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A Bounded Additive Model for Efficiency Evaluation in Two-Stage Production Systems with Negative Data

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Abstract

Data Envelopment Analysis (DEA) is a method for assessing the efficiency of Decision Making Units (DMUs). Traditional DEA models do not examine the potential differences between two stages caused by intermediate operations. As a result, DEA has been extended to evaluate the efficiency of two-stage processes. In these processes, all outputs of the first stage are intermediate operations that comprise the inputs of the second stage. The input data in real-world applications may have negative values. In this study, considering the importance of network production processes, we deal with the efficiency evaluation of two-stage production units with negative data. Also, we extend CRS (constant returns to scale) bounded additive model for the efficiency evaluation of the two-stage units in the presence of negative data. For illustration, we evaluate the efficiency and ranking of 36 airlines by applying the new model.

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INTRODUCTION

Data Envelopment Analysis (DEA) is a non-parametric method to evaluate the efficiency of Decision Making Units (DMUs). Farrell introduced this method in 1957, and Charnes et al. extended it in 1978. Everybody knows that the optimal use of available resources and precise efficiency evaluation can improve organizational operations. To achieve these objectives, researchers proposed various models and methods.

In 2004, Halkos and Salamouris used DEA in a practical research study. They evaluated the performance efficiency in the Greece Bank sector in a specific period from 1997 to 1999. They managed to design a model and compare efficient banks with inefficient ones [12]. In DEA, the input and output data are positive, but in the real world, the data could be negative. Therefore, many researchers addressed the negative data in this method. Pastor (1994), Lovell (1995), as well as Seiford and Zhu (2002) employed a transfer method to apply negative data in DEA. Others utilized a minute positive value instead of the negative output [26,16,31]. Portela et al. (2004) investigated the negative data using the directional distance function [27]. Sharp et al. (2007) presented the modified slack-based measure (MSBM) model to evaluate the efficiency in the presence of negative data [32]. Kazemi Matin and Azizi (2011) introduced an adjusted additive model with negative data in DEA. Most importantly, they used bank data in their model for comparing and drawing conclusions [13]. In 2013, Vencheh and Esmaeilzadeh introduced a super-efficiency model for ranking DMUs. Their model could also determine the efficiency for each DMU [34]. Lin et al. (2019) offered a successful slacks-based super-efficiency model to handle the negative data [24]. Lin (2019) investigated the evaluation of cross-efficiency in the presence of negative data [25].

Traditional DEA models treat any system as a black box. In the black box, the internal structures are ignored, and the inputs and outputs are considered for estimating the system's overall efficiency. Thus, one of the weak points of traditional or classic DEA models is disregarding the internal structure, while the internal structures of many systems generate intermediate products. In other words, DMUs could have two-stage structures where the inputs of one stage are outputs of the previous stage. The Network DEA (NDEA) models have resolved this defect by considering intermediate values. This notion is the difference between traditional and network views. As a result, it is necessary to consider the internal processes of a DMU. In this regard, banks could have a two-stage network structure.

In recent decades, network models have gained interest, and several models have been proposed in this area. The two-stage network models have extended over time. Besides, the efficiency results of the models in the two-stage network show that they could better evaluate the units. The two-stage network models could be applied more in the structural area. The two-stage DEA model can yield the overall efficiency for the whole process and efficiency scores of each stage perfectly well. NDEA has been introduced to academic society by the famous article of Fare & Grosskopf, entitled “Network Data Envelopment Analysis.” However, this topic had already been investigated by Fare (1991), Fare & Grosskopf (1996), and Lathgren and Tambour (1999). Recently, DEA has been extended with using network systems. There are several articles on network systems.

Lewis and Sexton (2004) showed that how DEA can be used to examine the internal DMUs. They also proved some of the theoretical features of NDEA [17]. Chen and Zhu (2004) used a two-stage production process to detect the efficient frontier [2]. Prieto and Zoflo (2007) proposed a multi-stage model. They applied this model to a group of OECD (The Organization for Economic Co-operation and Development) countries [28]. Chen et al. (2009) proposed an approach of additive efficiency decomposition in a two-stage DEA [4]. Chen et al. (2009) examined the correlation and equivalence in measuring efficiency in two-stage systems [3]. Chen et al.

(2010) developed DEA models under non-discretionary inputs to measure efficiency in two-stage network processes [5]. Chen et al. (2010) proposed an approach under two-stage DEA to determine the frontiers for inefficient units [6]. Paradi et al. (2011) used a two-stage DEA to mark the operating units in different scales simultaneously. In the end, they proposed a modified measurement slacks-based model. They evaluated this model by using the data of a prominent Canadian bank with 816 branches. They examined three dimensions of productivity, profitability, and intermediation. This method enables the bank managers to find their weak and strong points clearly [29]. Premachandra et al. (2012) presented a two-stage DEA that decomposes the efficiency of DMU into two parts. They used this model to assess the relative efficiency of the family budget in the USA during 1993-2008 [30]. Chen et al. (2013) proved some issues about the envelopment and multiplier points in NDEA [8]. Lin and Chiu (2013) used an overall model to evaluate the efficiency of Banks in Taiwan through decomposing the independent components analysis (ICA) and assessing based on network slacks (NSBM). In their work, three dimensions of efficiency: production efficiency, service efficiency, and profiting efficiency have been discussed [18]. Kao (2014) conducted a review study on network systems [14]. Wanke and Barros (2014) evaluated the efficiency of Brazilian banks using a two-stage model [35]. Liu et al. (2015) proposed the two-stage DEA models with undesirable inputs and intermediate outputs. Also, they applied two-stage models in some data of China banks [19]. Li et al. (2016) extended a centralized model to measure the efficiency in two-stage processes. They tested their model on 17 branches of a bank in Anhui Province, China [21]. Lim and Zhu (2016) studied the double standard of DEA in two-stage systems [20]. Wanke et al. (2016) used a two-stage model that simultaneously included both costs and efficiency of learning and evaluated the performance problems in state high schools in Australia[36]. Fukuyama and Matousek (2017) examined the efficiency of bank performance in a two-stage network (NDEA). Besides, they applied the Nerlove model to detect the inefficiencies in banks. Their article shows that local banks in Japan have not reached their desired target in production processes [10]. Li et al. (2018) proposed a network model with some important features in their article [22]. Lu et al. (2019), in their article, evaluated the efficiency of the car industry in Taiwan from 2010 to 2014 using a dynamic NDEA model. They showed that the overall growth production depends partly on the market production stage and partly on the production stage. Motivated by the benefits of combining multiple energies [23]. Cheng et al. (2020) designed a two-stage method at two levels. They wanted to achieve an optimal energy supply among multi-energy systems (MES) [9].

The rest of the articles are organized as follows. The section 2 discusses the two-stage systems with negative data model. In the fourth section, CRS-bounded additive model is examined. Also, In the section CRS-bounded additive model is examined. The section 3 evaluated this Two-stage bounded additive model. In the section 4 , a numerical and practical example is solved. Conclusions are presented in Section 5.

TWO-STAGE SYSTEMS WITH NEGATIVE DATA

Several articles have been written on two-stage systems with negative data. For example, Tavana et al. (2018) examined the RDM and the dynamic models on two-stage systems with negative data [33].

In 2020, Kao defined a set of general production possibilities with the negative data. Their proposed model could identify unrealistic production processes and assess the efficiency in the presence of negative data. It was the simplest two-stage system [15]. Cooper et al. (2011) proposed the BAM model [7]. BAM model had many advantages. The model not only could accept negative data but also distinguish between strong and weak units, which increased its credibility. The BAM model performs well in the variable returns to scale (VRS) efficiency mode and does not experience inconsistencies in the constant returns to scale (CRS). However, the two-stage CRS-boundary collective model has not been explored so far. We aimed to evaluate a two-stage CRS-boundary collective model (BAM-CRS).

CRS-bounded additive model

In the analysis of efficiency, inputs are tried to decrease and outputs increased, so we use $\underline{x_i}$ = min{ x_{ij}, j=1,…,n} lower bound for each input and

$\overline{y_r}$ = max{ y_{rj} , j=1,…,n} higher bound for each output in the DEA model.

The set of bounded production probability under CRS is as follows:

$T = \{(x,y)\epsilon R_+^m \times R_+^s \ (x,-y)= \sum_{j=1}^{n} \lambda_j \ (x_j,-y_j);\ \lambda_j \geq 0\ ,\forall j\ ;\ x_i \geq \underline{x_i}, \forall i; y_r \leq \overline{y_r}\ , \forall r\}$

It is assumed that for the evaluation of n units in DEA, m inputs and s outputs are defined for each unit. Fig. 1 shows the bounded frontier for CRS technology.

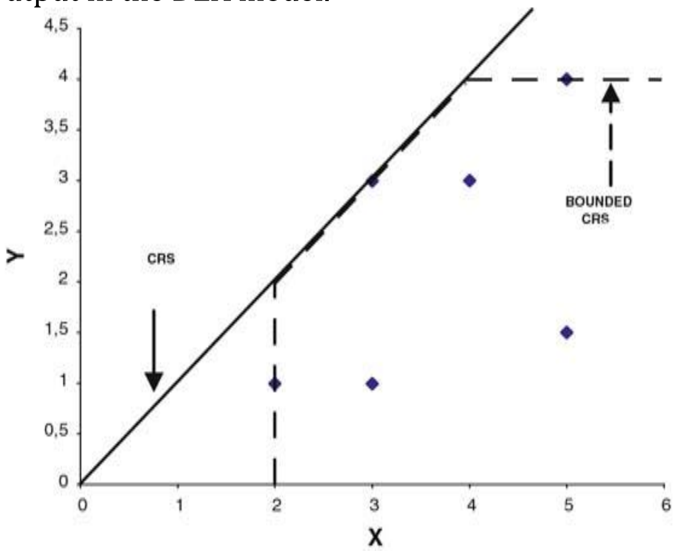

Fig. 1. Bounded frontier in CRS technology

The CRS-bounded additive model (for more information, refer to Cooper et al. [2011]) is presented as Model 1:

$Max\ (\sum_{i=1}^{m} s_{io}^{-} + \sum_{r=1}^{s} s_{ro}^{+})$

s.t.

$\sum_{j=1}^{n} \lambda_j\ x_{ij} + s_{io}^{-} = x_{io}\quad ,\quad i = 1, \dots, m$

$\sum_{j=1}^{n} \lambda_j\ y_{rj} - s_{ro}^{+} = y_{ro}\quad ,\quad r = 1, \dots, s$

$\sum_{j=1}^{n} \lambda_j\ x_{ij} \geq \underline{x_i}\quad ,\quad i = 1, \dots, m$

$\sum_{j=1}^{n} \lambda_j\ y_{rj} \leq \overline{y_r}\quad ,\quad r = 1, \dots, s$

$\lambda_j \geq 0\ ,\ j = 1, \dots, n\ ;\ s_{io}^{-} \geq 0, \forall i\ ;\ s_{ro}^{+} \geq 0\ , \forall r$

(1)

Model 1 can be revised as envelopment Model 2:

$Min\ \beta$

s.t.

$\beta x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}\quad ,\quad i = 1, \dots, m$

$y_{ro} \leq \sum_{j=1}^{n} \lambda_j\ y_{rj}\quad ,\quad r = 1, \dots, s$

$\sum_{j=1}^{n} \lambda_j\ x_{ij} \geq \underline{x_i}\quad ,\quad i = 1, \dots, m$

$\sum_{j=1}^{n} \lambda_j\ y_{rj} \leq \overline{y_r}\quad ,\quad r = 1, \dots, s$

$\lambda_j \geq 0\ ,\ j = 1, \dots, n\ ;$

(2)

Mode 2 is an envelopment CRS bounded additive model.

The value of $\Gamma_{BAM-CRS}$ efficiency is obtained from

$\Gamma_{BAM-CRS} = 1 - \beta$

Which has the following terms:

1- $0 \leq \Gamma_{BAM-CRS} \leq 1$
2- $\Gamma_{BAM-CRS} = 1 \Leftrightarrow$ DMU_O is fully efficient.
3- $\Gamma_{BAM-CRS} = 0 \Leftrightarrow$ DMU_O is fully inefficient.
4- $\Gamma_{BAM-CRS}$ is invariant to units of measurement of inputs and outputs.
5- $\Gamma_{BAM-CRS}$ is translation invariant.

6- $\Gamma_{\text{BAM}-\text{CRS}}$ is monotonic.

TWO-STAGE BOUNDED ADDITIVE MODEL

Several studies have proposed better and more precise models in the area of two-stage models literature. In this section, we evaluate the two-stage CRS-bounded additive model.

It is assumed that for each $\text{DMU}_j(j = 1,\ldots,\text{n})$, the $(\text{i} = 1,\ldots,\text{m})$ x_{ij} input is used for $z_{\text{dj}}(\text{d} = 1,\ldots,\text{D})$ intermediate products. Then, the output of the first stage is used as the input of another process to produce $y_{r\text{j}}(\text{r} = 1,\ldots,\text{s})$ outputs of the second stage. As shown in Figure 2.

In Model 3, the lower bound input of $\underline{x_i} = \min\{x_{ij}, \text{j}=1,\ldots,\text{n}\}$ and higher bound output of $\overline{y_r} = \max\{y_{rj}, \text{j}=1,\ldots,\text{n}\}$ have been employed.

In the two-stage DEA model, the lower and higher bounds of intermediate products are considered as follows:

$\overline{z_d} = \max\{z_{dj}, \text{j}=1,\ldots,\text{n}\}$ و $\underline{z_d} = \min\{z_{dj}, \text{j}=1,\ldots,\text{n}\}$.

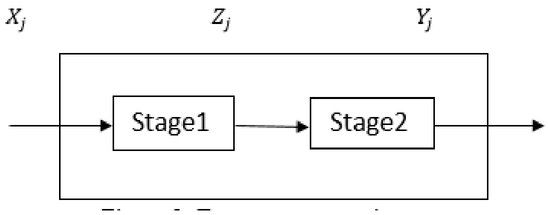

Fig.2. Two-stage network system

The two-stage CRS-bounded additive is obtained as Model 3, using Model 2 .

$$
\begin{aligned}
& Min\ \ \beta \\
& s.t. \\
& \beta x_{io} \geq \sum_{j=1}^{n} \lambda_j^1 x_{ij} \qquad , \quad i = 1, \ldots, m \\
& z_{do} \leq \sum_{j=1}^{n} \lambda_j^1 z_{dj} \qquad , \\
& d=1,\ldots,D \\
& z_{do} \geq \sum_{j=1}^{n} \lambda_j^2\, z_{dj} \qquad , \quad d=1,\ldots,D \\
& y_{ro} \leq \sum_{j=1}^{n} \lambda_j^2\, y_{rj} \qquad , \quad r = 1, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j^1\, x_{ij} \geq \underline{x_i} \qquad , \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j^1\, z_{dj} \leq \overline{z_d} \qquad , \quad d=1,\ldots,D \\
& \sum_{j=1}^{n} \lambda_j^2\, z_{dj} \geq \underline{z_d} \qquad , \quad d=1,\ldots,D \\
& \sum_{j=1}^{n} \lambda_j^2\, y_{rj} \leq \overline{y_r} \qquad , \quad r = 1, \ldots, s \\
& \lambda_j^k \geq 0 \quad , \quad j = 1, \ldots, n \quad , \quad k = 1, 2 \quad ;
\end{aligned}
\tag{3}
$$

The two-stage Model 3 is a set of variables with two sets of constraints related to Z_{do} intermediate products that make the model more efficient and, as a result, bigger. In this model, the intermediate product is of high importance.

In continuation, Model 3 is revised as Model 4.

$$
\begin{aligned}
& Min\ \ \beta \\
& s.t. \\
& \beta x_{io} \geq \sum_{j=1}^{n} \lambda_j^1 x_{ij} \qquad , \quad i = 1, \ldots, m \\
& z_{do} \leq \sum_{j=1}^{n} \lambda_j^1 z_{dj} \qquad , \quad d=1,\ldots \\
& z_{do} \geq \sum_{j=1}^{n} \lambda_j^2\, z_{dj} \qquad , \quad d=1,\ldots,D \\
& y_{ro} \leq \sum_{j=1}^{n} \lambda_j^2\, y_{rj} \qquad , \quad r = 1, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j^1\, x_{ij} \geq \underline{x_i} \qquad , \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j^1\, z_{dj} \leq \overline{z_d} \qquad , \quad d=1,\ldots,D \\
& \sum_{j=1}^{n} \lambda_j^2\, z_{dj} \geq \underline{z_d} \qquad , \quad d=1,\ldots,D \\
& \sum_{j=1}^{n} \lambda_j^2\, y_{rj} \leq \overline{y_r} \qquad , \quad r = 1, \ldots, s \\
& \underline{x_i} = \min\{x_{ij}, \text{j}=1,\ldots,\text{n}\} \quad , \quad i = 1, \ldots, m \\
& \underline{x_i} = \min\{x_{ij}, \text{j}=1,\ldots,\text{n}\} \quad , \quad i = 1, \ldots, m \\
& \overline{z_d} = \max\{z_{dj}, \text{j}=1,\ldots,\text{n}\} \quad , \quad d=1,\ldots,D \\
& \underline{z_d} = \min\{z_{dj}, \text{j}=1,\ldots,\text{n}\} \quad , \quad d=1,\ldots,D \\
& \overline{y_r} = \max\{y_{rj}, \text{j}=1,\ldots,\text{n}\} \quad , \quad r = 1, \ldots, s \\
& \lambda_j^k \geq 0 \quad , \quad j = 1, \ldots, n \quad , \quad k = 1, 2 \quad ;
\end{aligned}
\tag{4}
$$

Model 4 is a two-stage network model.

The dual form of Model 4 is Model 5:

$$
\begin{aligned}
& Max \quad w^1 z_{do} - w^2 z_{do} + u y_{ro} + \text{v}\, \underline{x_i} - w^1\, \overline{z_d} + w^2\, \underline{z_d} - u\, \overline{y_r} \\
& s.t. \\
& x_{io} v_i^1 - \sum_{j=1}^{n} v_i^1\, x_{ij} + \sum_{j=1}^{n} w_d^1 z_{dj} - \sum_{j=1}^{n} w_d^2 z_{dj} + \sum_{j=1}^{n} u_r^2\, y_{rj} + \sum_{j=1}^{n} v_i^1\, x_{ij} - \sum_{j=1}^{n} w_d^1\, z_{dj} + \sum_{j=1}^{n} w_d^2\, z_{dj} - \sum_{j=1}^{n} u_r^2\, y_{rj} \leq \beta \\
& \underline{x_i} = \min\{x_{ij}, \text{j}=1,\ldots,\text{n}\} \quad , \quad i = 1, \ldots, m \\
& \overline{z_d} = \max\{z_{dj}, \text{j}=1,\ldots,\text{n}\} \quad , \quad d=1,\ldots,D \\
& \underline{z_d} = \min\{z_{dj}, \text{j}=1,\ldots,\text{n}\} \quad , \quad d=1,\ldots,D \\
& \overline{y_r} = \max\{y_{rj}, \text{j}=1,\ldots,\text{n}\} \quad , \quad r = 1, \ldots, s \\
& v_i^1, u_r^2, w_d^1, w_d^2 \geq 0 \quad ;
\end{aligned}
\tag{5}
$$

THE NUMERICAL AND APPLIED EXAMPLE

In this section, a numerical and applied example is presented. Today, airlines have a considerable share in the economic development of countries worldwide. One of the most important issues that managers of airlines are interested in is to know the relative status and performance rank of their companies compared to other companies and competitors. Designing and evaluating the efficiency process is one of the issues that managers should attend to. The involved criteria in this process could be evolved and changed. The improved service qualities and customers' satisfaction could increase the benefits of airlines. Evaluation of services is one of the challenging issues in these companies. Many studies have been conducted on evaluating the efficiency of airlines.

Gillen and Lall (1997) used the DEA to evaluate the efficiency of airports. They used the data of 21 airports in the USA for 5 years [11].
Adler and Golany (2001) used the DEA to find the most efficient airlines in west Europe. They analyzed the significant components using an applied program [1].

In this study, we evaluated the efficiency of 36 airlines using the two-stage bounded additive model. As shown in Table 1, fuel cost, maintenance expenses, labor expenses, and fleet size are input values; available ton miles and available seat miles are intermediate values, and revenue passenger mile, revenue ton mile and net income are the output values.

Table 1: Data of 36 airlines

		Inputs				Intermediate		Outputs		
	Year	**Fuel cost (000,000)**	**52990-Total flight maintance (000,000)**	**Salary (000,000)**	**Fleet Size**	**Available ton miles (000,000)**	**Available seat miles (000,000)**	**Revenue px mile (000,000)**	**Revenue tone mile(000,000)**	**Net income (000)**
1	2010	1,006.21	424.83	1,564.18	708	20688	85710	96985	11322	-171,152.00
2	2010	1,184.42	448.65	1,593.98	708	21440	90038	107388	12419	40,724.00
3	2010	1,383.72	540.19	1,587.04	708	21646	91228	111076	12729	-160,325.00
4	2010	1,383.93	490.48	1,685.57	708	20115	85256	99682	11692	-600,966.00
5	2011	1,285.09	489.93	1,637.66	685	20109	85505	99047	11472	-105,857.00
6	2011	1,491.71	488.28	1,594.80	685	21128	89204	110573	12750	279,479.00
7	2011	1,546.64	504.35	1,607.41	685	21126	89065	109148	12592	730.00
8	2011	1,262.24	494.37	1,626.65	685	19963	84278	99605	11717	-10,476.00
9	2012	1,228.19	480.53	1,577.07	660	19653	83384	97726	11348	51,144.00
10	2012	1,428.64	484.36	1,563.40	660	20171	85297	107007	12315	270,608.00
11	2012	1,518.99	502.24	1,632.71	660	20400	86545	108872	12434	146,574.00
12	2012	1,634.30	494.77	1,641.50	660	19873	84596	101761	11825	-112,545.00
13	2013	1,801.96	548.82	1,557.63	648	19306	82106	97467	11267	-343,910.00
14	2013	2,118.50	562.22	1,575.60	648	19581	83439	103200	11926	-1,462,732.00
15	2013	2,402.01	544.75	1,549.94	648	19720	83933	103477	11883	-396,172.00
16	2013	1,637.73	530.41	1,653.24	648	18280	77597	91141	10494	-327,938.00
17	2014	1,142.87	545.22	1,604.12	606	17766	75567	85783	9695	-365,661.00
18	2014	1,163.95	548.49	1,619.21	606	18219	77135	94697	10673	-389,835.00
19	2014	1,270.37	564.24	1,621.42	606	18122	77087	97061	10959	-376,810.00
20	2014	1,279.90	568.91	1,640.12	606	17242	73769	89727	10392	-343,475.00
21	2015	1,301.97	602.49	1,615.08	614	17180	73691	86102	9956	-488,786.00
22	2015	1,434.21	594.43	1,624.56	614	17934	76829	96650	11106	-6,818.00
23	2015	1,386.61	575.35	1,642.70	614	18673	79885	100642	11499	128,583.00
24	2015	1,431.01	552.37	1,614.89	614	17627	76085	93075	10766	-102,197.00

25	2016	1,589.85	556.58	1,627.01	615	17481	75703	87499	10073	-430,511.13
26	2016	1,895.49	591.36	1,672.87	615	18167	78458	98367	11219	-284,356.00
27	2016	1,946.78	611.31	1,682.04	615	18590	79870	101692	11510	-153,152.00
28	2016	1,720.95	576.12	1,689.51	615	17237	74615	91918	10522	-1,097,082.00
29	2017	1,865.09	581.53	1,685.29	606	17600	75837	89879	10331	-1,676,208.00
30	2017	1,903.10	578.73	1,687.26	606	17853	76576	97757	11149	-263,749.00
31	2017	1,882.49	559.54	1,696.17	606	18159	77909	99905	11271	-256,922.00
32	2017	1,857.85	577.09	1,464.05	606	17610	74932	91671	10483	270,778.00
33	2018	1,879.91	587.61	1,390.91	612	17731	74784	90415	10280	-252,848.00
34	2018	1,825.11	569.39	1,360.92	612	18628	77444	98552	11269	227,756.00
35	2018	1,894.67	538.77	1,458.99	612	19387	80163	102233	11584	289,839.00
36	2018	1,814.91	555.06	1,422.19	612	18377	76602	94032	10899	-1,790,454.00

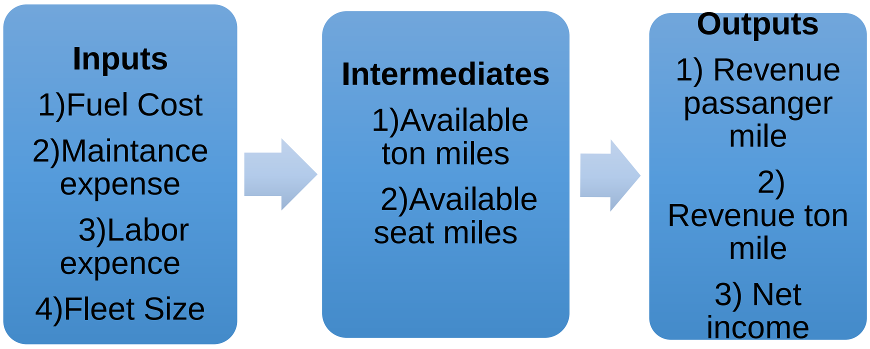

Fig.3 The two-stage graph of 36 airlines

We simulate this example using the two-stage non-parametric Model 4. This model is presented as Model 6.

$$\begin{aligned}
& Min\ \ \beta \\
& s.t. \\
& \beta x_{io} \geq \sum_{j=1}^{36} \lambda_j^1 x_{ij} \qquad , \quad i = 1\ ,2,3,4 \\
& z_{do} \leq \sum_{j=1}^{36} \lambda_j^1 z_{dj} \qquad , \quad d=1,2 \\
& z_{do} \geq \sum_{j=1}^{36} \lambda_j^2\ z_{dj} \qquad , \quad d=1,2 \\
& y_{ro} \leq \sum_{j=1}^{36} \lambda_j^2\ y_{rj} \qquad , \quad r = 1\ ,2,3 \\
& \sum_{j=1}^{36} \lambda_j^1\ x_{ij} \geq \underline{x_i} \qquad , \quad i = 1\ ,2,3,4 \\
& \sum_{j=1}^{36} \lambda_j^1\ z_{dj} \leq \overline{z_d} \qquad , \quad d=1,2 \\
& \sum_{j=1}^{36} \lambda_j^2\ z_{dj} \geq \underline{z_d} \qquad , \quad d=1,2 \\
& \sum_{j=1}^{36} \lambda_j^2\ y_{rj} \leq \overline{y_r} \qquad , \quad r = 1,2,3 \\
& \underline{x_i} = \min\{\ x_{ij}, \text{j=1,…,36}\} \quad , \quad , \quad i = 1\ ,2,3,4 \\
& \overline{z_d} = \max\{\ z_{dj}\ , \text{j=1,…,36}\}\ , \quad d=1,2 \\
& \underline{z_d} = \min\{\ z_{dj}\ , \text{j=1,…,36}\}\ , \quad d=1,2 \\
& \overline{y_r} = \max\{\ y_{rj}\ , \text{j=1,…,36}\} \quad , \quad r = 1\ ,2,3 \\
& \lambda_j^k \geq 0 \quad , \quad j = 1\ ,\ldots\ ,36 \quad , \quad k = 1\ ,\ 2 \quad ;
\end{aligned} \tag{6}$$

Model 6 is a two-stage CRS-bounded additive model. The efficiency values of Model 6 have been calculated by Lingo, and the results are presented in the third column of Table 2. In the fourth column, the efficiency of units is specified. Also, the overall efficiency values are ranked in the fifth column. Figure 4 displays the diagram of efficiency values based on Model 6.

As seen in Table 2 and Figure 4, there are efficient units in 36 airlines. Also, DMU1, DMU2, DMU6, DMU10, DMU14, DMU18, DMU22, DMU26, DMU30, DMU34 are efficient. Their efficiency values are equal to 1. However, DMU7 has the worst performance efficiency among these 36 airlines. Its efficiency value is 0.3367. According to the diagram in Figure 3, this unit has obtained the lowest efficiency.

Based on the Table, the other units are ranked as follows:

DMU12 < DMU16 < DMU8 < DMU4 < DMU24
< DMU20 < DMU32 < DMU28 < … < DMU7

Table 2: Results of efficiency values of 36 airlines

DMU	year	Efficiency two-stage additive bounded CRS model	Rank
1	2010	1.0000	1
2	2010	1.0000	1
3	2010	0.3406	25
4	2010	0.9365	5
5	2011	0.4315	11
6	2011	1.0000	1
7	2011	0.3367	27
8	2011	0.9380	4
9	2012	0.4242	12
10	2012	1.0000	1
11	2012	0.3420	24
12	2012	0.9478	2
13	2013	0.3851	19
14	2013	1.0000	1
15	2013	0.3400	26
16	2013	0.9400	3
17	2014	0.3983	15
18	2014	1.0000	1
19	2014	0.3679	22
20	2014	0.9179	7
21	2015	0.3860	18
22	2015	1.0000	1
23	2015	0.3873	17
24	2015	0.9226	6
25	2016	0.3474	23

26	2016	1.0000	1
27	2016	0.3958	16
28	2016	0.9053	9
29	2017	0.4238	13
30	2017	1.0000	1
31	2017	0.3692	21
32	2017	0.9113	8
33	2018	0.4161	14
34	2018	1.0000	1
35	2018	0.3788	20
36	2018	0.9014	10

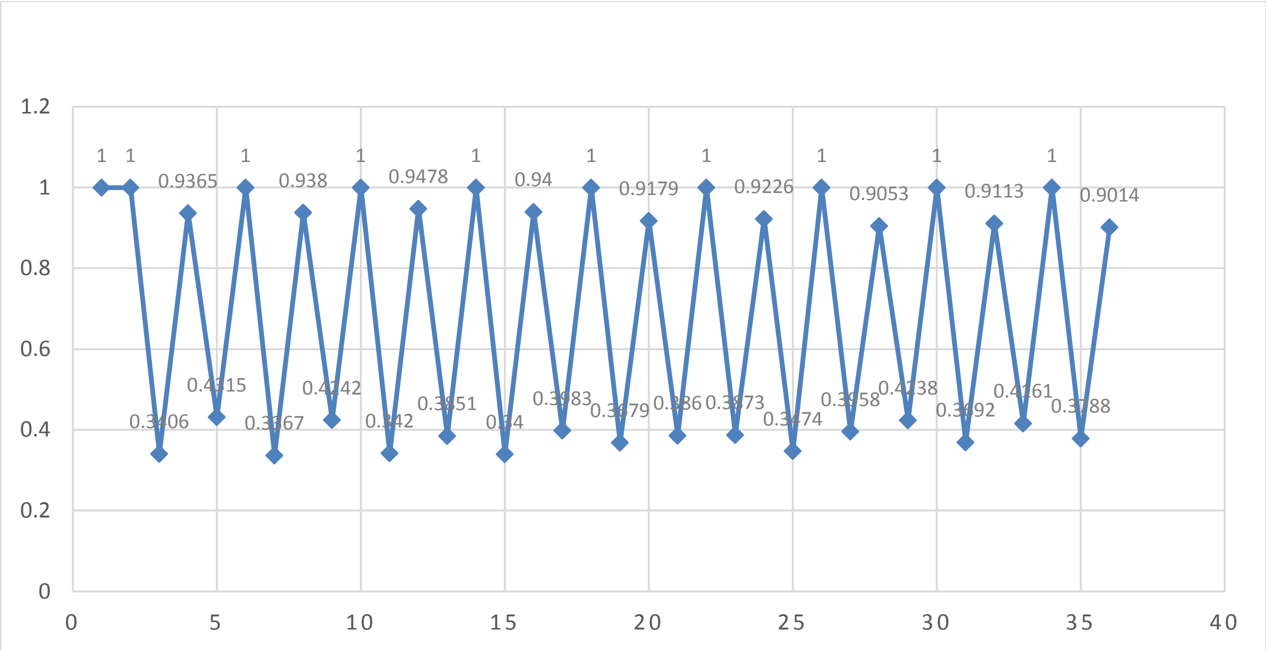

Fig. 4. Diagram of overall efficiency values based on Model 6

CONCLUSION

This article examined a two-stage CRS-bounded additive model with negative data. It also evaluated the efficiency of units in a two-stage network. It was found that a two-stage model could better assess the units by considering the intermediate values. Also, the overall efficiency of 36 airlines was examined, and DMU7 was found to have the Lowest performance. Two-stage models are variables with two sets of constraints that become more efficient by considering the intermediate products. In other words, the intermediate products become very important. For future studies, the two-stage methods could be used in DEA. The research areas and DEA techniques in two-stage models could evaluate the units with negative data in the best way.

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