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Determining the Optimum Investment Portfolios in the Iranian Banking Network Base on Bi-level Game using the Markowitz Optimization Model by Firefly Algorithm

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Abstract

The present study is presented in order to determine the optimal investment portfolios between a bank and its customers, in the form of a two-level game by Stackelberg (leader-follower). The game is based on the Markowitz mean-variance model. Leader player portfolios (Bank 3) have included deposit portfolios in rival banks (Banks 1 and 2), investment in the real estate market, investment in the stock market and investment in the foreign exchange market. Also follower player portfolios (Bank 3 customers), including deposits in rival Banks(1,2), investment in the coin and gold market, investment in the foreign exchange market, investment in the housing and real estate market, investment in the car market, investment in the stock market. The data related to the mentioned assets covered 2009-2017, where the optimal investment portfolios of the players was first determined using GAMS software. Next, the problem was solved again using the meta-heuristic algorithm of Firefly in Matlab Software. Eventually, the optimal technique was chosen. Finally, the results of the study showed that the optimal investment portfolios for the leader player include investing in the real estate market and investing in the stock market, respectively. Also, the optimal investment portfolios of the following player include depositing in Bank 2, investing in the coin and gold market, investing in the stock market and investing in the real estate market, respectively.

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Introduction

Selecting investment portfolios is one of the most common issues that various investors with different levels of capital always face. This can cover relatively small portfolios with few stocks, properties, etc. managed by typical investors to very large portfolios that include different types of assets and are managed by professional investors. (Mishra et al ,2016) proposes the key issue in selecting an asset portfolio is choosing the best possible combination of assets and determining the proper weight of each of them. (Mishra et al,2016) say that Selecting the optimal portfolio is difficult because of two reasons: i) investors have to first consider maximizing the efficiency of their portfolio while also managing the risk that is inherent in their chosen assets; ii) any investor in the portfolio selection process should consider different requirements in their investment decision-making. (Rezaei et al, 2019) propose that Portfolio optimization is one of the most important problems in investment. Further, considering the progressive development and complexity of financial markets, prediction methods are among the important factors in determining the profit and loss of investors.

In the today's challenging world, economic enterprises are also heavily competing with which other, and cannot make proper decisions only using traditional decision-making methods under certainty, risk, and uncertainty conditions in order to cope with both internal and foreign competitors. Thus, novel techniques should be identified under conflict conditions to compete with competitors. Use of such techniques helps economic evaluation of investment in investment portfolios and risk management, and aids the investor in better decision-making. Since investment in developing countries is associated with numerous risks because of the large and unknown variables and the many across a portfolio of assets and not merely one asset.

Stock portfolio is a type of investment consisting of several stocks. (Vesiani et al ,2020) believe that the aim of a stock portfolio is to minimize the investment risk and maximize the investment efficiency. The concepts of stock portfolio optimization and diversification can be analogous to a means for developing and understanding financial markets and decision-making. (Bahrisales et al, 2018) say that since Markowitz published his model, this model caused extensive changes and improvements in the way people viewed investment and start portfolio, and was employed as an efficient tool for optimizing the stock portfolio.

(Deng et al,2012) conclude that portfolio optimization is considered the main goal in risk management. Furthermore, the expected efficiencies and risk are the most important variables in portfolio optimization. Generally, investors prefer to maximize the efficiency while minimizing the risk. Nevertheless, high efficiencies are usually associated with high risk.

The level of risk and efficiency of financial assets are two important components in decision-making for investors in financial markets. Rationally, investors seek to maximize the efficiency at certain level of risk while minimizing the risk at a specific level of efficiency. Portfolio optimization refers to choosing the best combination of financial assets such that it would lead to the most desired investment portfolio efficiency plus minimized portfolio risk. In order to determine the optimal portfolio in the financial economics literature, two theories including modern portfolio theory and optimal portfolio determination theory based on undesired risk measures can be used. In the modern portfolio theory, optimal allocation of assets and identifying the optimal portfolio are performed according to optimization based on the mean and variance of patients (Khajezadeh et al, 2020). In another theory, optimal allocation of assets and identification of optimal portfolio occur based on the relationship between efficiency and undesired risk criteria (Paytakhti Oskouei et al, 2019). (Barkhordari & Rezaei , 2015) believe that rational investors seek for an efficient portfolio, since such portfolios would maximize the desired efficiency for a certain level of risk while minimizing the risk for a specific expected efficiency With these explanations, in this research which has been conducted based on Markowitz model and use of past data, firefly meta-heuristic algorithm has been used as the novelty of this research. Its usage can be due to the fact that firefly optimization algorithm is one of the suitable methods for solving multi-objective evolutionary problems. Since Markowitz model has been established on expected efficiency and portfolio risk indicators of investors, and according to Markowitz theory, an efficient investment portfolio is the one with the maximum efficiency and minimum risk, thus Markowitz model is a multi-objective optimization problem, in which maintaining the distribution diversity is important. Accordingly, firefly algorithm has been chosen as the optimization algorithm.

Therefore, (Vanayi et.al,2019) propose the utility of investors is a function of the expected efficiency and risk, with these two factors being the major parameters of decisions related to investment. (Eslami Bidgoli & Tayebi Thani , 2014) in their research used the criterion of risk value as an alternative index of variance, the basis of the stock portfolio selection model in Markowitz model. (Shadabfar &Cheng, 2020) made the optimal investment portfolio among the 7 Shanghai stocks over a 5-years period defined and probabilistically using the mean-variance method and the Monte Carlo simulation. The results of both definite and probabilistic methods were the same. (Bayat &Asadi, 2017) employed particle swarm optimization algorithm and Markowitz model for portfolio selection, and compared these two methods which each other. The comparison indicated that the particle swarm algorithm had less error in selecting the optimal investment portfolio compared to the Markowitz model. (Jiang et al, 2020) by using a differential approach, selected options for investing in tourism projects. (Sharma &Habib, 2019) examine the issue of creating a network between the shares of a market on the Indian Stock Exchange in 2014. They use the Markowitz model to show that selected environmental stocks using reciprocal information perform significantly better than stocks selected using correlation.

(Yiling Huang, 2020) proposes a method of optimizing investment in the portfolio of securities using the stochastic differential equation. In this paper, they use the Markowitz model to show that selected environmental stocks using reciprocal information perform significantly better than stocks selected using correlation.

(Shinzato & yasuda, 2015) used replica analysis and belief propagation algorithm for the portfolio optimization problem. The research results showed that the answers of the mean-variance model are consistent with the answers replica analysis and the belief propagation algorithm. (Wang et al, 2019) based on the Markowitz mean- variance model, discuss the portfolio selection problem in an uncertain environment. The results show that their proposed method is better and more practical than the E-V usual method. (Vanayi et.al,2019) developed a mathematical model for the production- the integrated distribution problem with a three-level supply chain including manufacturing factories, distribution centers, and customers for several types of products and in the course of several time periods. In order to consider the uncertainties in real problems, in the problem examined in this research, again some parameters including the costs in the model were converted to uncertain format using the Markowitz model, and eventually the model was solved with probable parameters using genetic algorithm. (Jurczyk et al,2016) study 37 major indicators of the US economy and use the mean-variance method to show that changes.

(Salehabadi et al, 2018) developed LMP-UPM model at different risk and potentiality levels using the indicators of all industries for the portfolio optimization. The optimized portfolio was compared against E-V model, and the performance was compared using Sharp ratio. The results indicated that LMP-Upm had a better performance. (Sina & fallahshams, 2019) performed their study entitled "optimizing the investment portfolio using extreme value theory in the securities exchange market of Tehran". They found that creating the optimal stock portfolio using extreme value theory does not have any significant difference with the Markowitz E-V model. (Rahnamay Roudposhti et al, 2017) presented a portfolio optimization model based on the sustainable Sharp ratio in Tehran securities exchange market. They found that the real efficiency in the Sharp model does not significantly differ with the real efficiency in Markowitz model.

(Tahmasbi, 2015) "estimated the risk of investment in an asset portfolio in Iran". In this study, the value at risk method was used to calculate the risk of investment in a household asset portfolio including bank deposits, corporate bonds, stocks, foreign-exchange, valuable coins, housing, and lands. Calculation of efficiency, efficiency standard deviation, and correlation coefficients between the efficiency of assets as well as the value at risk of each asset were extracted by applying the mean variance model of the optimal combination of assets. The results indicated that within the 14-year time horizon, the maximum risk of portfolio occurs for those with high-risk taking characteristics, while the individuals with low risk-taking levels would not experience any risk at any confidence level within this period. Further, within the one-year time horizon, the maximum risk of portfolio belongs to those with high-risk taking levels, and the minimum risk was found for those with low risk-taking degrees. (Mashayekhi & Omrani , 2016) conducted a study entitled "selecting multi-objective portfolio by combining Markowitz model and cross data envelopment analysis". They found that the proposed model significantly enhanced the efficiency as compared to the Markowitz model, while the portfolio efficiency diminishes slightly.

(Bayat & Abcher, 2015) investigated the relationship between decision-making models and expectations of investors about risk and investment efficiency in financial tools based on Markowitz model. They concluded that there is a positive relationship between the expected efficiency and tend to risk among investors. (Gholizadeh &Tahurimatin, 2011) conducted a study that the selection of the household asset portfolio was examined regarding the housing market for the first time in Iran. For that purpose, the data related to assets including stocks, foreign-exchange, valuable coins, banking deposits, securities, and housing were examined within the period of 1991 to 2006. After calculating the efficiency, risk, and correlation coefficients of the assets within the intended period, by applying the mean-variance model, the results indicated that housing is an important asset in the asset portfolio within the housing boom period, which would cause the efficiency boundary transfer. (Mushikhian &Najafi,2015) also developed Markowitz model and introduced mean-semi variance- skewness three-criterion model.

(Tehrani et al.,2018) performed stock portfolio optimization using Krill Herd Metaheuristic Algorithm using different criteria of risk in Tehran stock market. The findings suggested that initially the efficient boundaries of efficient portfolios have been drawn based on various risk, semi-variance, and expected fall criteria. The relative similarity of three efficient boundaries suggested stability of the algorithm in finding it. Next, the Sharpe ratios are obtained from the krill herd method were compared against particle swarm and imperialist competitive methods, which indicated that the krill herd method is preferred over them.

(Rahmani et al, 2020) performed stock portfolio selection by applying Artificial bee colony algorithm and compared it with genetic and ant algorithms. They found that the Sharpe criterion of the stock portfolio created through the Artificial bee colony algorithm had better performance compared to the genetic and ant algorithms.(Vesiani et al, 2020) performed portfolio optimization using priority index and genetic algorithm. Their results showed that increasing the value of the scale parameters does not always lead to enhanced mean efficiency. (Masoum Alishahi & Azami , 2018) conducted a research entitled optimization of stock portfolio based on Markowitz model. They observed that usage of heuristic methods, the performance of the portfolios created, and increased value of the input information for portfolio creation would guide the company towards investment, which is effective for increasing the efficiency for company.

(Deng et al., 2012) in their paper entitled "selecting start portfolio based on Markowitz model", concluded that particle swarm algorithm had better performance over genetic algorithm in selecting the stock portfolio.

As observed in the review of literature, most studies conducted on Markowitz model both in Iran and worldwide are related to determining the optimal investment portfolio in the stock portfolio, and limited research has been associated with determining the optimal investment portfolios in markets other than the stock market. Further, none of the researches detailed above have analyzed Markowitz model in a bi-level game between leader and follower.

With this explanation, in this study, using mean variance algorithm attributed to Markowitz (the mean as a criterion for efficiency, and variance as a criterion of risk, the optimal combination of assets in Iran (including banking investment, foreign-exchange market, gold and valuable coin market, housing and property, car, and stock market) is extracted for 2009-2017 using meta-heuristic algorithm in MATLAB software as well as GAMS software for two players:

leader (Bank 3) and follower (customers of Bank 3) in a bi-level game

2. Portfolio Optimization Models

The algorithms that exist to solve the optimization problems can be divided into two categories: precision algorithms and approximate algorithms. Exact algorithms are able to find optimal solutions accurately, but approximate algorithms are able to find near optimal solutions for difficult optimization problems and are divided into three categories of heuristic, meta-heuristic, and hyper-heuristic. The two main problems of the heuristic algorithms are their local optimality, and their inability to apply them to various problems. (zanjirdar,2020) propose that the meta-heuristic algorithms presented to solve the heuristic algorithms are a variety of approximate optimization algorithms that have local optimization solutions that are applicable to wide range of problems In (Table 1) Portfolio optimization models are showed.

Table 1. Portfolio optimization models (zanjirdar,2020)

Meta- Heuristic Models			Mathematical Models		
Particle swarm Algorithm	Krill Herd Algorithm	Genetic Algorithm	Markowitz Model	Value at risk (VAR)	Conditional Value at risk (CVAR)
Firefly Algorithm	Ant colony Algorithm	Artificial Neural Network	Fuzzy Approach	Linear Programming	Game Theory
Bee colony Algorithm	Whale optimization Algorithm	Simulated Annealing	----	-----	-----

On the table1, in this paper Markowitz model answers in Mathematical Models is compared to Markowitz model answers by Firefly Algorithm and the optimal answer is selected.

The present research is a library research based on the approach of examining theoretical principles. Based on real data, it deals with an economic evaluation of investment options in available portfolios between the leader player (bank 3) and follower player (bank 3 customers). Evidently, each of the players seeks to maximize their utility in the available markets including banking deposits, foreign-exchange market, valuable coins and gold market, real estate market, car market, and securities or stock market. In this section, the profit and loss rate of deposits in bank 3 and its competitors (banks 1 and 2, which have been the strongest competitors of Bank 3 on gaining profit based on their financial statements in Codal website) as well as the loss and profit rate of parallel markets of depositing in the banking system including the real estate market, foreign-exchange market, valuable coins and gold market, car market, and stock market for investment of customers in 2009-2017 have been investigated. Specifically, the strategies of bank 3 customers alongside their loss or profit status are calculated.

The investment strategies of customers in the investment portfolios are in the form of Figure 1:

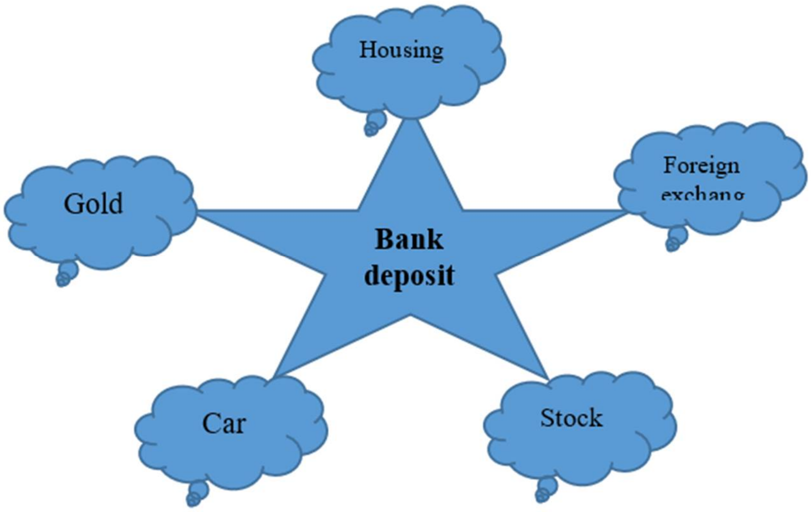

Figure 1. Investment strategies of customers in the investment portfolios (Iran)

Table 2 reports the interest rate on banking deposits from the three banks of interest in this research based on their financial statements in 2009-2017.

Table 2. The interest rate on banking deposits across the Iranian banking systems in 2009-2017

No.	Bank name	(%) The interest rate of timed deposits within 2010-2017							
1		2010	2011	2012	2013	2014	2015	2016	2017
2	A	2.1	2	12.8	13.65	16.05	16.64	15.3	14.14
3	B	10.67	9.94	11.68	10.97	15.2	15.63	15.7	13.55
4	C	7.53	8.05	11.61	14.87	16.47	17.66	16.38	19

Table 3. The loss and profit rate of parallel markets with banking deposits, including the housing market, foreign-exchange market, valuable coins and gold market, car market, and stock or securities market in 2009-2017

Year	Loss/profit rate in parallel markets (%)				
	Capital market	Car market	Valuable coins and gold market	Housing market	Foreign exchange market
2009	68	2.5	24	-13	4
2010	87	2.44	51	-7	10
2011	0	3.57	45	12	64
2012	60	95.4	44	28	111
2013	108	8.82	16	31	-5
2014	-21	10	-9	10	-1
2015	28	4.18	-3	-8	1
2016	-4	6.13	25	3	-5
2017	29	4.89	37	12	24

2.1. Introducing Markowitz model

Markowitz model is a nonlinear programming model based on the mean and variance of efficiency of assets, which is based on the presumption of normal distribution of asset efficiency. Based on this model, risk is associated with efficiency fluctuations, and the fluctuations are measured based on the efficiency variance. The efficiency rate of a portfolio consisting of different assets is obtained based on the weighted mean of the individual assets constituting that portfolio according to Equation 1:

$$E(R_P) = \sum W_i \, E(R_i) \qquad (1)$$

In Relation (2), $E(R_P)$, represents the portfolio efficiency rate, R_i indicates the asset efficiency rate, and Wi denotes the weight of assets in the portfolio. The intended portfolio risk is also obtained by Relation (2)

$$Min\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \,\sigma_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j SR_i SR_j r_{ij} \; (2)$$

In Relation (2), σ_p^2 represents the portfolio efficiency variance, SR_i and SR_j denote the standard deviation of efficiency of assets i and j respectively, σ_{ij} indicates the covariance between the efficiency of assets, W_i and W_j show the weight of assets i and j in the portfolio respectively, and n indicates the number of assets present in the portfolio.

Based on this model, individuals maximize the expected efficiency of the portfolio considering a fixed risk; alternatively, they minimize the risk of portfolio considering a fixed expected efficiency. Thus, we use the nonlinear programming model as follows (Relation 3):

$$Min\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \,\sigma_{ij}$$
$$= \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j SR_i SR_j r_{ij} \qquad (3)$$

S.t:

$$E(R_P) = \sum W_i \, E(R_i) \;\; (4)$$

$$\sum_{i=1}^{n} W_i = 1 \;\; (5)$$

$$W_i \geq 0 \;\; (6)$$

Where, $E(R_i)$ indicates the expected efficiency rate of each asset, $E(R_p)$ shows the expected efficiency rate of the portfolio, σ_{ij} denotes the covariance between the efficiency of i_{th} and j_{th} assets, and W_i represent the share of each asset in the portfolio.

Relation (4) indicates the expected efficiency of the portfolio, while Relation (5) shows that the entire budget of the person is invested. Relation (6) represents the positive weights on each asset in the portfolio, suggesting no short sell. By solving this model, W_i and W_j (weight of assets), which are the decision variables, are identified (vaezi et al, 2020).

Accordingly, the main assumptions of Markowitz model are:

- Investors are risk-averse and have an incremental expected utility, and the final utility curve of their wealth is diminishing.

- Investors choose their investment portfolio based on the expected mean and variance of efficiency. Hence, their indifference curves are a function of the expected variance and efficiency rate.
- Every investment option is divisible ad infinitum.
- The time horizon of all investors is the same and one period
- Investors prefer a higher level of efficiency at a certain level of risk, and vice versa for a certain level of efficiency, they prefer to have the minimum risk.

2.2. Introducing Bi-level programming problems

(Talizadeh et al,2013) explain that in general, decisions in the supply chain are made in two forms: centralized and decentralized. In a centralized supply chain, a single decision maker or core member who has access to sufficient information in the supply chain and has the necessary decision-making power makes policy-making for all members of the chain. In this case, the members work together in line with the defined policy. In the subject literature, standard models of mathematical programming have answered such problems.

But since the members of the supply chain are often separate organizations and independent enterprises, despite the benefits of integrated decision-making in practice, reluctant to follow decisions. They do not have it adopted for all members and try to optimize their goals instead of the goal of the whole system. (Pontrandolfo & Giannoccaro,2004). Thus, in many real-world problems, the decentralized supply chain is a practical model close to reality.

In the literature Review, a Hierarchical Decision-Making (HDM) system has been used to model decentralized supply chain management issues. In this system, first the high level member decides and then the low level member determines his optimal strategy by observing the decision made. Assuming complete information about how a low-level member makes decisions, the high-level decision-maker will be able to predict the low-level decision-maker's reaction before deciding on her strategy and make a decision accordingly. In this method, the top level decision maker is known as the leader and the low level decision maker is known as the follower. The hierarchical decision-making process in economics goes back to Stackelberg's famous game in game theory, first introduced in 1952 by (Von Stackelberg, 1952).

(Koh, 2013) explains that hierarchical programming is a mathematical framework for displaying Stackelberg games in which several optimization problems at different levels are considered simultaneously. (Amir taheri et al, 2016) explain that In a special case, if the problem has two levels, it is called a Bi-level problem or a Bi-level programming problem. The Bi-level programming problem is a special case of mathematical programming models in which one optimization problem falls within the constraints of another optimization problem.

Each of the two decision makers (leader and follower) tries to determine the values of the decision variables under their control, optimize their objective function without considering another objective function. But the decision of each player affects the selectable options and the objective function of the other decision maker (player). Thus, the leader, as a high-level decision maker, will be able to influence the behavior of the follower and control her behavior simultaneously without complete control over her (Amir Taheri et al ,2016) Equation 7 is one of the bi-level problems.

$$\min_{x \subseteq x,y} F_{(x,y)}$$
$$G_{(X,Y)} \leq 0$$
$$st: MIN_Y f_{(x.y)}$$
$$g_{(x,y) \leq 0}$$

3. Theory/Calculation

Accordingly, (Table 4) reports the characteristics of the symbols of the leader- follower bi-level game modeling based on the Markowitz model between bank 3 and its customers.

Table 4. The characteristics of the symbols for modeling the leader - follower bi-level game based on the Markowitz model between bank 3 and its customers

Index (symbol)	Definition of symbols
N	The number of portfolios of the leader player
M	The number of portfolios of the follower player
j , i	The index of portfolios of the leader player
t , k	The index of portfolios of the follower player
ER_i	Investment efficiency in the investment portfolios by the leader player
SR_i	Risk (SD) of investment in the investment portfolios by the leader player
ER_k	Investment efficiency in the investment portfolios by the follower player
SR_k	Risk (SD) of investment in the investment portfolios by the follower player
r_{ij}	Correlation coefficient between the efficiency of i_{th} and j_{th} assets of the leader player
r_{kt}	Correlation coefficient between the efficiency of k_{th} and t_{th} assets of the follower player
W_i	Weight of assets of the leader player’s portfolio
V_i	Weight of assets of the follower player’s portfolio
$E_{(}RP_{L)}$	Expected efficiency rate of the leader player’s portfolio
$E_{(}RP_{F)}$	Expected efficiency rate of the follower player’s portfolio
δ_L^2	Efficiency variance of the leader player portfolio
δ_F^2	Efficiency variance of the follower player portfolio
$\delta_T^2 = \delta_L^2 + \delta_F^2$	Total variance of the portfolio efficiency

Overall, such multilevel problems are considered to be NP-hard, for which precise methods cannot be used. To solve such problems, authors and researchers apply meta-heuristic methods based on optimizing hybrid problems. As such, this research has used Firefly algorithm to solve the research model. Thus, in this research, Firefly algorithm optimization and mathematical model have been used to solve the research model. Eventually, the obtained solutions are compared with each other, and then the optimal portfolios of investment for the leader and follower players have been chosen.

Firefly algorithm (FA) is one of the important tools in swarm intelligence and has extensive applications in various areas of optimization. This algorithm first propounded by (Yang , 2014) and employed has been inspired by the behavior of fireflies.

The behavior of fireflies

Fireflies appear in warm climates and in the sky of summer nights. So far, around 2000 different species of fireflies have been recorded worldwide. Fireflies emit the energy stored in them as patterns of light using chemical mechanisms, with the patterns also called flashing light. Most fireflies generate rhythmic and short flashing lights. The type of the flesh pattern also differs across different species, and fireflies only respond to the patterns of their own species. Typically, each type of firefly species generates unique flashing patterns. The flashing lights emitted by fireflies are a result of a biological process known as bioluminescence, causing the noctilucence among the fireflies. Accordingly, the two fundamental functions of the bioluminescence process and its resulting noctilucence are as follows:

- absorbing the opposite sex for mating and reproduction (the communication function) of the bioluminescence. In the attraction process, the firefly is attracted to another firefly who has a stronger flash among the others. The intensity of the flashing light in this algorithm is in proportion to the value of objective function that should be optimized (Bahrampour, 2016).
- Extracting the possible preys towards them

One of the points that should be considered about the light flashing pattern of fireflies is that the light intensity at a specific distance, r, from the light source, follows inverse square law. In other words, the light intensity, I, decreases with elongation of the distance, r, according to the relation $I\propto1/r^2$. In addition, the air absorbs the light causing the light intensity to attenuate further with increasing distance. The flashing light generated by fireflies can be formulated such that it would match an objective function to be optimized by optimization algorithms. This would allow researchers to formulate and implement new optimization algorithms.

The concepts related to the firefly algorithm

In this section, in order to implement the firefly inspired optimization algorithm, the characteristic features related to the behavior of fireflies and the flashing light pattern generated by them will be formulated. In order to simplify the formulation of this algorithm, the following rules are applied:

1. All fireflies are unisex. This means that fireflies, irrespective of their gender, will be attracted to other fireflies present in the problem space.
2. In the firefly algorithm, the attractiveness of a firefly using proportion with the brightness of that bug. In other words, per every two flashing fireflies, the one with less brightness would be attracted to the bug with more brightness. Thus, the attractiveness of the firefly will be in line with its brightness.

- When the distance between two fireflies increases, the extent of attractiveness and brightness of them diminishes. In other words, when the distance between two fireflies from each other grows, both the attractiveness for each other and their brightness (visible) for each other decrease. In case the brightness of a specific firefly is higher than that of other bugs, it would move across the environment randomly (it would not be attracted towards any of the other fireflies).

3. The brightness of a firefly is affected by the characteristic features of the objective function, or it would be identified through it. In maximization problems, brightness can be identified in line with the value of the fitness function. Note it is possible to define the brightness of fireflies in a

similar way to the fitness function in genetic algorithms.

Firefly algorithm (FA)

- Objective function. $f(x), x=(x_1,.......,x_d)^T$.
- Generation of the primary population from the fireflies. $x_i,(i=1,2,3,.....,n)$.
- The light intensity parameter I_i in x_i is obtained through replacing the value of x_i in $f(x_i)$.
- Defining the light absorption coefficient or γ.
- Loop: while (t <Max Generation)
- For loop (for i=1 to n) per all n of the fireflies available in the population
- For loop (for j=1 to i) per all n fireflies available in the population
- If condition: if ($I_j > I_i$), then firefly i would be driven towards the firefly j in the d-dimensional space of the problem (end of condition if).
- The value of attractiveness parameter is calculated and updated based on distance r and according to relation exp(–γr)
- The new solutions (candidate solutions) are evaluated and the parameter of light intensity is updated.
- End of for loop
- End of for loop
- End of while loop
- The final solutions are postprocessed and visualizations related to the final output are performed.
- Attractiveness in FA algorithm
- When formulating the FA algorithm, two important issues should be considered: "variation of light intensity" and "formulating the mutual attractiveness of fireflies. To simplify the formulation process of FA algorithm, it can be assumed that the attractiveness of a firefly is identified based on the brightness of this agent (firefly); the brightness of the firefly is in turn associated with the encoded objective function to solve a special problem.
- In the simplest case and for a maximization problem, the value of the brightness parameter I of a firefly, located in special place X, can be obtained through $I(X) \propto f(x)$ relation. Meanwhile, the value of attractiveness parameter β of a firefly is relative and should be identified by other fireflies (i.e. the distance between fireflies plays a direct role in their attractiveness (getting attracted towards each other). Here, the more attractive firefly will attract the one with less attractiveness. This means that poorer solutions in the problem would move towards the stronger solutions of the problem (Bahrampour ,2016). Thus, the attractiveness parameter β, will vary based on the distance r_{ij} between firefly i and firefly j.
- In addition, as the distance with the light source increases, the light intensity would decline. Further, when passing through media such as air, the light would be absorbed by them. Thus, the FA should be formulated each such that the value of attractiveness parameter would change in line with different degrees of absorption.
- In the simplest possible case, the light intensity parameter I(r) would also change according to the inverse square law:

$$I_{(r)} = \frac{I_s}{r^2}$$

- In this law, I_s parameter indicates the light intensity in the source. Having a medium (such as air) with a constant light absorption coefficient, γ, the light intensity I would change based on distance r and according to the following relation:

$$I = I_0 e^{-\gamma r}$$

- In this relation, I_0 represents the main light intensity. To prevent incidence of singularity in the relation I_s/r^2 and at point r=0, the combined effect of inverse square law and the light absorption principle by the air medium can be approximated using the following function which is known as Gaussian form:

$$I_{(r)} = I_0\, e^{-\gamma r^2}$$

- Sometimes, a function is required that would decrease steadily and at a lower rate. In such a case, the following approximation can be used:

$$I_{(r)} = \frac{I_0}{1+\gamma r^2}$$

- Within a short distance (when the distance between two fireflies is very low), the two forms provided by I(r) function are almost equal to each other. Such a phenomenon is due to the fact that for r=0, series expansions of these two functions would be equivalent to each other up to order O(r3):

$$\frac{1}{1+\gamma r^2} \approx 1-\gamma r^2+\frac{1}{2}\gamma^2 r^4+\cdots$$

$$e^{-\gamma r^2} \approx 1-\gamma r^2+\frac{1}{2}\gamma^2 r^4+\cdots$$

- Since the attractiveness of a firefly is in proportion with the light intensity observed by the neighboring fireflies, the attractiveness parameter β of a firefly can be defined by the following relation:

$$\beta_{(r)} = \beta_0\, e^{-\gamma r^2}$$

- In this relation, β_0 represents the attractiveness of firefly at r=0. Since calculation of $1/(1+r^2)$ relation is faster than calculating an exponential function, instead of the above function, the following one can be employed, and the value of attractiveness parameter can be calculated by the following relation:

$$\beta_{(r)} = \frac{\beta_0}{1+\gamma r^2}$$

- The above relation defines a parameter known as characteristic distance or length scale as $\Gamma=\frac{1}{\sqrt{\gamma}}$, through which the attractiveness parameter would considerably change from β_0 form to $\beta_0 e^{-1}$.

- When implementing the firefly algorithm (and its different versions), β(r) function can also be represented as monotonically decreasing functions such as the following generalized form:

$$\beta_{(r)} = \beta_0\, e^{-\gamma r^m} \text{ (m≥1)}$$

- In case parameter γ is assumed constant, when m→∞, the characteristic distance or length scale parameter would change into $\Gamma=\gamma^{-1/m}$. Meanwhile, by knowing the value of the characteristic distance or length scale in an authorization problem, Γ parameter can be in the form of $\gamma=\frac{1}{\Gamma^m}$ and as a typical initial value.

The distance and movement in the FA

The distance between two fireflies i and j, located at coordinates x_i snd x_j, is equal to the Cartesian distance between them as follows:

$$r_{ij} = \|x_{i-}\, x_j\| = \sqrt{\sum_{d}^{k=1}(x_{i,k}-x_{j,k})^2}$$

In this relation, xi,k is equal to the k-th component of the special coordinate associated with the i-th firefly (xi). In a two-dimensional search space, the distance between two fireflies i and j is calculated by the following relation:

$$r_{ij} = \|x_{i-} x_j\| = \sqrt{(x_i - x_j)^2 - (y_i - y_j)^2}$$

The firefly movement (for example firefly i) attracted to a more attractive firefly (brighter), is calculated by the following relation:

$$X_i = X_i + \beta_0 \, e^{-\gamma r_{ij}^2}(x_j - x_i) + \alpha(rand - \frac{1}{2})$$

Here, the second term on the right-hand of the relation indicates the effect of attractiveness parameter on calculating the value of movement by firefly i. Further, the third term is employed for randomization, in which α plays the role of randomization parameter. rand term is also a random number generator, whereby the random values generated by it follow a uniform distribution within [0,1].

In the implementations performed from the firefly algorithm to solve optimization problems, the value of β_0 has been considered 1, while the value of α is chosen from within $\alpha \in [0,1]$. In addition, the value of the randomization term (the third term in the above relation) can be calculated based on a normal distribution as N(0,1) or any other desired distribution.

Furthermore, in conditions when numerical scales of the problem variables have considerable differences with each other, for example when the numerical scale of one of the problem variables is $[10^{-5},10^{5}]$ while the numerical scale of another variable is [[−0.01,0.01], then it is better to replace parameter α in the randomization term with αS_k parameter. The components of S_k in this parameter are called scaling parameters. In case the optimization problem of interest is d-dimensional, that scaling parameters S_k(K=1,2,...,d) should be identified based on the real scales of the optimization problem.

With regards to identifying the movement of fireflies, parameter γ specifies the variations in the attractiveness of fireflies in FA. Furthermore, the value of this parameter plays a significant role in specifying the convergence rate, behavior of the firefly algorithm in the search of the problem space, and solving the given optimization problem. In theory, the value of parameter γ is identified through $\gamma \in [0,\infty)$, but in practice this parameter is initialized by γ=0(1) . This value is specified through the characteristic distance or length scale parameter (Γ parameter) of the system to be optimized. Hence, in most applications, this parameter adopts a value between 0.01 and 100.

3.1. Leader- follower bi-level model using Markowitz model

In order to determine the leader-follower bi-level model of investment between bank 3 and its customers using Markowitz model, first the data related to this model, covering the data of 2009-2017, were extracted according to (Table 5) and (Table 6). Table 5 reports the data over the leader- follower equation of the bank and customers in order to determine the optimal investment portfolio using Markowitz model within 2009-2017.

3.2. Determining the leader-follower model of investment between Bank (3) and customers based on Markowitz model

Leader-follower bi-level model of investment between bank (3) and its customers, according to Markowitz model was developed based on the study by Venayi et al. [5] as Equation 8:

$$Min\delta_T^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j SR_i SR_j r_{ij} + \sum_{k=1}^{m}\sum_{t=1}^{m} V_k V_t SR_k SR_t r_{kt} \quad (8)$$

S.t:

$$E(RP_L) = \sum_{i=1}^{n} W_i \, E(R_i)$$

$$\sum_{i=1}^{n} W_i = 1$$

$$E(RP_F) = \sum_{k=1}^{m} V_k \, E(R_k)$$

$$\sum_{k=1}^{m} V_k = 1$$

$$W_i \geq 0, V_k \geq 0$$

Eventually, after solving the above model using genetic algorithm in MATLAB software as well as GAMS, the answer of the unknowns of the problem has been obtained according to Table 7.

Table 5. The data of leader- follower equation for the bank and customers in order to determine the optimal investment portfolio using Markowitz model (data: 2009-2017)

no	Leader (bank) data	Portfolio No. n=1 to 4	Symbol	Value	Follower (customers)	Portfolio No. n=1 to 8	Symbol	Value
1	Efficiency of investment in stocks market	1	ER_1	0.081	Efficiency of investment in stocks market	1	ER_1	0.350
2	Risk of investment in stocks market	1	SR_1	0.132	Risk of investment in stocks market	1	SR_1	0.385
3	Efficiency of investment in other banks	2	ER_2	0.48	Efficiency of investment in real estate	2	ER_2	0.199
4	Risk of investment in other banks	2	SR_2	0.356	Risk of investment in real estate	2	SR_2	0.232
5	Efficiency of investment in foreign exchange currency (US dollar)	3	ER_3	0.991	Efficiency of investment in valuable coins/gold market	3	ER_3	0.258
6	Risk of investment in foreign exchange currency (US dollar)	3	SR_3	0.664	Risk of investment in valuable coins/gold market	3	SR_3	0.212
7	Efficiency of investment in real estate	4	ER_4	0.957	Efficiency of investment in foreign exchange currency (US dollar)	4	ER_4	0.248
8	Risk of investment in real estate	4	SR_4	0.121	Risk of investment in foreign exchange currency (US dollar)	4	SR_4	0.390
9					Efficiency of investment in car market	5	ER_5	0.169
10					Risk of investment in car market	5	SR_5	0.298
11					Efficiency of investment in bank (1)	6	ER_6	0.782
12					Risk of investment in bank (1)	6	SR_6	1.887
13					Efficiency of investment in bank (2)	7	ER_7	0.047
14					Risk of investment	7	SR_7	0.166

					in bank(2)			
15					Efficiency of investment in bank (3)	8	ER_8	0.151
16					Risk of investment in bank (3)	8	SR_8	0.154

Table 6. Correlation coefficients and covariance of the efficiency of investment portfolios of leader and follower players

Player's name	Name/index of the first market	Name/index of the second market	Covariance	Correlation coefficient
Leader	Stocks	Stocks	COV_{11}= 0.019	R_{11}=1
	Stocks	Banking deposition	COV_{12}= -0.015	R_{12}= -0.287
	Stocks	Foreign exchange	COV_{13}= 0.009	R_{13}= 0.377
	Stocks	Real estate	COV_{14}= -0.007	R_{14}= -0.324
	Banking deposition	Banking deposition	COV_{22}= 0.142	R_{22}=1
	Banking deposition	Foreign exchange	COV_{23}= 0.184	R_{23}=0.846
	Banking deposition	Real estate	COV_{24}= 0.017	R_{24}=0.377
	Foreign exchange	Foreign exchange	COV_{33}= 0.588	R_{33}=1
	Foreign exchange	Real estate	COV_{34}= 0.034	R_{34}=0.286
	Real estate	Real estate	COV_{44}= 0.017	R_{44}=1
Follower	Stocks	Stocks	COV_{11}= 0.191	R_{11}=1
	Stocks	Property	COV_{12}= 0.057	R_{12}=0.523
	Stocks	Gold coin	COV_{13}= 0.035	R_{13}=0.350
	Stocks	Foreign exchange	COV_{14}= -0.001	R_{14}= -0.005
	Stocks	Car	COV_{15}= 0.013	R_{15}=0.096
	Stocks	Bank 1 deposit	COV_{16}=0.148	R_{16}=0.203
	Stocks	Bank 2 deposit	COV_{17}= -0.027	R_{17}= -0.422
	Stocks	Bank 3 deposit	COV_{18}=0.038	R_{18}=0.634
	Property	Property	COV_{22}= 0.062	R_{22}=1
	Property	Coins	COV_{23}= 0.006	R_{23}=0.108
	Property	Foreign exchange	COV_{24}= 0.047	R_{24}=0.454
	Property	Car	COV_{25}= 0.050	R_{25}=0.632
	Property	Bank 1 deposit	COV_{26}=0.267	R_{26}=0.602
	Property	Bank 2 deposit	COV_{27}=0.001	R_{27}=0.028
	Property	Bank 3 deposit	COV_{28}=0.031	R_{28}=0.860
	Coins	Coins	COV_{33}= 0.051	R_{33}=1
	Coins	Foreign exchange	COV_{34}= 0.055	R_{34}=0.582
	Coins	Car	COV_{35}= 0.02	R_{35}=0.287
	Coins	Bank 1 deposit	COV_{36}=0.158	R_{36}=0.415
	Coins	Bank 2 deposit	COV_{37}= -0.172	R_{37}= -0.512
	Coins	Bank 3 deposit	COV_{38}=0.01	R_{38}=0.307
	Foreign exchange	Foreign exchange	COV_{44}= 0.174	R_{44}=1
	Foreign exchange	Car	COV_{45}= 0.107	R_{45}=0.809
	Foreign exchange	Bank 1 deposit	COV_{46}=0.639	R_{46}=0.820
	Foreign exchange	bank 2 deposit	COV_{47}=0.004	R_{47}=0.058
	Foreign exchange	Bank 3 deposit	COV_{48}=0.039	R_{48}=0.620
	Car	Car	COV_{55}= 0.101	R_{55}=1
	Car	bank 1 deposit	COV_{56}=0.589	R_{56}=0.999
	Car	bank 2 deposit	COV_{57}=0.019	R_{57}=0.360
	Car	Bank 3 deposit	COV_{58}=0.038	R_{58}=0.785
	Bank 1 deposit	Bank 1 deposit	COV_{66}=3	R_{66}=1
	Bank 1 deposit	Bank 2 deposit	COV_{67}=0.11	R_{67}=0.351
	Bank 1 deposit	Bank 3 deposit	COV_{68}=0.227	R_{68}=0.781
	Bank 2 deposit	Bank 2 deposit	COV_{77}=0.027	R_{77}=1
	Bank 2 deposit	bank 3 deposit	COV_{78}=0.024	R_{78}=0.153

Table 7. The answer of unknowns for the leader-follower bi-level problem using Markowitz model

Unknowns of leader (bank 3)	Portfolio No. n=1 to 4	Symbol	Value	Follower (customers)	Portfolio No. m=1 to 8	Symbol	Value
The expected efficiency rate for the leader portfolio		$E_{(}RP_{L)}$	0.528	The expected efficiency rate for the follower portfolio		$E_{(}RP_{F)}$	0.145
Weight of assets of the leader portfolio	(1) Stocks	W_1	0.473	Weight of assets of the follower portfolio	(1)Stocks	V_1	0.08
	(2)Banking deposits	W_2	0.0284		(2)Property	V_2	0.022
	(3)Foreign exchange	W_3	0		(3)Valuable coins and gold	V_3	0.335
	(4)Property	W_4	0.497		(4)Foreign exchange	V_4	0
Variance of leader's portfolio efficiency		δ^2_L	0.0053		(5)Car	V_5	0
					(6)Bank deposit (1)	V_6	0
					(7)Bank deposit (2)	V_7	0.562
					(8)Bank deposit(3)	V_8	0
			Variance of follower's portfolio efficiency			δ^2_F	0.0073
Total variance of the portfolio efficiency		δ^2_T	$\delta^2_T = \delta^2_L + \delta^2_F$				**0.0126**

As shown in (Table 7), based on the results of this table, the optimal portfolio for the leader (bank 3) regarding investment contains investment in real estate (W_4=0.497), investment in securities market (W_1=0.473), and investment in other banks (W_2=0.0284), respectively, while investment in the foreign exchange market is not economically justified. On the other hand, the optimal portfolio for the follower player (customers of bank 3) regarding investment include investment in bank 2 (V_7=0.562), investment in valuable coins and gold market (V_3=0.335), investment in securities market (V_1=0.08), and investment in real estate (V_2=0.022), while investment in other parallel markets (foreign-exchange and car) or investment in banks (1) and (3) are not economically justified in this model. The final and optimal value of the objective function, 0.0126, indicates the minimum total variance of the efficiency of investment portfolio for both leader and follower players.

Next, after solving the above model using FireFly algorithm in MATLAB software, the unknowns of the problem have been obtained according to (Table 8).

Table 8. The findings of unknowns of the leader-follower bi-level problem using Markowitz model and Firefly algorithm solution method

Unknowns of leader (bank 3)	Portfolio No. n=1 to 4	Symbol	Value	Follower (customers)	Portfolio No. m=1 to 8	Symbol	Value
The expected efficiency rate for the leader portfolio		$E\,(RP_L)$	0.5283	The expected efficiency rate for the follower portfolio		$E\,(RP_F)$	0.1427
Weight of assets of the leader portfolio	Stocks(1)	W_1	0.4739	Weight of assets of the follower portfolio	Stocks(1)	V_1	0.08
	Banking deposits(2)	W_2	0.0285		Property(2)	V_2	0.0178
	Foreign exchange(3)	W_3	0		Valuable coins and gold(3)	V_3	0.3260
	Property(4)	W_4	0.4976		Foreign exchange(4)	V_4	0
Variance of leader's portfolio efficiency		δ^2_L	0.0053		Car(5)	V_5	0
					Bank (1) deposit (6)	V_6	0
					Bank (2) deposit (7)	V_7	0.5762
					Bank (3) deposit (8)	V_8	0
			Variance of follower's portfolio efficiency			δ^2_F	0.007
Total variance of the portfolio efficiency		δ^2_T	$\delta^2_T = \delta^2_L + \delta^2_F$				**0.0123**

As shown in (Table 8), based on the results of this table, the optimal portfolio for the leader (bank 3) regarding investment contains investment in real estate (W_4=0.4976), investment in securities market (W_1=0.4739), and investment in other banks (W_2=0.0285), respectively, while investment in the foreign exchange market is not economically justified. On the other hand, the optimal portfolio for the follower player (customers of bank 3) regarding investment include investment in bank 2 (V_7=0.5762), investment in valuable coins and gold market (V_3=0.3260), investment in securities market (V_1=0.08), and investment in real estate (V_2=0.0178), while investment in other parallel markets (foreign-exchange and car) or investment in banks (1) and (3) are not economically justified in this model. The final and optimal value of the objective function, **0.0123**, indicates the minimum total variance of the efficiency of investment portfolio for both leader and follower players.

Figure 2 indicates the convergence of FireFly algorithm for the leader-follower bi-level problem using Markowitz model. As can be observed, in this diagram, after around 20 iterations, the value of objective function reaches its saturated or optimal state.

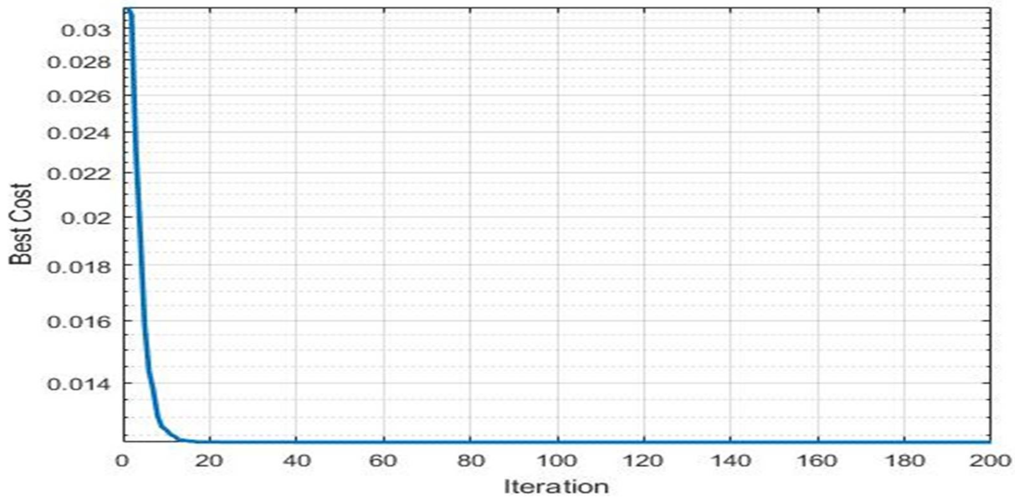

Figure 2. Convergence of FireFly algorithm for the leader-follower bi-level problem using Markowitz model

4. Results

After solving the leader-follower problem for investment between bank (3) and its customers according to Markowitz model, using mathematical model by GAMS software and FireFly algorithm by Matlab sofware, the results show that the objective function of the firefly algorithm has a better and more appropriate response.

Comparison of results mathematical model by software Gams and results FireFly algorithm by Matlab Software is presented in Table 9.

According to the results of (Table 9), the return on investment in the bi-level Markowitz model using the firefly algorithm is higher than mathematical modeling, and also the total investment risk in the bi-level Markowitz model using the firefly algorithm is less than mathematical modeling.

Therefore, the optimal answer to the problem is the same as the answers of the firefly algorithm, and the final optimal answer will be equal to 0.0123.

Also, Useful information about the model and solution process have been presented in Table 10.

4.1. Investigating the research hypotheses

As noted, the firefly algorithm has a better answer than mathematical model. But there have been no significant differences between the weights of investment portfolios of the leader and follower players, to examine the research hypotheses. The results obtained from solving the problem of investment portfolios of the leader and follower players based on the Firefly algorithm and mathematical model have been presented in Table 11.

Hypothesis 1: The strategy of bank 3 (leader) in not investing in competitor banks is optimal for the profitability of this bank. Based on the results of (Table 10), it can be concluded that since investment in the real estate market is considered the most profitable action to be taken by the leader player (bank 3), followed by investment in the securities market as well as in other banks, hence the strategy of Bank (3) as not investing in competitor banks is optimal for its profitability, and thus the first research hypothesis is also confirmed.

Hypothesis 2: The strategy of customers (followers) in banking deposition/investment for their profitability is optimal.

Based on the results of (Table 11), we find that since investment in bank (2) is considered the most profitable act by the follower players, investment of customers in banks is better than investment in other markets, and brings less risk to the customers. Thus, the strategy of customers (follower) in banking deposition is optimal for their profitability, and thus the second research hypothesis is confirmed.

Table 9. Comparing the results obtained from mathematical model and FireFly algorithm in solving the leader- follower model problem for investment between bank (3) and its customers according to Markowitz model

NO	Variable	Variable response with FireFly algorithm	Variable response with Mathematical model
1	$E_{(}RP_{L)}$	0.5283	0.528
2	W_1	0.4739	0.473
3	W_2	0.0285	0.0284
4	W_3	0	0
5	W_4	0.4976	0.497
6	δ^2_L	0.0053	0.0053
7	$E_{(}RP_{F)}$	0.1427	0.145
8	V_1	0.08	0.08
9	V_2	0.0178	0.022
10	V_3	0.3260	0.335
11	V_4	0	0
12	V_5	0	0
13	V_6	0	0
14	V_7	0.5762	0.562
15	V_8	0	0
16	δ^2_F	0.0070	0.0073
17	$\delta^2_T= \delta^2_L+ \delta^2_F$	0.0123	0.0126

Table 10. Useful information about the model and solution process

CPU time	Number of variables	Number of constrains	Max iteration
10.46(s)	17	6	200

Table 11. Prioritization of the optimal portfolios in solving the leader-follower model for investment between bank (3) and its customers according to Markowitz model and using FA and Mathematical Model

No.	Variable	Prioritization of portfolios using FA	Prioritization of portfolios using Mathematical model
1	Leader player	Properties	Properties
2		Securities exchange market	Securities exchange market
3		Investment in other banks	Investment in other banks
4	Follower player	Investment in bank (2)	Investment in bank (2)
5			
6		Investment in gold and valuable coins market	Investment in gold and valuable coins market
7		Investment in securities exchange market	Investment in securities exchange market
8		Investment in properties	Investment in properties

5. Discussion

Regarding investment in portfolios (investment portfolios) by the bank (3) and its customers, the research results indicated that the optimal investments by bank (3) include investment in real estate, securities market, and competitor banks respectively (in the order of priority), while investment in the foreign exchange market within the studied years did not prove to be profitable. Regarding the investment priorities by customers in investment portfolios, the research results indicated that optimal investments by customers, in the order of priority, have been "investment in bank (2)", "investment in coin and gold market", "investment in stock market", and "investment in real estate".

Selection of valuable coins and gold as well as securities markets as the second and third optimal options for investment in the customer's portfolios concurs with the results obtained by Gholizadeh and Tahurimatin [4] regarding investment portfolios during the housing recession period. Also, the observational investment option in the securities market is in line with the results of the same research regarding investment portfolios within the housing boom period. Regarding the order of priority of investment in bank deposits, securities market, and real estate, the research results concur with the results of Tahmasbi [21] regarding estimating the risk of investment in an asset portfolio in Iran. In this research, three major portfolios for investment for the individuals included banking deposits, land, and stocks. Based on (Table 6), the maximum negative correlation was observed between the asset efficiency of "valuable coins and gold as well as investment in bank (2)". This means that the combination of these two assets in one portfolio significantly reduces the risk. Thus, for the people and investors seeking a lower level of risk, this point can be notable. Nevertheless, since the goal of investors is to achieve an optimal combination of risk and efficiency (the maximum expected utility), they are recommended to consider several markets as their investment target concurrently to achieve this aim.

6. Conclusions

The main reason for the optimality of real estate investment by the bank (3) is that the municipality, as one of the shareholders of this bank, owes a lot of money to the bank and in order to compensate these debts, it transfers its property to the bank. However, most of the claims of the bank (3) are due to the non-implementation of facilities granted to municipalities and related organizations. Of course, this issue does not cause disruption and interruption in the bank's investments (3) in urban projects and fulfilling social responsibility and increasing social welfare at the community level.

Bank (3)'s investment in the stock market is based on the notifications of the Central Bank and the approval of the Monetary and Credit Council. According to the law on removing barriers to production, banks can not invest in the stock market and can only enter the capital market to provide facilities. It is worth mentioning that the entry of any bank into investing in the stock exchange is not long-term and the bank, after buying a share in the stock exchange, can sell the shares at the appropriate time for a maximum of two years, as a result of which this operation is not considered an investment. The main reason for the inefficiency of investment in the foreign exchange market is the issue of extensive sanctions against Iran and its banking system. Therefore, investing in this area is not beneficial for the bank unless there is a reasonable balance between supply and demand for foreign currency due to sanctions. However, Bank (3) has been one of the rare and successful banks in the country in the face of sanctions, which has been able to grant foreign facilities and obligations to its customers.

The main reason for the optimal deposit of Bank (3) customers in Bank (2), despite the higher interest rates on deposits in Bank 3 compared to Bank (2), is due to the high range of interest rate changes on deposits in Bank 3; So that the range of interest rate changes in deposits in Bank (3) during the research period was twice the range of changes in interest rates on deposits in Bank (2).

With these explanations, it can be stated that the main reason and philosophy of investing in parallel banking deposit markets in Iran can be related to the devaluation of the national currency of Iran (Rial) over the past 4 decades. So that the value of Iran's national currency against the US dollar has decreased by almost 1/3 every 8 years. This devaluation of the national currency, together with the double-digit inflation in the society, leads to the feeling of loss of assets in the people and investors, and forces them to invest in the parallel markets of bank deposits to prevent further depreciation of their assets.

At the end of this research, in order to improve and be effective monetary and fiscal policies in the country, the researcher provides recommendations and suggestions to the central bank, banking network and investors:

1. Bank deposit is one of the least risky investment markets. But one of the reasons for encouraging people to invest in markets parallel to bank deposits is the mismatch of interest rates on deposits with the prevailing rate of inflation in the society, which is also due to the central bank's policies in determining interest rates on deposits without proportionality to inflation in Economics has been around for research period.

2. In advanced economies, commodities such as housing and cars are “consumer goods” and not “capital goods”. Meanwhile, in Iran, due to unbalanced economic policies in the supply and demand of housing and cars, these goods are considered as capital goods. Therefore, the government should reconsider its policies regarding the supply and provision of housing and cars.

3. Banks' investment in real estate is one of the causes of inflation and price increases in this commodity. It also leads to an increase in “toxic assets” of banks, which is detrimental to the bank. Therefore, in order to improve their financial indicators, banks should sell their real estate while improving the capital adequacy ratio by at least 8%, which will lead to a reduction in real estate prices.

4. In order to prevent investors from entering the parallel markets, the most important points that should be considered by the government and the Central Bank of the Islamic Republic are first increasing the value of the national currency, then reducing the budget deficit and finally controlling inflation and liquidity in the economy. Achieving these priorities is not achieved solely through economic and domestic monetary and fiscal policies; Rather, it requires the strengthening of macro-domestic and foreign policies and how to interact with the world's economic powers, and ultimately the lifting of extensive sanctions against Iran.

5. Basically, among investment markets, any market that is more profitable will have a higher risk. Meanwhile, the stock market, which is one of the markets with a higher average and risk, is not a good place for banks to invest; Because the sale and purchase of banks in the capital market is associated with the creation of money and rising inflation, and the registration of risky assets in the balance sheets of banks, creates monetary problems for the country's economy.

Encouraging ordinary people to invest in the stock exchange in order to support domestic production and economic growth is also justified and reasonable, provided that the companies present in the main and sub-halls of the stock exchange have accurate audited financial statements and this company, Factories and manufacturing industries in the country can operate with the maximum nominal capacity of production and services. This depends on their activity with sufficient capital and in an efficient and effective manner, which leads to an increase in their income and profitability. Otherwise, even if the overall stock market index has an upward trend, this is just a “bubble” and this bubble will sooner or later be emptied and will lead to huge losses for investors in the stock market.

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