Attitude Tracking Control of Autonomous Helicopter using Polytopic-LPV Modeling and PCA-Parameter Set Mapping

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Abstract: This paper presents a new method for modeling and Attitude Control of Autonomous Helicopters (A.H.) based on a polytopic linear parameter varying approach using parameter set mapping with the Principal Component Analysis (PCA). The polytopic LPV model is extracted based on angular velocities and Euler angles, that is influenced by flopping angles, by generating a set of data over the different trim points. Because of the high volume of trim data, parameter set mapping based on (PCA) is used to reduce the parameter set dimension. State feedback control law is proposed to stabilize the system by introducing a Linear Matrix Inequality (LMI) set over the vertices models. The proposed controller is performed for an Autonomous Helicopter in different scenarios. All the scenarios are investigated with the PCA algorithm as a technique for reducing the computational volume and increasing the speed of solving the LMI set. The simulation results of implementing the planned controller on the nonlinear model of an autonomous helicopter in different scenarios show the effectiveness of the proposed scheme.

Keywords: Attitude Control, Linear Matrix Inequality, Linear Parameter Varying, Principal Component Analysis

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Biographical notes: Reza Tarighi is now a PhD candidate degree student at South Tehran Branch, Islamic Azad University, Tehran, Iran. His current research interests include control analysis image processing, robotics, and aerospace. **Amir Hooshang Mazinan** is currently an Associate Professor and also the Director of Control Engineering Department at Islamic Azad University, South Tehran Branch, Tehran, Iran, since 2009. He is now acting as the Associate Editor in Transactions of the Institute of Measurement and Control (Sage publisher) and Computers & Electrical Engineering Journal (Elsevier Publisher), as well, **Mohammad Hosein Kazemi** is currently an Associate Professor in the Department of Electrical Engineering at the Shahed University, Tehran, Iran.

Research paper

1 INTRODUCTION

One of the most essential control modes in flight systems is the attitude control of the flight system. That is, the more stable is the attitude control system's performance, the more reliable the flight control system will be Euler angles and angular velocities. Nowadays, due to the unique features of the unmanned flight systems, their low maintenance costs as well as their high performance, they have received more attention. The autonomous helicopter flight system is used at different centers due to the specific flight characteristics and flight superiority in specific maneuvers, including side-to-side movement, backward, hover mode, etc. Examples include geology, aerial imaging, emergency and medical services, and so on. Different research has been done on helicopter control, but less research has been done on attitude control and also attitude control tracking, especial focus on of the attitude control with the selection of the proposed technique (PLPV) and PCA algorithm. In this study, it has been attempted to apply a novel method of attitude control with nonlinear modeling and PCA algorithm, which comparatively performs tracking. Both of the investigated methods show excellent stability. Based on the attitude control issue for Autonomous Helicopter, we review the related works and examine the tracking and techniques used. Regarding attitude control, the following studies can be referred: robust control with the neural network [1], control using nonlinear dynamics in [2], In [3], the fuzzy control is studied with the use of gain scheduler; a nonlinear mathematical model of the helicopter with fuzzy logic is used in [4]. In [5], the adaptive PID for attitude control is investigated, and in [6], adaptive attitude control is mentioned. The following is a brief, overview of the articles dealing with status control tracking Including [7]. In this reference, the robust control in attitude control tracking is investigated in two modes: one of them is the full–state feedback, and the other is the output state feedback form. In [8], the robust nonlinear control for a laboratory helicopter attached to a desk is investigated using output feedback that relies solely on position sensors and the design is based on the Lyapunov function. Attitude control tracking with an adaptive approach and considering uncertainty parameters can be seen in [9]. In [10], Attitude Control of the Autonomous Helicopter (A.C.A.H) tracking in the adaptive form has been analyzed with the exception that it considers dead zones of the actuator. The design is based on adaptive fuzzy control with the application of external disturbances. [11] applies A.C.A.H tracking with the geometric structure approach with the description that the investigative procedure is implemented in two situations. However, a bout helicopter research with focusing on the linear parameter varying can be referred to the following references mention. In [12], trajectory tracking of nonlinear unmanned rotorcraft based on Polytopic modeling, and in [13], velocity control of nonlinear unmanned rotorcraft using Polytopic modelling, is presented. Also, as mentioned earlier in [14], the attitude control of an unmanned helicopter by Fuzzy Gain-Scheduler is done. A Polytopic Linear Parameter Varying (LPV) system with Takagi–Sugeno has been expanded. This design is based on the Polytopic model and is one of the good works. But the change is expressed as a set of only maximum four rules. This concept can be expressed as follows; the nonlinear model is approximated by a TS-fuzzy model, which boils down to convex combination of linear sub models. Although, the aim of the research in this direction has not been achieved. But some LPV models were investigated for quadrotors before. Some references are about using LPV models in UAVs, which can be briefly mentioned. In [15], Robust LPV attitude control of a quadrotor unmanned with state constraints and input saturation and wind disturbance has been studied. In this reference, the control scheme was based on L2-gain notion of the H-Infinity norm, Polytopic representation of saturation function, and ellipsoids as invariant sets have been analyzed. Although the designed model approach has a more favorable output response compared to the nonlinear approach, the performance of the closed loop system of the Euler angles does not fully meet the ultimate goal. The first approach uses the angular velocities and the flap angle modes, and the second approach uses the inherent damping in helicopter dynamics. Also, for further investigation, you can refer to [16-17]. One of the scenarios has been compared, this scenario is introduced in [18]. In [18], a robust control system is designed based on method and is evaluated in real flight tests. The states and inputs considered for inner loop in this research are two cyclic inputs, two angular velocities, Euler angles, and also the main rotor flapping angles. The strength of this article is the use of experimental results on a testbed in order to evaluate control system performance, but the proposed linear robust control system can be improved with more novel control algorithms. After this section, a nonlinear model and LPV designed model and then PCA model fit to an LPV model are introduced, respectively. Then, different scenarios are presented in two modes: without approximate and with approximate external disturbance and without disturbance. The simulated results are compared with each other, and finally, the conclusion is presented.

2 PROBLEM STATEMENT AND PRELUDES

In this section, it is exclusively aimed to briefly review the completed nonlinear dynamic model of A.H and introduce some introductory science about PLPV and determined performance-based control method.

2.1. A.H Model

In general, due to the specific complexities of the nonlinear model of helicopters, their control performance is one of the notable challenges of active researchers in this field, which of course, these challenges have led to scientific advances. The modeling and the introduction of the complete system of rotors and forces and the momentum and flapping of helicopters can be found in various references, most notably in references[19-22]. In "Fig. 1", the coordinates and angular velocities and Euler angles and linear velocities are presented.

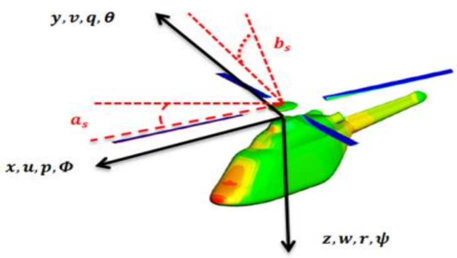


Fig. 1 Introduced distribution of helicopter forces.

The helicopter image is used from [23]. The helicopter has four inputs that are normalized between (1 and -1). In general, helicopter movement equations can be expressed as force, momentum, kinematic, and navigation equations in [21], As shown in "Fig. 1". The angles of the (Euler) attitude are yaw-pitch-roll, which the roll angle of ϕ is in the X direction, and the yaw angle of θ is in the Y direction and ψ pitch is in the Z direction.

$$\dot{P} = R^{-1}{}_{B} V_{B} \tag{1}$$

$$R_B = \begin{pmatrix} C\theta C\psi & C\theta S\theta s\phi - C\phi S\psi & C\theta c\psi S\theta + S\phi S\psi \\ C\theta S\psi & S\psi S\theta S\phi + C\phi C\psi & C\theta s\psi S\theta - S\phi S\psi \\ -S\theta & C\theta S\phi & C\theta C\phi \end{pmatrix} \tag{2}$$

Where, $P = (x, y, z)^T$ represents the position and $V_B = (u, v, w)^T$ represents the linear velocity and R_B is the rotation matrix and it is parameterized concerning to the three Euler angles. This introduction $C = \cos(.); S = \sin(.)$. Hence the kinematic equation can be obtained as follows.

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = S^{-1}\omega_B \tag{3}$$

In "Eq. (3)", ω_B represents the vector of the angular velocity and S^{-1} is the lumped transformation matrix in [20], which is introduced in the "Eq. (4)".

$$S^{-1} = \begin{pmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{pmatrix} \tag{4}$$

In designing the control model of the attitude control system, external and momentum forces are approximated as a linear combination of control modes and inputs or the use of control derivatives and stability. The overall structure can be shown in the format by equations (5)- (9) in [24] .The details and values used are presented in "Table 1".

$$\begin{aligned} \dot{u} &= vr - wq - g\sin\theta + X_u u + X_a a \\ \dot{v} &= wp - ur + g\sin\phi\cos\theta + Y_v v + Y_b b \\ \dot{w} &= uq - vp + g\cos\phi\cos\theta + Z_w w + Z_{col} D_{col} - g \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{p} &= -S_p qr + L_a.a + L_b.b \\ \dot{q} &= -S_q pr + M_a.a + M_b.b \\ \dot{r} &= -S_r pq + N_r.r + N_{col}.D_{col} + N_{ped} D_{ped} \end{aligned} \tag{6}$$

In"Eq.(5)", S_p, S_q, S_r express $I^{-1}_{xx}(I_{yy}-I_{zz}), I^{-1}_{yy}(I_{zz}-I_{xx}), I^{-1}_{zz}(I_{xx}-I_{yy})$, respectively. That they are the main momentums of helicopter inertia. In "Eq.(6)", N_{ped}, N_{col} as, pedal control derivative and collective control derivative are introduced. Also, N_r is indicant damping derivative for yaw moment in reference [24]. It is important to note that there is no coupling effect in the I_{yz}, I_{xz} directions. Because in most flight systems, the X-Z direction will be symmetric in [21].

$$\begin{aligned} \dot{a}_s &= -(q + a\tau^{-1}) + \tau^{-1} A_{lat} D_{lat} + \tau^{-1} A_{lon} D_{lon} \\ \dot{b}_s &= -(p + b\tau^{-1}) + \tau^{-1} B_{lat} D_{lat} + \tau^{-1} B_{lon} D_{lon} \end{aligned} \tag{7}$$

The Dynamics of the main rotor are gained by $(a_s \quad b_s)^T$ flapping angles in which the τ value is constant. If we approximate "Eq. (7)", as "Eq. (8)". Also $A_{lat}, A_{lon}, B_{lat}, B_{lon}$ are reagent cross-coupled and control derivatives for the longitudinal flapping angle, control and cross-coupled derivatives for the lateral flapping angle, respectively.

$$\begin{aligned} a_s &= -\tau q + A_{lat} D_{lat} + A_{lon} D_{lon} \\ b_s &= -\tau p + B_{lat} D_{lat} + B_{lon} D_{lon} \end{aligned} \tag{8}$$

The basic helicopter state and input vector modes are shown as the relation below:

$$x = [x \quad y \quad z \quad u \quad v \quad w \quad p \quad q \quad r \quad \phi \quad \theta \quad \psi]^T$$
$$u = \left[D_{lat} \quad D_{lon} \quad D_{ped} \quad D_{col}\right]^T \tag{9}$$

Table 1 Introduction of abbreviations and values [25].

symbol	deal	symbol	deal
A_{lat}, A_{lon}	12.50;141.08	L_b, M_a	55.86 ,345.19
B_{lat}, B_{lon}	180.98, 10.29	L_a, M_b	55.86;-23.03
I_{xx}, I_{yy}, I_{zz}	0.305; 0.684;0.787	N_{ped} , N_{col}	2095.16;256.42
g	9.8	N_r	-11.445

Now one can define the design system in a new way to design attitude control. It means become specified a new state of input and state parameters as defined in “Eq. (10)” .

$$x_{atit} = \left(p \quad q \quad r \quad \phi \quad \theta \quad \psi\right)^T$$
$$u_{attit} = \left(Dlat \quad Dlon \quad Dped\right)^T \tag{10}$$

2.2. The LPV generalized plant

The general state of the nonlinear equations can be shown as:

$$\dot{\mathbf{x}}(\mathbf{t}) = f\ (x(t), u(t), \sigma(t))$$
$$\mathbf{y}(\mathbf{t}) = g\,(x(t), u(t), \sigma(t)) \tag{11}$$

Where, $x(t) \in \Re^m, y(t) \in \Re^n, u(t) \in \Re^l, \sigma(t) \in \Re^{m_\sigma}$, are state variable, output, input and is a measurable exogenous parameter vector, called the scheduling parameter, respectively. That can be σ called the parameter vector and this variable parameter is usually constrained to be bounded in a box like form and assumed to be convex and known compact subset $\tilde{\sigma} \subset \Re^{m_\sigma}$ in [26]. By introducing:

$$\tilde{\mathbf{S}} = \left\{\sigma \in \Re^+ \to \Re^{m\mu} : \sigma(t) \in \tilde{\sigma}, \dot{\sigma} \in \dot{\tilde{\sigma}}\, \forall t \geq 0\right\} \tag{12}$$

That $\dot{\sigma}$, the rates of the parameter variation, is expressed. Also $\tilde{s}$ is determined as the collection of acceptable trajectories.

Hypothesis.1 [26]

Let $(\tilde{x}(\sigma), \tilde{u}(\sigma))\, \forall \sigma \in \tilde{S}$ that is referred as a family of Trimming Points (T.P) or Equilibrium Points (E.P).

$$\mathbf{f}(\tilde{\mathbf{x}}(\sigma), \tilde{\mathbf{u}}(\sigma), \sigma) = 0$$
$$\tilde{\mathbf{y}}(\sigma) = \mathbf{g}(\tilde{\mathbf{x}}(\sigma), \tilde{\mathbf{u}}(\sigma), \sigma) \tag{13}$$

However, we can have the nonlinear relationship presented in “Eq. (11)” around the trim point based on the Taylor series and conversion Jacobin, Linearization is introduced in the following form:

$$\mathbf{\Delta x} = x - \tilde{\mathbf{x}}(\sigma)$$
$$\mathbf{\Delta u} = u - \tilde{\mathbf{u}}(\sigma) \tag{14}$$

According to Differentiation rules, it can be concluded:

$$\dot{\mathbf{\Delta}}\mathbf{x} = \dot{x} - \dot{\tilde{\mathbf{x}}}(\sigma) \therefore$$
$$\dot{\mathbf{\Delta}}\mathbf{x} = \mathbf{f}(\tilde{\mathbf{x}}(\sigma), \tilde{\mathbf{u}}(\sigma), \sigma) - \dot{\tilde{\mathbf{x}}}(\sigma) \tag{15}$$

In the meantime, $\dot{\tilde{\mathbf{x}}}(\sigma)$ is the phrase to be created Caused by the time variation in σ. Now, with the development of linearization performance around a family of trim points in “Eq. (11)” , it can be stated that:

$$\Delta\dot{x} = A(\sigma)\Delta x(t) + B(\sigma)\Delta u(t)$$
$$\Delta y = C(\sigma)\Delta x(t) + D(\sigma)\Delta u(t) \tag{16}$$

$$A(\sigma) = \sum_{i=1}^{n} \frac{\partial f}{\partial x}(\mathbf{x}^i, \mathbf{u}^i)\mu_i$$
$$B(\sigma) = \sum_{i=1}^{n} \frac{\partial f}{\partial x}(\mathbf{x}^i, \mathbf{u}^i)\mu_i \tag{17}$$

With this explanation $\left\{\mathbf{x}^i, \mathbf{u}^i\right\} = \left\{\mathbf{x}(\sigma^i), \mathbf{u}(\sigma^i)\right\}$ in [27]. Indeed, an LPV system based on the convex interpolation, that is a method of linearization, is used. The Polytopic systems can also be considered in the scope of robust control. [28] is a suitable and well-founded article in this field as well as [29], expressing the concept of LPV, a Polytopic system is expressed as follows:

$$\sum_{i=1}^{N} \sigma_i(t) \begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} = \begin{pmatrix} A(\sigma(t)) & B(\sigma(t)) \\ C(\sigma(t)) & D(\sigma(t)) \end{pmatrix} \tag{18}$$

The Polytopic term comes from the fact that the vector $\sigma(t)$ evolves over the unit simplex, which is a Polytope (A form in n-dimensional geometry corresponding to a polygon or polyhedron), by:

$$\eta = \left\{ \underset{i}{col}(\sigma_i(t) \therefore \sum_{i=1}^{N} \sigma_i(t) = 1, \sigma_i(t) > 0 \right\} \tag{19}$$

In which, η is clearly a set of Polytope vertices.

$$\vartheta = \bigcup_{i=1}^{N} \vartheta_i$$
$$\vartheta_i = \begin{pmatrix} 0_{(i-1)\times 1} \\ 1 \\ 0_{(N-1)\times 1} \end{pmatrix} \tag{20}$$

In the above expression, the term ϑ is the convex hull and $hull[\vartheta]$ value coincides with η [30]. So in summary, it can be said that Polytopic systems are verily distributed in robust analysis and robust control.

2.3. Grid-based LPV Methodology

The following form is provided for a PLPV model [31]. In this paper, writers are fond in the grid based LPV methodology for the angular velocities and Euler angles of A.H using model by "Eq. (11) and (16)" , and are presented as the following relationship. The variation condition for the model designed for angular velocities (p,q,r) and Euler angles (ϕ,θ) is (± 1), $(\pm\pi/6)$, respectively.

$$A_{plpv\,(m,n)} = \begin{pmatrix} \alpha_{plpv\,(1,1)} & \alpha_{plpv\,(1,2)} & \alpha_{Pplpv\,(1,3)} & \alpha_{plpv\,(1,4)} & \alpha_{plpv\,(1,5)} & \alpha_{plpv\,(1,6)} \\ \alpha_{plpv\,(2,1)} & \alpha_{plpv\,(2,2)} & \alpha_{plpv\,(2,3)} & \alpha_{plpv\,(2,4)} & \alpha_{plpv\,(2,5)} & \alpha_{plpv\,(2,6)} \\ \alpha_{plpv\,(3,1)} & \alpha_{plpv\,(3,2)} & \alpha_{plpv\,(3,3)} & \alpha_{plpv\,(3,4)} & \alpha_{plpv\,(3,5)} & \alpha_{plpv\,(3,6)} \\ \alpha_{plpv\,(4,1)} & \alpha_{plpv\,(4,2)} & \alpha_{plpv\,(4,3)} & \alpha_{plpv\,(4,4)} & \alpha_{plpv\,(4,5)} & \alpha_{plpv\,(4,6)} \\ \alpha_{plpv\,(5,1)} & \alpha_{plpv\,(5,2)} & \alpha_{plpv\,(5,3)} & \alpha_{plpv\,(5,4)} & \alpha_{plpv\,(5,5)} & \alpha_{plpv\,(5,6)} \\ \alpha_{plpv\,(6,1)} & \alpha_{plpv\,(6,2)} & \alpha_{plpv\,(6,3)} & \alpha_{plpv\,(6,4)} & \alpha_{plpv\,(6,5)} & \alpha_{plpv\,(6,6)} \end{pmatrix}$$
$$B_{plpv\,(m,n)} = \begin{pmatrix} b_{plpv\,(1,1)} & b_{plpv\,(1,2)} & b_{plpv\,(1,3)} \\ b_{plpv\,(2,1)} & b_{plpv\,(2,2)} & b_{plpv\,(2,3)} \\ b_{plpv\,(3,1)} & b_{plpv\,(3,2)} & b_{plpv\,(3,3)} \\ b_{plpv\,(4,1)} & b_{plpv\,(4,2)} & b_{plpv\,(4,3)} \\ b_{plpv\,(5,1)} & b_{plpv\,(5,2)} & b_{plpv\,(5,3)} \\ b_{plpv\,(6,1)} & b_{plpv\,(6,2)} & b_{plpv\,(6,3)} \end{pmatrix} \tag{21}$$

$$\begin{aligned}
\alpha_{plpv\,(1,1)} &= \tan\theta(q\cos\phi - r\sin\phi) \\
\alpha_{plpv\,(1,2)} &= \sec^2\theta(q\sin\phi + r\cos\phi) \\
\alpha_{plpv\,(1,4)} &= 1 \\
\alpha_{plpv\,(1,5)} &= \sin\phi\tan\theta \\
\alpha_{plpv\,(1,6)} &= \cos\phi\tan\theta \\
\alpha_{plpv\,(2,1)} &= (q\sin\phi - r\cos\phi) \\
\alpha_{plpv\,(2,5)} &= \cos\phi \\
\alpha_{plpv\,(2,6)} &= -\sin\phi \\
\alpha_{plpv\,(3,1)} &= \sec\theta(q\cos\phi - r\sin\phi) \\
\alpha_{plpv\,(3,2)} &= \sec\theta\tan\theta(q\sin\phi + r\cos\phi) \\
\alpha_{plpv\,(3,5)} &= \sin\phi\sec\theta \\
\alpha_{plpv\,(3,6)} &= \cos\phi sce\theta \\
\alpha_{plpv\,(4,4)} &= -S_2 \\
\alpha_{plpv\,(4,5)} &= -S_1 - S_p.r \\
\alpha_{plpv\,(4,6)} &= -S_p.q \\
\alpha_{plpv\,(5,4)} &= -S_4 - S_q.r \\
\alpha_{plpv\,(5,5)} &= -S_3 \\
\alpha_{plpv\,(5,6)} &= -S_q.p \\
\alpha_{plpv\,(6,4)} &= -S_r.q \\
\alpha_{plpv\,(6,5)} &= -S_r.p \\
\alpha_{plpv\,(6,6)} &= -N_r \\
b_{plpv(4,4)} &= L_a A_{lat} + L_b B_{lat} \\
b_{plpv(4,5)} &= L_a A_{lon} + L_b B_{lon} \\
b_{plpv(5,4)} &= M_a A_{lat} + M_b B_{lat} \\
b_{plpv(5,5)} &= M_a A_{lon} + M_b B_{lon} \\
b_{plpv\,(6,5)} &= N_{col} \\
b_{plpv\,(6,6)} &= N_{ped}
\end{aligned} \tag{22}$$

Which in "Eq. (21)" , the parameters are introduced as bellow. All of the unspecified parameters presented in "Eq. (21)" and not brought in "Eq. (22)" are equal to zero. The same vertices of the Polytopic model are made. In "Fig. 2" , Algorithm Design and the overall structure of the controller model are provided.

$$\begin{aligned} \boldsymbol{A}_{plpvi\,(m,n)} &= (*)_{18522\times 6} \\ \boldsymbol{B}_{plpvi\,(m,n)} &= (*)_{18522\times 3} \end{aligned} \tag{23}$$

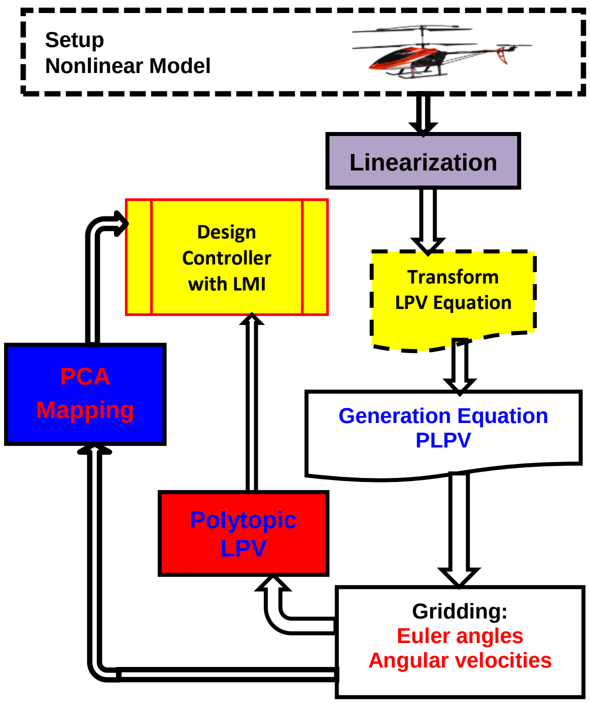


Fig. 2 Algorithm Design of the controller model.

2.4. LMI Conditions

In this part, a few LMI conditions for Hurwitz stabilizability is offered. In the introduction, with the help of the relationship defined in equation (18), the following general form PLPV system is considered:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \sum_{i=1}^{N} \sigma_i(\boldsymbol{t})\boldsymbol{A}_i(\boldsymbol{t}) \tag{24}$$

Theorem.1 [26]
The PLPV "Eq. (24)" is quadratically stable if and only if there exists a matrix $\mathrm{P,W}$ so that:

$$\boldsymbol{A}_i\,\mathrm{P} + \mathrm{P}\boldsymbol{A}_i^{\boldsymbol{T}} + \boldsymbol{B}_i\,\mathrm{W} + \mathrm{W}\boldsymbol{B}_i^{\boldsymbol{T}} < 0 \tag{25}$$

Proof: According to Lyapunov stability theory, the systems (25) exist, If and only if a matrix K and a matrix P>0, for the following condition have establishment.

$$\begin{aligned} &(\boldsymbol{A_i} + \boldsymbol{B_i K})\mathrm{P} + \mathrm{P}(\boldsymbol{A_i} + \boldsymbol{B_i K})^{\boldsymbol{T}} = \boldsymbol{A_i}\,\mathrm{P} + \mathrm{P}\boldsymbol{A_i}^{\boldsymbol{T}} - \gamma \boldsymbol{B_i B_i}^{\boldsymbol{T}} < 0 \\ &\text{for } \boldsymbol{i} = 1,\ 2,\ 3,\ 4 \ldots \boldsymbol{N};\gamma > 0 \end{aligned} \tag{26}$$

Also with designation the Lyapunov Function (LF) $\mathrm{V(x(t)) = x(t)^T\,P\,x(t)}$ and a symmetric positive definite (SPD) matrix $P > 0$, the time derivative of the LF is calculated along trajectories of system (19) propel to:

$$\dot{\mathrm{V}}(\mathrm{x(t)}) = \mathrm{x(t)^T}(\boldsymbol{A}(\sigma(\boldsymbol{t})^{\mathrm{T}}\,\mathrm{P} + \boldsymbol{A}(\sigma(\boldsymbol{t}))\,\mathrm{x(t)} \tag{27}$$

Letting:

$$\sum_{i=1}^{N} \sigma_i(\boldsymbol{t})(\boldsymbol{A}_i^{\boldsymbol{T}}\,\mathrm{P} + \mathrm{P}\,\boldsymbol{A}_i) \tag{28}$$

As a result, it can be said that the LMI of System (28) could be a collection of NLMI with the explanation that:

$$\mathrm{K = WP^{-1}} \tag{29}$$

The value of gain control designed from the solving the above LMI equations are obtained for each two method that is provided in Appendix.

3 PCA TECHNIQUES

By definition $E(t) = \tilde{G}(\mu(t));\ \tilde{G}:\Re^{\Omega} \to \Re^{s}$ as a continuous mapping function to express PLPV. The transformational Principal Component Analysis (PCA) can be shown in vector space to reduce the size of the data produced. In LPV systems, the number of data generated is often large, and its computational analysis is practically lengthy and difficult at some stages. The relation presented in equation (18) can be approximated by the following format based on the PCA approximation algorithm.

$$\begin{aligned} \Delta\dot{x} &= \tilde{A}(E(t))\Delta x(t) + \tilde{B}(E(t))\Delta u(t) \\ \Delta y &= \tilde{C}(E(t))\Delta x(t) + \tilde{D}(E(t))\Delta u(t) \end{aligned} \tag{30}$$

Refer to references [29], [32], for a more detailed discussion of this algorithm. The relationship introduced in "Eq. (30)" is an appropriate approximation of "Eq. (18)" . We present a time frame for this subject $t = \{1,2,3,\ldots,N\}$, which in fact, it is the producer of the produced $l \times N$ matrix data and $\Theta = (\upsilon_1\ \ \upsilon_2\ \ \upsilon_3\ \ \ldots\upsilon_N)$. In this introduction, the Θ_i matrix rows are normalized by Π_i Affine's law for the generation of the generated data in the form is denoted with the mean standard deviation and the unit of zero with the explanation that

$\Theta_i^j = \Pi_i(\Theta_i)$ and $\Theta_i = \Pi_i^{-1}(\Theta_i^{\,j})$. See more details in reference [29-30]. The normalized matrix data form follows

$$\Theta^j = \Pi(\Theta) \tag{31}$$

Definition 1

$[A]_{n\times d}$ If the matrix exists with the rank r and $n \geq d$, then $A = UWV^T$ which in $[W]_{n\times d}$, it is a diagonal matrix and $[U]_{n\times n}, [V]_{d\times d}$ are orthogonal matrices. By explaining that $[AA^T]_{n\times n}$ is the positive semi definite symmetric matrix for $[A^TA]_{d\times d}$. Singular right vectors are $u_1, u_2, .., u_i; i = [1, n]$ and singular left vectors are $v_1, v_2, .., v_j; j = [1, d]$. It is now possible to use the square of the singular values, which the result of their multiplication is (PSD). It should be noted that the singular values are not zero. Hence, it can be said that $A^T u_i = z_i$ that is the singular vector of A^TA. The following equation can now be presented in [34].

$$Az_i = \sigma^2{}_i u_i$$
(32)

With the explanation that z_i is not necessarily a vector of uniqueness.

$$\|z_i\| = \left(A^T u_i, A^T u_i\right) = \left(AA^T u_i, u_i\right) = \sigma^2{}_i (u_i, u_i) = \sigma^2{}_i \tag{33}$$

According to the definition 1, now Singular Value Decomposition (SVD) can be represented in the following relation:

$$\tilde{\Theta}^j = \begin{pmatrix}\tilde{U}^T & U^T\end{pmatrix} = \begin{pmatrix}\tilde{\sigma} & 0 & 0 \\ 0 & \sigma & 0\end{pmatrix}\begin{pmatrix}\tilde{V} \\ V\end{pmatrix} \tag{34}$$

If s is the answer of the effective value for $\tilde{U}, \tilde{V}, \tilde{\sigma}$, then the following approximation can be considered:

$$\tilde{\Theta}^j = \tilde{U}\tilde{\sigma}\tilde{V}^T \cong \Theta^j \tag{35}$$

Where, $\tilde{\Theta}^j$ is introduced as the appropriate approximation of the generated data:

$$\begin{aligned} &E(t) = \tilde{G}(\mu(t)) = \tilde{U}\,\Pi((G(\mu(t)) \therefore \\ &E(t) = \tilde{U}\,\Pi(\upsilon(t)) \end{aligned} \tag{36}$$

As a result:

$$\begin{aligned} &\tilde{\eta}(E) = \begin{pmatrix}\tilde{A}(E(t)) & \tilde{B}(E(t)) \\ \tilde{C}(E(t)) & \tilde{D}(E(t))\end{pmatrix} \\ &\tilde{\eta}(E) = \begin{pmatrix}A(\tilde{\upsilon}(t)) & B(\tilde{\upsilon}(t)) \\ C(\tilde{\upsilon}(t)) & D(\tilde{\upsilon}(t))\end{pmatrix} \end{aligned} \tag{37}$$

In fact, it can be said:

$$\begin{aligned} &\tilde{\upsilon}(t) = \Pi^{-1}(\tilde{U}E(t)) \therefore \\ &\tilde{\upsilon}(t) = \Pi^{-1}(\tilde{U}\tilde{U}^T\,\Pi(\upsilon(t))) \end{aligned} \tag{38}$$

Where, Π^{-1} denotes row-wise rescaling. According to the relation (19), it can be stated that, by these terms, new system PLPV to with vertex number $\bar{\mathrm{N}} = 2^s$ can be explained.

$$\tilde{\eta}(\tilde{\upsilon}(t) \in convex\{\tilde{\eta}(\tilde{\upsilon}_1), ..., \tilde{\eta}(\tilde{\upsilon}_N)\} = \sum_{i=1}^{\bar{N}} \alpha_i \tilde{\eta}(\tilde{\upsilon}_i)$$
(39)

The appropriate value of the approximation for the design can be introduced by this form in "Eq. (40)" with the help of the relation (34) and the definition 1, if to be defined Γ_{ftv} fraction of total variation in [33].

$$\Gamma_{ftv} = \frac{\sum_{i=1}^{\beta 1} \sigma^2{}_i}{\sum_{i=1}^{l} \sigma^2{}_i} \tag{40}$$

According to the first produced data that equals 3087, the SVD matrix will equal $(U)_{16\times 16}; (V)_{3087\times 3087}; (S)_{16\times 3087}$ respectively. Also, according to the PCA approximation algorithm, the matrices $\hat{A}_{PCA}; \hat{B}_{PCA}$ will be as follows.

$$\begin{aligned} &\hat{A}_{PCA(m,n)} = (*)_{96\times 6} \\ &\hat{B}_{PCA(m,n)} = (*)_{96\times 3} \end{aligned} \tag{41}$$

4 SIMULATIONS RESULTS

In the first part, the simulation of the results without approximation is performed (the steps are examined in both non-disturbance and external disturbance mode) and then it is performed by applying the same reference inputs to the scenarios to simulate in the PCA approximation part.

4.1. PLPV Simulation without disturbance

In this scenario, the square inputs are applied. In this scenario, the determined target of the applied attitude control angles (ϕ,θ,ψ) are (10, 20, 30) degrees respectively, which ϕ is black and θ is blue and ψ is red in the reference input.

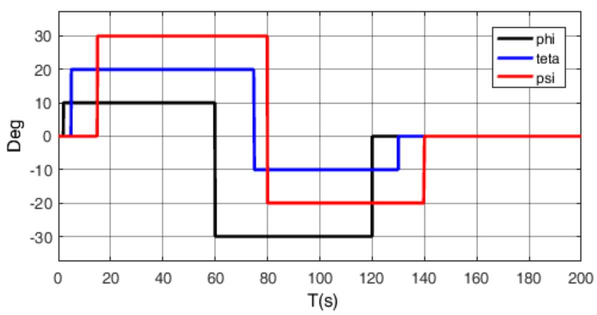
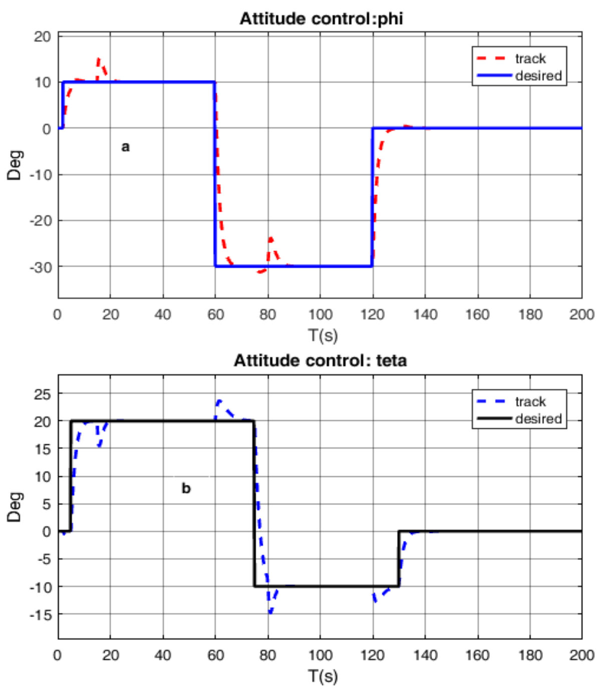


Fig. 3 Input of a square reference in the first scenario without Disturbance.



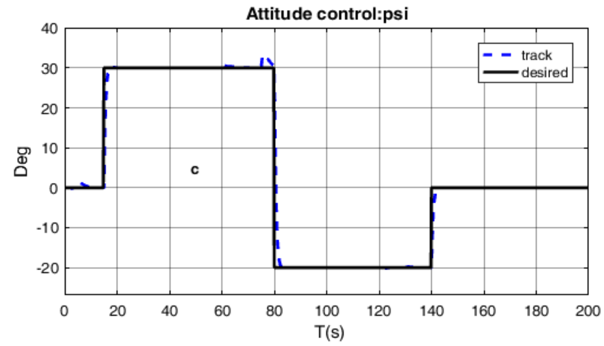


Fig. 4…Attitude control tracking of the angle (ϕ, θ,ψ) : (a,b,c) the first scenario, respectively.

In "Fig. 4 " , for part (a), discrete red line of tracking and continuous blue line of the reference input and part (b) Discrete blue line of tracking and continuous black line of the reference input and part (c), Discrete blue line of tracking and continuous black line of the reference input are presented. Also in "Fig. 4" , as is obvious, the tracking condition is well done. In "Fig. 5 " , the control inputs in the first scenario are shown. The control inputs $D_{lat}, D_{lon}, D_{ped}$ are red, black and blue, respectively.

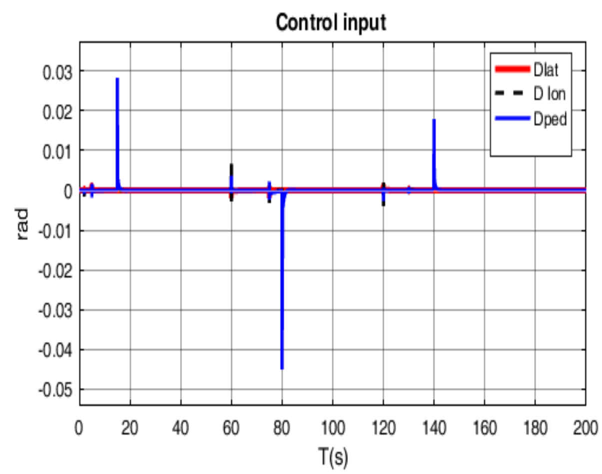


Fig. 5 Control input of the first scenario.

4.2. PLPV Simulation with disturbance

In this section, all the modes of the pervious section with external disturbance are applied. For the noise interference, a Dryden wind model is provided in Simulink MATLAB based on the reference[35]. The selected wind velocity range is 1 m/s. In this regard,the different effect of wind speed is also specified in this scenario. "Fig. 6 " show the angular rates models of wind turbulence.

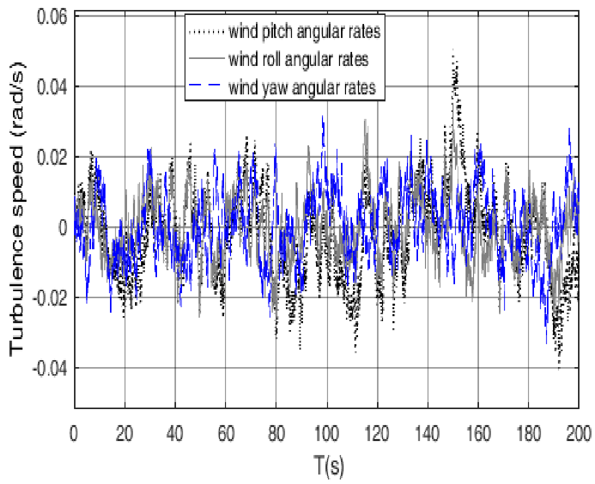


Fig. 6 Turbulence angular rates in the three body axes.

Figures 7 to 9 present the attitude control tracking with disturbance to the system. In " Fig. 7 " , the reference input tracking is presented for the angle ϕ and in "Fig. 8", it is presented for θ and in "Fig. 9 " , it is presented for the angle ψ . As is obvious, it also performs tracking in this mode completely. Also, "Fig. 10 " shows the control inputs in the scenario.

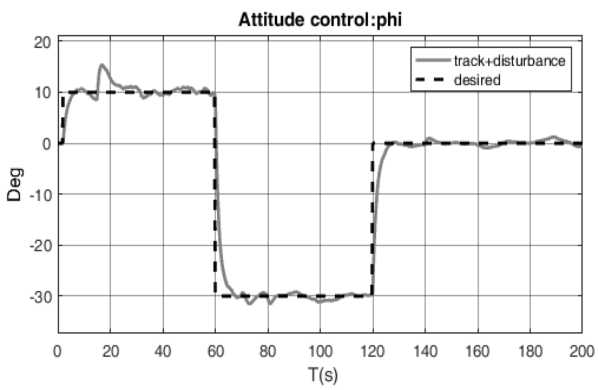


Fig. 7 Attitude control tracking of the angle ϕ with disturbance. \The continuous line of tracking and discrete line of the reference input.

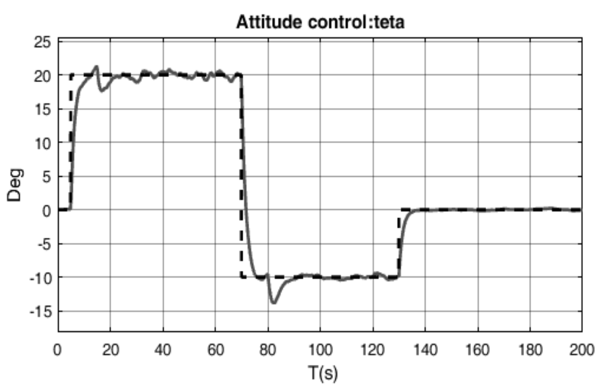


Fig. 8 Attitude control tracking of the angle θ with disturbance. The continuous line of tracking and discrete line of the reference input.

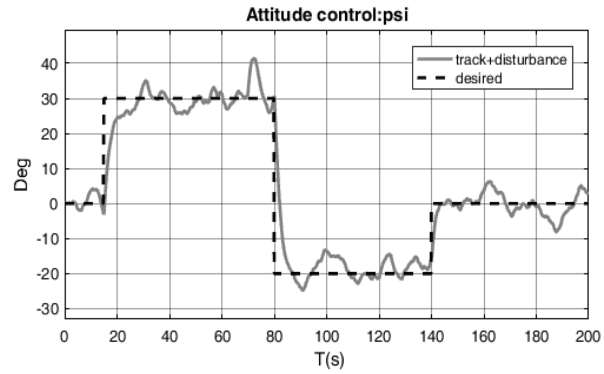


Fig. 9 Attitude control tracking of the angle ψ with disturbance. The continuous line of tracking and the discrete line of the reference input.

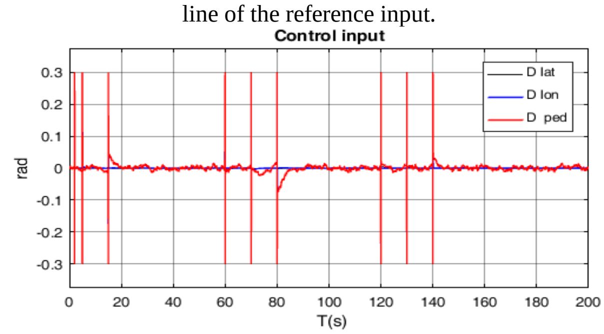


Fig. 10 Control input of the first scenario with Disturbance.

4.3. Simulation scenario of PCA results with disturbance

In this section, we have done all the simulations of the results of the previous section with the same defined and introduced scenarios with disturbance. The approximation value, based on the PCA model is presented by "Fig. 11" . As can be seen in this figure, approximation determined it by the algorithm PCA about, at 72%.

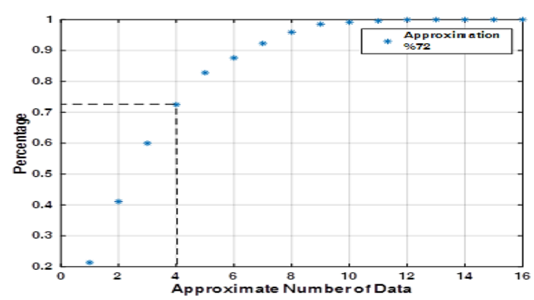
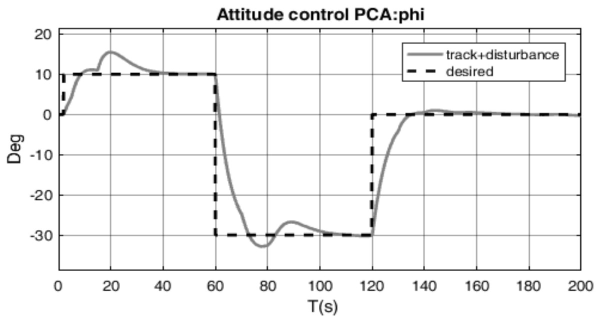
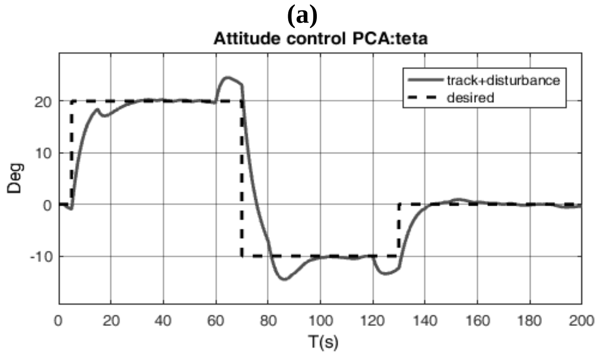


Fig. 11 Approximate percentage of PCA model.

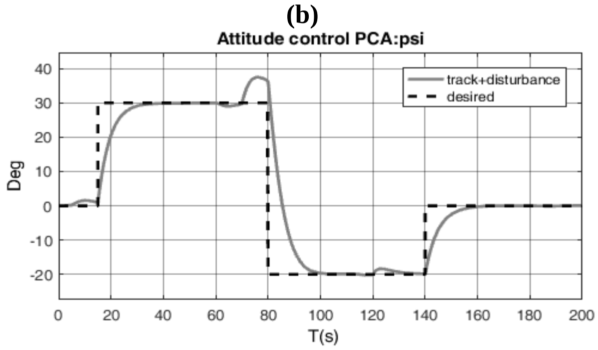
In "Fig. 12 " , for part (a) continuous line of tracking and discrete line of the reference input and part (b) continuous line of tracking and discrete line of the reference input and part (c), continuous line of tracking and the discrete line of the reference input are shown. Compared to "Fig.s. 7 to 9", it is quite clear that this approximation has maintained its performance and tracking the trajectory is excellently done. This means that in both the PLPV method and PCA method, the tracking is performed well and the desired result is obtained.



(a)



(b)



(c)

Fig. 12 Attitude control tracking of the angle (ϕ, θ, ψ) of PCA algorithm (a, b, c), respectively.

In addition, "Fig. 13" shows control input. According to theorem one and the solution of LMI equations, the value of equals with gamma 2.3 and for PLPV method value of equals with gamma 0.601. Also, the control gain matrix is shown in the appendix.

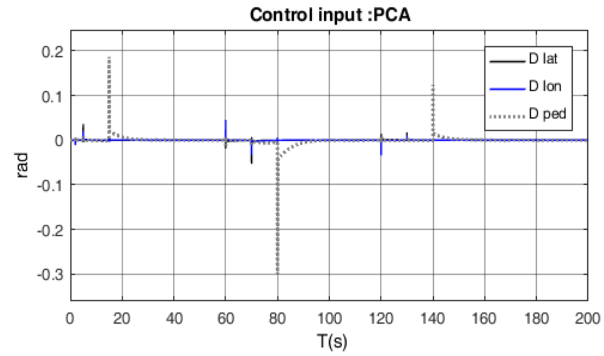


Fig. 13 Control input of the PCA scenario with Disturbance.

4.4. Simulation scenario of PCA, PLPV, Robust control H∞ results with disturbance

In this section, a comparison with the reference [18] that model is one of the strongest recent works in which modeling is done using robust control. Also for turbulence, the wind speed of the previous section is used. "Fig. 14" shows a comparative presentation of the method introduced in the article and the method in [18] is presented. As it is clear, given the tracked path with the noise application introduced at the beginning of this section, the tracking function is well done. The abbreviation for perturbation (D), is shown in Figure below.

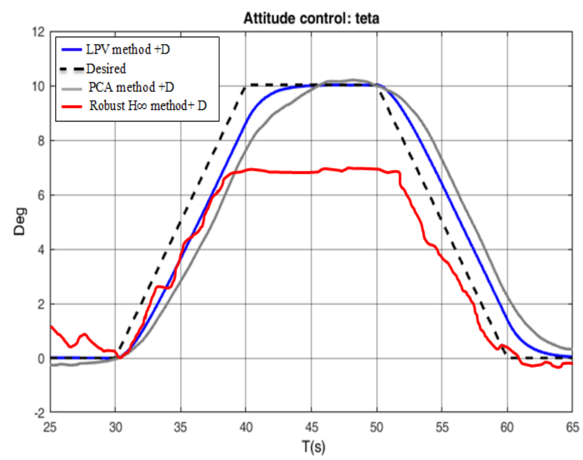


Fig. 14 Attitude control tracking of the angle θ. The continuous red line robust method in [18]. The continuous blue line: LPV method and continuous black line: PCA method, that introduced in this article.

This comparison properly shows a good improvement in tracking performance with less overshoot than the reference model that was reviewed, so that in the reference model there is a difference of about 5 degrees in the model.

5 CONCLUSIONS

In general, attitude control is one of the most critical stages of stability for a flight system. For this purpose, in this paper, an approximation algorithm for tracking attitude control of a helicopter is proposed to deal with the increase in the number of data produced and to reduce computational volume. The PCA algorithm showed excellent performance in this study. This is particularly well illustrated in trajectory tracking. Also, in this study different scenarios were examined. In both cases, with approximation (PCA) and without approximation (PLPV), the problem of stability in tracking is well demonstrated. In the design of the LPV controller presented, both modes of investigation are the same. That was one of the other challenges to solve. it is clear that although the huge number of vertexes in Polytopic model (extracted from linearization of nonlinear helicopter dynamic model) will increase the model dimensions and size, this technique is an acceptable way to track Euler angles and will have a lot of functionality in wide envelope flight control system design. Also, in comparison with the method introduced in the modeled reference [18], each of the two methods introduced in this paper had a much more acceptable performance than the compared model.

APPENDIX

$$K_{PLPV} = \begin{pmatrix} 0.0470 & 0.6921 & -0.0883 & 0.0021 & 0.3927 & -0.0764 \\ -1.7426 & 0.0900 & -0.0148 & -0.8535 & 0.0038 & 0.0114 \\ 0.1588 & 0.0313 & -0.0037 & 0.0766 & 0.0220 & -0.0054 \end{pmatrix}$$

$$\gamma_{PLPV} = 0.601$$

$$K_{PCA} = \begin{pmatrix} 0.2131 & 2.2974 & -0.2992 & -0.0055 & 0.7322 & 0.0029 \\ -1.5043 & 0.1667 & 0.0375 & -0.3364 & 0.0179 & 0.0391 \\ -0.0992 & 0.1182 & 0.1465 & -0.0220 & 0.0258 & 0.0555 \end{pmatrix}$$

$$\gamma_{PCA} = 2.3$$

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