Analytical Adjustment of Predictive Stabilizer Parameters in Interconnected Industrial Systems: Three Reservoir System

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Abstract:

Regarding the daily increasing development of process and chemical industries and the requirement to improve energy consumption and increasing the output, advanced process control strategies are utilized effectively. Model predictive control as a successful developed process control has been proposed to deal with problems having constraints and multivariable systems. The successful administration of this strategy requires proper adjustment of its' parameters. In this research, three reservoir system has been modeled as a laboratory plant and predictive control algorithm for first order system with delay has been designed in the form of an unlimited case. The delayed first order model has been achieved though the implementation of a white noise signal into the plant to recognize the system. Then parameters were adjusted using an analytic method. Results of simulations showed that model predictive control has represented an optimal performance through proper adjustment of the parameters.

Keywords: Advanced Process Control, Model Predictive Control, Three Reservoir System, White Noise

Introduction

During the latest two decades there has been a great deal of interest on the part of industry and academic environments towards advanced process control [1]. Considering the growth and spread of process industries a need is felt to access products with higher qualities, products with better performance, rapid adjustment with changes in the market, and a need to access successful control design which can work strongly in theory and practice [2]. Model predictive control has been extensively used in process industries such as oil refinery, chemistry engineering and metalogics for more than two decades as an optimal method based on a clear use of process model [3-14]. The idea of model predictive control method could be attributed to output behavior prediction or a process state within a limited time horizon, the calculation of input signal for future in each time step through minimizing cost function in the presence of restrictions and the application of the very first constituent of the controlling input vector. Therefore, having a system model is known as a prerequisite condition to design a model predictive control approach.

Model predictive control has some advantages compared with other control methods and some of the most important items in this regard are the objectivity of the concepts, working with it in the industry, working with complicated dynamics, systems with delay, non-minimized phase, multivariable, and the application of simple prerequisites [8]. In [15], the model predictive control has been applied for a connected two reservoir system. In [16], and using Laguerre functions, predictive control has been designed and implemented for a four reservoir system. In [17], a generalized predictive control for a coupled four tank MIMO system using a continuous-discrete time observer has been designed and implemented. One of the challenges of using model predictive control is how to adjust its parameters. To reach consistency and optimal system performance we need to adjust predictive control parameters properly. In [18], the adjustment methods proposed during the years between 1980 and 1994 have been represented. In this research and through applying a white noise signal into the plant and recognizing it, the first order model with delay has been achieved. Then an analytical method [19], has been utilized to adjust predictive control parameters. Model predictive control has been designed to control the liquid level in the third tank of a three reservoir system and its parameters have been adjusted using an analytical method. The present paper has been divided into five sections. In second section, the system has been modeled and has been written in the form of state space. In third section, a predictive model control for first order model with a delay in the system has been designed and its control signal has been achieved. In section four, the adjustment of parameters and simulation results have been investigated. In part five, the conclusion has been represented.

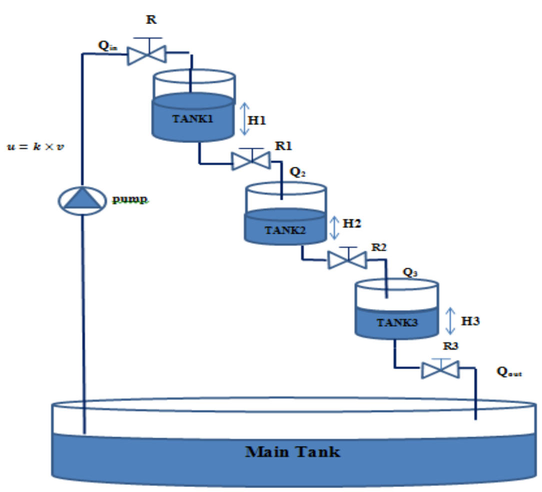

Fig. 1.Three Reservoir System

Three Reservoir System Modeling

In this part the system model in the form of space state has been represented. According to figure 1, water with a fluid rate of Qin is sucked from the main tank and enters the system and exits from it with a fluid rate of Qout. The system input Qin and its output is known as water height in the third tank (H3).

Through writing a mass balance equation for each tank, we would have the following equations:

Tank 1:

$$\rho A_1 \frac{dh_1}{dt} = \rho Q_{in} - \rho Q_2 \tag{1}$$

$$Q_2 = \frac{h_1}{R_1} \tag{2}$$

Tank 2:

$$\rho A_2 \frac{dh_2}{dt} = \rho Q_2 - \rho Q_3 \tag{3}$$

$$Q_3 = \frac{h_2}{R_2} \tag{4}$$

Tank 3:

$$\rho A_3 \frac{dh_3}{dt} = \rho Q_3 - \rho Q_{out} \tag{5}$$

$$Q_{out} = \frac{h_3}{R_3} \tag{6}$$

Through placing equation 2 in 1 and equation 2 in 3 and equations 4 and 6 in 5, the system space state model is achieved as follows:

$$\dot{x} = Ax + Bu \tag{7}$$

$$y = Cx \tag{8}$$

$$x = [h_1 \quad h_2 \quad h_3]^T \tag{9}$$

$$A = \begin{bmatrix} \frac{-1}{A_1R_1} & 0 & 0 \\ \frac{1}{A_2R_1} & \frac{-1}{A_2R_2} & 0 \\ 0 & \frac{1}{A_3R_2} & \frac{-1}{A_3R_3} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{A_1} \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0 \quad 0 \quad 1] \tag{10}$$

Through changing the amounts above using Laplace transfer in equations 1 to 6 and applying the numbers, the system exchange function with output h3 is gained as follows:

$$\frac{H_3(s)}{Q_{in}(s)} = \frac{R_3}{(A_1R_1s+1)(A_2R_2s+1)(A_3R_3s+1)} \quad (11)$$

Predictive model control design for first order system with delay

In this part, a predictive model control in the form of space state model [20] for first order system with delay has been designed. Discrete time model with sampling time Ts has been represented as equation (12) below:

$$G_d(z^{-1}) = \frac{k_p(1-a)z^{-k-1}}{1-az^{-1}} \quad (12)$$

where, $a = e^{-T_s/\tau}$, $k = \theta/T_s$ The discrete time space state model is explained as follows:

$$x_m(k+1) = A_m x_m(k) + B_m u(k), \quad (13)$$

$$y(k) = C_m x_m(k) \quad (14)$$

u is the input variable and y is the output in the process. Using subtraction operation in both sides of the equation 13 and equations 15 and 16, result in equation 13 modified as equation 17.

$$\Delta x_m(k+1) = x_m(k+1) - x_m(k) \quad (15)$$

$$\Delta u(k) = u(k) - u(k-1) \quad (16)$$

$$\Delta x_m(k+1) = A_m \Delta x_m(k) + B_m \Delta u(k) \quad (17)$$

Also for output, we would have:

$$\begin{aligned} y(k+1) - y(k) &= C_m(x_m(k+1) - x_m(k)) \\ &= C_m \Delta x_m(k+1) \quad (18) \end{aligned}$$

Through placing numbers in equations 17 and 18, the output is achieved as equation 19:

$$y(k+1) = C_m A_m \Delta x_m(k) + C_m B_m \Delta u(k) + y(k) \quad (19)$$

The new state variable is defined as equation 20 below:

$$x(k) = [\Delta x_m(k)^T \quad y(k)]^T \quad (20)$$

The completed space state model is explained as follows:

$$x(k+1) = Ax(k) + B\Delta u(k)$$

$$y(k) = Cx(k)$$

$$x(k+1) = \begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix},$$

$$A = \begin{bmatrix} A_m & o_m^T \\ C_m A_m & 1 \end{bmatrix}, B = \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}$$

$$C = [o_m \quad 1], o_m = \overbrace{[0\ 0 \dots 0]}^{n_1} \quad (21)$$

The identification of optimal control signal through solving the cost function below is determined:

$$\min_{u(n)} \left(w(n) - y(n)\right)^T Q(w(n) - y(n)) + \left(\Delta u(n)\right)^T R(\Delta u(n))$$

$$u_{min} \le u(n+i|n) \le u_{max} \quad (22)$$

$$y_{min} \le \hat{y}(n+j|n) \le y_{max}$$

Where,

$$w(n) = \begin{bmatrix} w(n) \\ w(n) \\ \vdots \\ w(n) \end{bmatrix}, \ y(n) = \begin{bmatrix} \hat{y}(n+N_1|n) \\ \hat{y}(n+N_1+1|n) \\ \vdots \\ \hat{y}(n+N_p|n) \end{bmatrix}$$

$$\Delta u(n) = \begin{bmatrix} \Delta u(n) \\ \Delta u(n+1) \\ \vdots \\ \Delta u(n+N_c-1) \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & q_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & q_p \end{bmatrix}_{P\times P},$$

$$R = k_p^2(1-a)^2 \begin{bmatrix} r_1 & 0 & \dots & 0 \\ 0 & r_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & r_M \end{bmatrix}_{M\times M} \quad (23)$$

In equations 23, lower level predictive horizon was $N_1 = k+1$ and the upper level of predictive horizon was $N_2 = k+P$. Also P represents predictive horizon and M represents control horizon. The amount of predicted output in time k is known as $y(n+k|n)$ where we possess the output until time n. If the output is represented as the vector y(n) in equations 23, the predicted output in the equation will be as follows:

$$y(n) = \emptyset \Delta u(n) + Fx(k) \tag{24}$$

where,

$$F = \begin{bmatrix} CA^{k+1} \\ CA^{k+2} \\ \vdots \\ CA^{k+P} \end{bmatrix}_{P\times(k+2)},$$

$$S = \begin{bmatrix} CA^{k}B & 0 & 0 & 0 \\ CA^{k+1}B & CA^{k}B & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{k+P-1}B & CA^{k+P-2}B & \dots & CA^{k+P-M}B \end{bmatrix}$$

(25)

The real system output prediction is gained as equation 26 below:

$$\hat{y}(n+k) = a^{k} y_m(n) + k_p(1 - a)(a^{k-1}u(\mathrm{n}-\mathrm{k}) + au(n-2) + u(n-1) + d(n))$$

$$d(n) = y_p(n) - y_m(n) \tag{26}$$

where y_p is the plant output and y_m is represented as model output.

We have an optimal control signal as below:

$$\Delta u(n) = K_y\big(w(n) - \hat{y}(n+k)\big) - K_x(\Delta\hat{y}(n+k)) \tag{27}$$

where,

$$K_x = \begin{bmatrix} K_{x1} \\ K_{x2} \\ \vdots \\ K_{xM} \end{bmatrix} = (R + S^T Q S)^{-1} S^T Q F$$

$$K_y = \begin{bmatrix} K_{y1} \\ K_{y2} \\ \vdots \\ K_{yM} \end{bmatrix} = (R + S^T Q S)^{-1} S^T Q 1_{P\times 1} \tag{28}$$

Using law of diminishing marginal utility through which the first row of control signal is implemented, equation 27 will be changed as follows:

$$G_m(s) = \frac{4.0224e^{-s}}{5.6355s+1} \tag{29}$$

The adjustment of parameters and results of simulations

In this section, the white noise input with variance 5 has been applied into the system regarding the amounts for three reservoir system parameters using MATLAB recognition software and the results were estimated through collecting input and output data into the first order model with delay. Having amounts related to system model in the form of first order with delay model using an analytic method (17), the parameters P and M and weight matrixes R and Q are adjusted and regarding their amounts, the predictive model control has been designed.

The sample amounts for three tank system are represented as follows:

$$R_1 = R_2 = 2\, m/(m^3/\mathrm{s})\,,\, R_3 = 4\, m/(m^3/\mathrm{s})$$

$$A_1 = A_1 = 1m^2\,, A_3 = 0.5m^2$$

$$\frac{\boldsymbol{H_3(s)}}{\boldsymbol{Q_{in}(s)}} = G_p(s) = \frac{\mathbf{4}}{(\mathbf{2s+1})(\mathbf{2s+1})(\mathbf{2s+1})} \tag{30}$$

White noise input with variance 5 and system output is represented as figure 2.

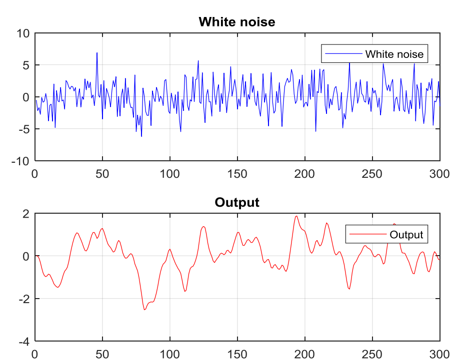

Fig. 2. White noise input and applying it into plant

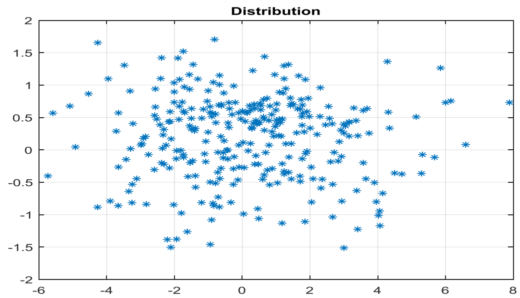

Fig. .3. System Distribution Spread

First order system with estimated delay is represented as follows:

$$G_m(s) = \frac{4.0224e^{-s}}{5.6355s+1} \qquad (31)$$

Figure 4 represents the recognized model output and plant regarding white noise input where model output, has followed main plant output properly.

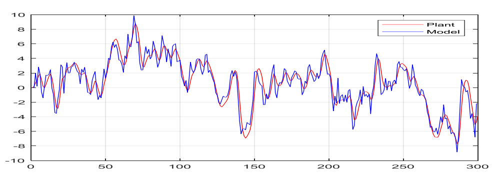

Fig. 4. Plant output and model regarding amounts of white noise input

Discrete time transfer function of process model with fixed time of 1 second is gained using equation 32:

$$G_m(z^{-1}) = \frac{.654z^{-2}}{1-.8374z^{-1}} \qquad (32)$$

Discrete time transfer function for plant is represented as follows:

$$G_p(z^{-1}) = \frac{.05755z^2+.1589z+.02718}{z^3-1.82z^2+1.104z-.2231} \qquad (33)$$

System closed loop transfer function is represented as equation 34:

$$G_{cl}(z^{-1}) = \frac{\acute{k}_{y1}(-.2276z^{-4}+.1387z^{-3}-.323z^{-2}+.5755z^{-1})}{\Delta_{cl}(z^{-1})} \qquad (34)$$

Where, $\Delta_{cl}(z^{-1})$ is represented as (35):

$$\Delta_{cl}(z^{-1}) = -.1868\acute{k}_{x1}z^{-6} + \left(.1868\left(\acute{k}_{x1} + \acute{k}_{y1}\right) + 1.3389\acute{k}_{x1} - .1222\right)z^{-5} + (-1.338\left(\acute{k}_{x1} + \acute{k}_{y1}\right) - 2.5873\acute{k}_{x1} + .8727)z^{-4} + \left(2.5873\left(\acute{k}_{x1} + \acute{k}_{y1}\right) + 2.6843\acute{k}_{x1} - 2.4693\right)z^{-3} + \left(-2.6843\left(\acute{k}_{x1} + \acute{k}_{y1}\right) - 1.4126\acute{k}_{x1} + 3.4567\right)z^{-2} + \left(1.4126\left(\acute{k}_{x1} + \acute{k}_{y1}\right) - 2.3919\right)z^{-1} + .6546 \qquad (35)$$

Stability area for equation 35 is known as figure 5 below:

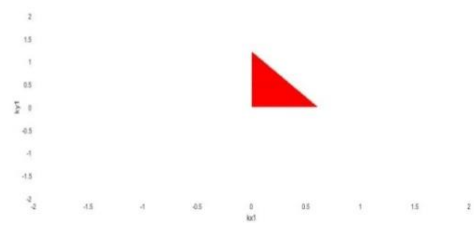

Fig. 5. Stability area for system multiple sentences

Through the selection of amounts $\acute{k}_{x1} = .5$, $\acute{k}_{y1} = .15$, the amount of prediction horizon is considered to be equal to 15. In this research, the control horizon has been considered to be equal to 2. X11, X12, Y11, Y22, and X22 were calculated as follows regarding article [17]:

$$X_{11} = 293.31\,, X_{12} = 274.5\,, X_{22} = 259.86$$

$$Y_{11} = 62.78\,, Y_{22} = 57$$

The amounts r1 and r2 were achieved as follows:

$$r_1 = 29.21\,, r_2 = 45.95$$

Through the application of these amounts in predictive control algorithm, state and output, control signal and control challenges could be represented as figure 6 below:

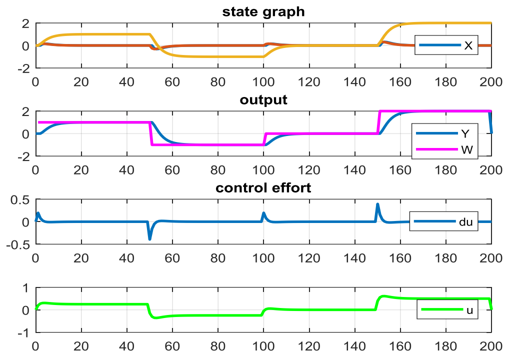

Fig. 6. Output and first order system control signal with estimated delay

Conclusion

In this paper, a three reservoir system was modeled and its recognition was calculated through the application of a white noise signal using plant to first order system with estimated delay through designing a predictive control. The requirement for optimal performance of the predictive control is the proper adjustment of its parameters. The predictive control parameters were adjusted using an analytic method. Results of an optimal performance of simulation of the predictive control represents height control of water level in third tank.

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