-c, and 4-d). The MTTFs of the systems for which the reliabilities are depicted in Figure 4 are presented in Table 1.

Table 1
The system MTTF based on the number of redundant components

Number of Components	Imperfect Switching	Perfect Switching
n	MTTF (Hour)	MTTF (Hour)
2	1,666.67	2,000
5	3,007.3	5,000
10	4,305.1	10,000
20	6,319.9	20,000

The numerical results in Table 1 show that as the number of redundant components in the system increases, the lifetime of the system swiftly decreases compared to when the switch is completely perfect. For instance, under imperfect switching, when the number of system components changes from 2 to 5 components, the system MTTF is approximately 1,340 hours longer. However, when the number of system components changes significantly (from 5 to 20 components), the system MTTF just increases to 3,312 hours. This is due to the imperfect switch with an increasing failure rate with respect to the number of switches.

5. Conclusion

In this paper, closed-form equations were derived to calculate the reliability and the mean time to failure of a non-repairable redundant system with an active component and (n-1) spare components in cold-standby redundancy under the imperfect switching using the Markov approach. While it was assumed that the failures of the components and the switch were independent of each other, a constant rate was considered for the components and an increasing rate was used for the switch as it was used more frequently.

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Appendix 1

Limiting reliability of a cold standby redundant system Without loss of generality, here the proof of Equation (16) is provided for a 3-component system.

$$\lim_{\lambda_{1C}\to 0}\lim_{\lambda_{2C}\to 0}\lim_{\lambda_{3C}\to 0} R_{sys}^3(t)'_{\lambda_{1C}\neq\lambda_{2C}\neq\lambda_{3C}}\left[\sum_{i=0}^{2}\left(\frac{\lambda^i\lambda_{(i+1)C}}{\prod_{j=1}^{i+1}\lambda_{jC}}\right)\right.$$

$$\left.-\left(\sum_{i=1}^{2}\sum_{l=1}^{i+1}\left(\frac{\lambda^i(\lambda_{(i+1)C}-\lambda_{lC})e^{-\lambda_{lC}t}}{\lambda_{lC}[\prod_{j=1,j\neq l}^{i+1}(\lambda_{jC}-\lambda_{lC})]}\right)\right)\right]$$

$$e^{-\lambda t}=\lim_{\lambda_{1C}\to 0}\lim_{\lambda_{2C}\to 0}\lim_{\lambda_{3C}\to 0}\left[e^{-\lambda t}+\lambda e^{-\lambda t}\left(\frac{1-e^{-\lambda_{1C}t}}{\lambda_{1C}}\right)\right.$$

$$+\lambda^2 e^{-\lambda t}\left(\frac{(\lambda_{2C}-\lambda_{1C})-\lambda_{2C}e^{-\lambda_{1C}t}}{\lambda_{1C}\lambda_{2C}(\lambda_{2C}-\lambda_{1C})}\right)$$

$$\left.+\lambda^2 e^{-\lambda t}\left(\frac{e^{-\lambda_{2C}t}}{\lambda_{2C}(\lambda_{2C}-\lambda_{1C})}\right)\right]$$

$$=\lim_{\lambda_{2C}\to 0}\left[e^{-\lambda t}+\lambda e^{-\lambda t}\lim_{\lambda_{1C}\to 0}\left(\frac{1-e^{-\lambda_{1C}t}}{\lambda_{1C}}\right)\right.$$

$$+\lambda^2 e^{-\lambda t}\lim_{\lambda_{1C}\to 0}\left(\frac{(\lambda_{2C}-\lambda_{1C})-\lambda_{2C}e^{-\lambda_{1C}t}}{\lambda_{1C}\lambda_{2C}(\lambda_{2C}-\lambda_{1C})}\right)$$

$$\left.+\lambda^2 e^{-\lambda t}\lim_{\lambda_{1C}\to 0}\left(\frac{e^{-\lambda_{2C}t}}{\lambda_{2C}(\lambda_{2C}-\lambda_{1C})}\right)\right]$$

However, each of the terms $\frac{1-e^{-\lambda_{1C}t}}{\lambda_{1C}}$ and $\frac{(\lambda_{2C}-\lambda_{1C})-\lambda_{2C}e^{-\lambda_{1C}t}}{\lambda_{1C}\lambda_{2C}(\lambda_{2C}-\lambda_{1C})}$ when λ_{1C} approaches 0, is $\frac{0}{0}$ ambiguity. Applying the L'Hospital's rule to remove the ambiguity, we have

$$\lim_{\lambda_{2C}\to 0}\left[e^{-\lambda t}+(\lambda t)e^{-\lambda t}+\lambda^2 e^{-\lambda t}\left(\frac{(\lambda_{2C}t)-1}{\lambda_{2C}^2}\right)\right.$$

$$\left.+\lambda^2 e^{-\lambda t}\left(\frac{e^{-\lambda_{2C}t}}{\lambda_{2C}^2}\right)\right]$$

$$=e^{-\lambda t}+(\lambda t)e^{-\lambda t}+\lambda^2 e^{-\lambda t}\lim_{\lambda_{2C}\to 0}\left(\frac{e^{-\lambda_{2C}t}+(\lambda_{2C}t)-1}{\lambda_{2C}^2}\right).$$

After removing the ambiguity by using the L'Hospital's rule twice, the limit of the term $\frac{e^{-\lambda_{2C}t}+(\lambda_{2C}t)-1}{\lambda_{2C}^2}$ when λ_{2C} approaches 0 is

$$\text{Lim}_{\lambda_{1C}\to 0}\lim_{\lambda_{2C}\to 0}\lim_{\lambda_{3C}\to 0} R_{sys}^3(t)'_{\lambda_{1C}\neq\lambda_{2C}\neq\lambda_{3C}}$$

$$=e^{-\lambda t}+(\lambda t)e^{-\lambda t}+\frac{(\lambda t)^2}{2!}e^{-\lambda t}$$

The reliability of a system with *n*-components is obtained similarly.

Appendix 2
Notation

n	Number of components in the system
x_i	i^{th} Component
t	Mission time
$R_{sys}^n(t)$	Reliability of the 1-out-of-*n* cold-standby system under imperfect switching at time t
λ_i	The failure rate of the ith component in active mode
λ_{iC}	The failure rate of the switch when it is connected to the ith active component
$P_i(t)$	The probability of the system being in the ith state at time t
$P_i(s)$	Laplace transformation of P_i
$\dfrac{dP_i(t)}{dt}$	Derivative of P_i with respect to time
s	Laplace transform variable