A Wedge-Shaped Invisibility Cloak

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ABSTRACT:
In this paper, a wedge-shaped invisibility cloak whose diameter gradually increases along the arms of the wedge is introduced. The constitutive parameters of the cloak are obtained using a proper transformation. By considering two special cases for incident fields, (TEz and TMz), and paying attention that the wedge structure is invariant in axial and radial directions, the reduced set of cloaking parameters are obtained spatially invariant and larger than one, keeping the PML (perfectly matched layer) condition for perfect cloaking performance. Spatially invariant and isotropic constitutive parameters lead to the physically realizable cloak in a wide band of frequency for this specific type of incident fields.

KEYWORDS: Microwave Cloak, Transformation Optics.

1. INTRODUCTION

In recent years, possibility of designing invisibility cloak has attracted much attention. The issue is based on the form invariant coordinate transformation of Maxwell's equations, known as transformation optics (TO) [1-4]. According to TO, two types of electromagnetic cloaks have been proposed; transmission cloak and reflection cloak [5].

The first invisibility cloak had a cylindrical shape, with anisotropic permittivity and permeability with components varying as a function of its radius, which made it very difficult to fabricate. In order to simplify the cloaking structure, some assumptions were made on incident electromagnetic fields. As a result, the material parameters obtained were relatively simple, but still anisotropic and inhomogeneous, making the cloak difficult to fabricate [2].

Afterwards, based on TO, several cloaking structures with different geometries have been proposed, such as spherical cloak, conical cloak, and cylindrical cloak with various cross sections [6-9]. However, achieving a perfect cloaking material with isotropic permittivity and permeability has been very challenging. Obtaining such anisotropic and inhomogeneous materials is feasible with metamaterials. Unfortunately, because of narrow bandwidth of metamaterials, these cloaks cannot perform in a wide range of frequency.

To the best of the author's knowledge, there are several papers concerning the wedge cloaking, but they are mostly based on carpet (reflection) cloak [10], which is a special case of cloaking, assuming the wedge is placed on a ground plane, and cloaking with nanostructured graphene meta-surface which only cancels the dominant scattering mode [11].

In this paper, the constitutive parameters of a wedge-shaped cloak are derived based on a linear transformation and they are obtained homogeneous and in some special cases they are even isotropic. The proposed cloak is simulated and its advantages and disadvantages are discussed.

2. MATERIAL PARAMETERS OF A WEDGE-SHAPED CLOAK

According to the form invariance of the Maxwell's equations under coordinate transformations, the material parameters of a wedge-shaped cloak can be obtained using the Jacobian matrix of the mapping from the original cylindrical (ρ, φ, z) to the new physical one (ρ', φ', z'). The original space is assumed to be free space with permittivity ε_0 and permeability μ_0, while the permitivitty $(\boldsymbol{\varepsilon}')$ and the permeability $(\boldsymbol{\mu}')$ tensors of the wedge cloak can be expressed in the transformed physical coordinate system (ρ', φ', z') as follows:

$$\boldsymbol{\varepsilon}' = \begin{bmatrix} \varepsilon'_\rho & 0 & 0 \\ 0 & \varepsilon'_\varphi & 0 \\ 0 & 0 & \varepsilon'_z \end{bmatrix}, \boldsymbol{\mu}' = \begin{bmatrix} \mu'_\rho & 0 & 0 \\ 0 & \mu'_\varphi & 0 \\ 0 & 0 & \mu'_z \end{bmatrix} \quad (1)$$

Compressing the solid wedge $(-\varphi_0 \le \varphi \le \varphi_0)$, as shown in Fig. 1(a), into a wedge cover ($-\varphi_0 \le \varphi' \le -\varphi_1$ and $\varphi_1 \le \varphi' \le \varphi_0$), shown in Fig. 1(b), the spatial components of the two coordinates can be related to each other as follows:

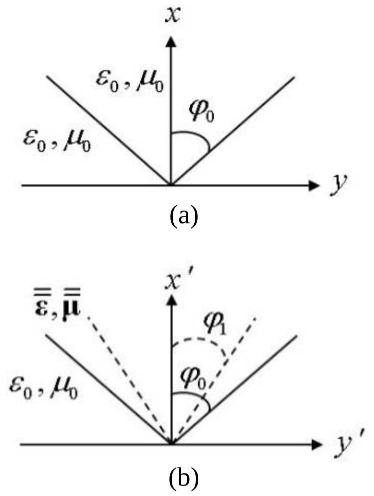

Fig. 1. (a) Geometry of the wedge in the original free space, (b) Geometry of the wedge-shaped cloak.

$$\rho' = \rho, z' = z$$

$$\text{for } 0 \le \varphi \le \varphi_0 : \begin{cases} \varphi = 0 \rightarrow \varphi'_R = \varphi_1 \\ \varphi = \varphi_0 \rightarrow \varphi'_R = \varphi_0 \end{cases}$$

$$\text{for } -\varphi_0 \le \varphi \le 0 : \begin{cases} \varphi = 0 \rightarrow \varphi'_L = -\varphi_1 \\ \varphi = -\varphi_0 \rightarrow \varphi'_L = -\varphi_0 \end{cases}$$

$$\left.\begin{aligned} &\text{for } 0 \le \varphi \le \varphi_0 : \varphi'_R = \frac{(\varphi_0 - \varphi_1)}{\varphi_0}\varphi + \varphi_1 \\ &\text{for } -\varphi_0 \le \varphi \le 0 : \varphi'_L = \frac{(\varphi_0 - \varphi_1)}{\varphi_0}\varphi - \varphi_1 \end{aligned}\right\} \quad (2)$$

$$\Rightarrow \frac{\partial \varphi'}{\partial \varphi} = \frac{(\varphi_0 - \varphi_1)}{\varphi_0}$$

The Jacobi matrix of the transformation is [1]:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\varphi_0 - \varphi_1}{\varphi_0} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The original space is assumed to be free space. Hence, the constitutive parameters of the transformed space are:

$$\boldsymbol{\mu}' = \frac{\mathbf{A}\boldsymbol{\mu}\mathbf{A}^T}{\det \mathbf{A}}, \boldsymbol{\varepsilon}' = \frac{\mathbf{A}\boldsymbol{\varepsilon}\mathbf{A}^T}{\det \mathbf{A}}$$

$$\boldsymbol{\varepsilon}' = \boldsymbol{\mu}' = \begin{bmatrix} \frac{1}{\Delta} & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \frac{1}{\Delta} \end{bmatrix}, \Delta = \frac{\varphi_0 - \varphi_1}{\varphi_0} \quad (4)$$

where

$$\varepsilon'_\rho = \mu'_\rho = \varepsilon'_z = \mu'_z = \frac{1}{\varepsilon'_\varphi} = \frac{1}{\mu'_\varphi} = \frac{1}{\Delta} : \text{PML condition} \quad (5)$$

$\varepsilon'_\rho, \varepsilon'_\varphi, \varepsilon'_z$ represent the relative permittivity components along ρ', φ', z' directions, while $\mu'_\rho, \mu'_\varphi, \mu'_z$ represent the relative permeability components along ρ', φ', z' directions, respectively. For a perfect cloak, the PML (perfectly matched layer) condition on the interface of the cloak and free space must be satisfied [4]. As can be seen in Eq. (5), this condition is satisfied too. Since $\varphi_0 > \varphi_1 > 0$, the relative magnitude of $\varepsilon'_\rho, \mu'_\rho, \varepsilon'_z, \mu'_z$ are larger than one, which is naturally available; but $\varepsilon'_\varphi, \mu'_\varphi$ are smaller than one, feasible with metamaterials, which may limit the operational bandwidth of the structure.

Beside the advantages of this cloak, there are some problems in preparing it for fabrication: anisotropic constitutive parameters, infinite length of the wedge arms, and the varying diameter of the cloak. The first issue can be tackled considering TMz and TEz polarizations separately.

2.1. Case 1: TMz ($H'_z = 0$)

In this case, the Maxwell's equations reduce to the following equations:

$$\left.\begin{aligned} &\nabla \times \mathbf{E}' = -j\omega\mu_0 \boldsymbol{\mu}'\mathbf{H}' \\ &\nabla \times \mathbf{H}' = j\omega\varepsilon_0 \boldsymbol{\varepsilon}'\mathbf{E}' \\ &\frac{\partial}{\partial \rho'} = \frac{\partial}{\partial z'} = 0, H'_z = 0 \end{aligned}\right\}$$

$$\Rightarrow H'_\varphi = E'_\rho = E'_\varphi = 0, \begin{cases} \frac{1}{\rho'}\frac{\partial E'_z}{\partial \varphi'} = -j\omega\mu_0\mu'_\rho H'_\rho \\ -\frac{1}{\rho'}\frac{\partial H'_\rho}{\partial \varphi'} = j\omega\varepsilon_0\varepsilon'_z E'_z. \end{cases} \quad (6)$$

Considering ε'_z is spatially invariant, $\varepsilon'_z E'_z$ can be substituted by D'_z . therefore, the above equations can be rewritten as follows:

$$\begin{cases} \frac{1}{\rho'}\frac{\partial D'_z}{\partial \varphi'} = -j\omega\mu_0\mu'_\rho\varepsilon'_z H'_\rho \\ -\frac{1}{\rho'}\frac{\partial H'_\rho}{\partial \varphi'} = j\omega\varepsilon_0 D'_z. \end{cases} \quad (7)$$

As can be seen, the dispersion will not change if $\varepsilon'_z\mu'_\rho$ remains equal to $1/\Delta^2$.

$$\mu'_{\rho 1}\varepsilon'_{z1} = \mu'_{\rho 2}\varepsilon'_{z2} = 1/\Delta^2. \quad (8)$$

Based on Eq. (7), the relationship among the electromagnetic wave components depend only on μ'_ρ and ε'_z . As a result, the other constitutive parameters of the cloaking medium ($\varepsilon'_\rho, \mu'_z, \varepsilon'_\varphi, \mu'_\varphi$), which are ineffective in the equations, can have any arbitrary value. According to Eqs. (5) and (8), by choosing $\mu'_{\rho 2} = \varepsilon'_{z2} = 1/\Delta$, we have:

$$\varepsilon'_{\rho 2} = \varepsilon'_{\varphi 2} = \varepsilon'_{z2} = \varepsilon', \mu'_{z2} = \mu'_{\varphi 2} = \mu'_{\rho 2} = \mu'. \quad (9)$$

actually we need an isotropic material supporting the PML condition with equal constant permittivity and permeability equal to $1/\Delta$ which is greater than one. Although natural materials usually have constitutive parameters larger than one but, it is generally very difficult to find natural materials with equal magnitudes for permittivity and permeability, especially in a wide range of frequency. Instead, we can choose $\mu'_{\rho 2} = 1$ and $\varepsilon'_{z2} = 1/\Delta^2$, easier to obtain in nature, but dismissing the PML condition.

2.2. Case 2: TE$_z$ ($E'_z = 0$)

The results of this case are similar to the TM$_z$ case. As a result, a unique cloak can deal with both TE$_z$ and TM$_z$ incident fields, perfectly.

In order to verify the performance of the truncated cloak, a finite PEC wedge with and without cloak under direct and oblique incident fields is simulated. The arms are truncated in 4.5 wavelengths. $\varphi_0 = 20^\circ$ and $\varphi_1 = 10^\circ$ are selected as an example. Considering the case TM$_z$, the wedge is illuminated by an electric field (E_z), as shown in Fig. 2.

Based on Eq. (9), the cloak is assumed to be perfect with $\varepsilon = \mu = 2$, but since it is truncated, we do not anticipate the ideal response. As simulation results show, Fig. 2(a-f), the incident field is less distorted when a PEC wedge is placed inside the cloak. The simulation results show the good and acceptable performance of the cloak even for a finite structure.

The simulation operating frequency is assumed to be 3 GHz. It is apparent that if the operating frequency changes, the electrical length of the wedge would differ, where in the ideal cloak with infinite arms, it would have no effect. Hence, it is expected that the ideal cloak would have infinite operating bandwidth, which is the anticipated performance of structures with geometries specified by angles [12]. However, because of truncation of the wedge arms, the bandwidth and also the efficiency of the cloak would be limited, but still it can be designed with acceptable properties in some practical applications.

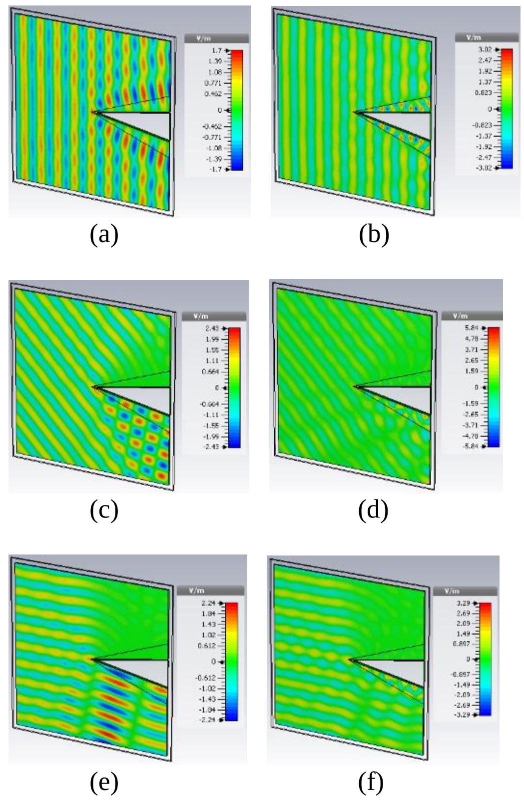

Fig. 2. (a) A PEC wedge in free space. The incident electric field is E_z and it propagates in *x* direction. (b) A PEC wedge inside a wedge-shaped cloak. The incident electric field is E_z and it propagates in *x* direction. (c) A PEC wedge in free space. The incident electric field is E_z and it propagates in ρ direction with $\varphi = 45^\circ$. (d) A PEC wedge inside a wedge-shaped cloak. The incident electric field is E_z and it propagates in ρ direction with $\varphi = 45^\circ$. (e) A PEC wedge in free space. The incident electric field is E_z and it propagates in y direction. (f) A PEC wedge inside a wedge-shaped cloak. The incident electric field is E_z and it propagates in *y* direction.

3. CONCLUSION

A wedge-shaped invisibility cloak is designed based on electromagnetic transformation. The constitutive parameters of the perfect cloak are obtained spatially invariant and anisotropic. It has been shown that for two special cases, (TEz and TMz) the constitutive parameters of the cloak can be set as spatially invariant and isotropic. Hence, it can perform in a wide band of frequency and it can be easily fabricated. To the best of our knowledge, the contributions of current paper can be summarized as follows:

1. A new cloak geometry with simple constitutive parameters has been proposed.

2. The simplified cloak is homogeneous isotropic for both TEz and TMz waves.
3. The cloak performance is independent of direction of wave propagation.

REFERENCES

[1] D. Kwon, and D. H. Werner, **“Transformation electromagnetics: an overview of the theory and applications,”** *IEEE Antennas and Propagation Magazine*, Vol. 52, No. 1, pp. 24-46, Feb. 2010.

[2] D. Schurig, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, **“Metamaterial electromagnetic cloak at microwave frequencies,”** *Science*, Vol. 314, pp. 977-980, 10 Nov. 2006.

[3] S. A. Cummer, B. Popa, D. Schurig, D. R. Smith, and J. Pendry, **“Fullwave simulations of electromagnetic structures,”** *Physical Review*, E 74, 036621, pp. 1 -5, 2006.

[4] J. B. Pendry, D. Schurig, and D. R. Smith, **“Controlling electromagnetic fields,”** *Sciencexpress*, 10.1126, science. 1125907, pp. 1 -4, 25 May 2006.

[5] X. Wang, **“Experimental and computational studies of electromagnetic cloaking at microwaves,”** PhD thesis, Michigan Technological University, Michigan, 2014.

[6] Q. Cheng, W.X. Jiang, and T. J. Cui, **“Investigations of the electromagnetic properties of three-dimensional arbitrarily shaped cloaks,”** *Progress In Electromagnetics Research*, PIER 94, pp. 105-117, 2009.

[7] Y. Luo, J. Zhang, B. Wu, and H. Chen, **“Interaction of an electromagnetic wave with a cone-shaped invisibility cloak and polarization rotator,”** *Physical Review B 78*, 125108, pp. 1 -9, 2008.

[8] S. Taravati, and A. Abdolali, **“A new three dimensional conical groundplane cloak with homogeneous materials,”** *Progress In Electromagnetics Research M*, Vol. 19, pp. 91 -104, 2011.

[9] A. Nikolet, F. Zolla, and S. Guenneau, **“Finite-element analysis of cylindrical invisibility cloaks of elliptical cross section,”** *IEEE Transactions on Magnetics*, Vol. 44, No. 6, pp. 1150-1153, June 2008.

[10] B. Zhang, Y. Luo, X. Liu, and G. Barbastathis, **“macroscopic invisibility cloak for visible light**", *Phys. Rev. Lett*. 106, 033901, pp. 1 -12, Jan., 2011.

[11] A. Forouzmand, and A. B. Yakovlev, **“Electromagnetic cloaking of a finite conducting wedge with a nanostructured graphene metasurface,”** *IEEE Trans. On Antennas and Propag.*, Vol. 63, No. 5, pp. 2191 -2202, May 2015.

[12] C. A. Balanis, **“Antenna Theory, Analysis and Design, ”**, *third edition, John Wiley and Sons*, 2005.