Robust State Estimation for Uncertain Switched Fuzzy Systems with Time-Varying Delays by Average Dwell Time Approach

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Received: Feb. 2015 Revised: May 2015 Accepted: July 2015

ABSTRACT:
Switched systems are an important class of hybrid systems. In recent years, such systems have drawn considerable attentions in control field. A switched fuzzy system is a switching system, for which all subsystems are fuzzy systems. This paper investigates the robust state estimation problem for a class of uncertain switched fuzzy systems with time-varying delays. By using appropriate switched Lyapunov functional approach, average dwell time scheme and H_∞ filtering theory, delay dependent sufficient conditions for the solvability of this problem are stablished in terms of linear matrix inequalities (LMIs). An illustrative example is provided to show the effectiveness of the proposed theoretical results.

KEYWORDS: Robust State Estimation, Average Dwell Time, Switched Fuzzy Systems, LMIs.

1. INTRODUCTION

Switched systems are a class of sophisticated nonlinear systems that consist of many subsystems and a switching strategy that governs switching between the subsystems [1]. Most of modern technological systems, such as water quality process [3], unmanned aerial vehicles [4] and automotive engine control [5], require several dynamical systems to describe their behaviour due to various environmental factors [2]. Using Takagi-Sugeno (T-S) approach, a nonlinear system can be represented by a set of local linear systems [6]. Switching between subsystems in switched systems can be assumed to be fast or slow. In stabilization context, specifying a dwell time is conservative [7]. On the other hand, time delay is very common in real applications because of mechanical structures, signal transmission over the network and so on. The existence of time delay in a system usually lead to instability or bad performance of the system [8]. Moreover, unknown inputs and model uncertainties are coupled in many practical systems. H_∞ filtering theory is used to solve this obstacle [9]. In [10], H_∞ control for asynchronous switched systems with mode dependent average dwell time is studied.

Lots of researches have been devoted to stability of switched systems. Stabilization of switched linear systems with unknown time-varying delays under arbitrary switching signal has been investigated in [11]. Authors in [12] have studied stability of switched systems with stable and unstable subsystems via average dwell time approach. In [13] stability of discrete time linear systems with a constant delay factor is considered which render the delay-independent results. The case of time-varying delay is addressed in [14] that cause to delay dependent results. Delay dependent approaches are more practical and yield less conservative results [15]. In [16], a delay-dependent stability criterion, based on an input-output approach has been studied such that the interconnected system is asymptotically stable. Moreover, there have been several studies in the field of switched fuzzy systems. In [17] fuzzy reliable controllers via observer switching for uncertain time-delay switched fuzzy systems are designed. Authors in [18] have designed state feedback controllers for switched fuzzy systems which make the closed loop system quadratically stable. To the best of our knowledge, the problem of state estimation for switched fuzzy systems, has not been considered yet, which motivated us to study this issue.

The aim of this paper is to study robust state estimation for uncertain time-delay switched fuzzy systems with time-varying state-delays, under arbitrary switching signal. The proposed approach uses switched Lyapunov functional method and average dwell time approach. A numerical example demonstrate the effectiveness of the proposed approach.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following switched fuzzy system that each subsystem is an uncertain time-delay fuzzy system:

$$R_\sigma^l : if\ \ \xi_1 \text{ is } \mathrm{M}_{\sigma 1}^l \text{ ... and } \xi_p \text{ is } \mathrm{M}_{ip}^l, \text{ then}$$

$$\begin{aligned} x_{k+1} &= (A_{\sigma l} + \Delta A_{\sigma l})x_k + (A_{d\sigma l} + \Delta A_{d\sigma l})x_{k-d} \\ &\quad + (\mathrm{B}_{\sigma l} + \Delta B_{\sigma l})\mathrm{u}_k + \mathrm{G}_{\sigma l}\ f_k + E_{\sigma l} d_k \\ y_k &= C_{\sigma l}x_k + C_{d\sigma l}x_{k-d} + Q_{\sigma l}u_k + D_{\sigma l}d_k + J_{\sigma l} f_k \end{aligned} \tag{1}$$

Where $x_k \in \mathbb{R}^{n_x}$, $y_k \in \mathbb{R}^{n_y}$ and $d_k \in \mathbb{R}^{n_d}$ are respectively the state, the measured output and the unknown input that belongs to $L_2[0,\infty)$. $\sigma = \sigma(x_k):[0,+\infty) \to \bar{M} = \{1,2,...,\mathrm{m}\}$ is the switching signal. $u_k \in \mathbb{R}^{n_u}$ is the control input and $f_k \in \mathbb{R}^{n_f}$ is the fault vector. The matrices $A_{il}, A_{dil}, B_{il}, G_{il}, E_{il}, C_{il}, C_{dil}, Q_{il}, D_{il}, J_{il}$ are constant with appropriate dimensions; ΔA_{il}, ΔA_{dil}, ΔB_{il} are norm-bounded unknown matrices representing parameter uncertainties, and are assumed to be of the form of (5). R_σ^l denotes the lth fuzzy plant rule in the σth subsystem. The global fuzzy model of the i-th switched subsystem is represented by:

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{N_i} \eta_{il}(\xi_k) \begin{pmatrix} (A_{il} + \Delta A_{il})x_k + (A_{dil} + \Delta A_{dil})x_{k-d} \\ +(\mathrm{B}_{il} + \Delta B_{il})\mathrm{u}_k + \mathrm{G}_{il}\ f(k) + E_{il}d_k \end{pmatrix} \\ y_k &= \sum_{i=1}^{N_i} \eta_{il}(\xi_k) \begin{pmatrix} C_{il}x_k + C_{dil}x_{k-d} + Q_{il}u_k + D_{il}d_k \\ +J_{il} f_k \end{pmatrix} \end{aligned} \tag{2}$$

$$0 \le \eta_{il}(\xi_k) \le 1, \quad \sum_{i=1}^{N_i} \eta_{il}(\xi_k) = 1, \quad \mathrm{i} = 1,...,\mathrm{m}$$

Where

$$\eta_{il}(\xi_k) = \Theta_{il}(\xi_k) / \sum_{l=1}^{N_i} \Theta_{il}(\xi_k)\ , \Theta_{il}(\xi_k) = \prod_{\rho=1}^{p} M_{i\rho}^l(\xi_{\rho k})$$

In which $M_{i\rho}^l(\xi_{\rho k})$ is the membership function. Since states of the system are not often measured directly, it is assumed that $\sigma = \sigma(\hat{x}_k)$ where $\hat{x}_k$ is the filter's state. Assume that operation space can be partitioned into m regions, i.e. $\bar{\Omega}_1 \cup \bar{\Omega}_2 \cup ... \cup \bar{\Omega}_m = \mathbb{R}^n$ and $\bar{\Omega}_i \cap \bar{\Omega}_j = \phi$, $\mathrm{i} \ne \mathrm{j}$. When $\hat{x}_k \in \bar{\Omega}_i$ the switching signal is $\sigma = \sigma(\hat{x}_k) = i$, which depends on $\bar{\Omega}_1, \bar{\Omega}_2, .., \bar{\Omega}_m$. When $\hat{x}_k \in \bar{\Omega}_i$ the switching signal is subjected to:

$$\mu_i(\hat{x}_k) = \begin{cases} 1 & , \hat{x}_k \in \bar{\Omega}_i \\ 0 & , \hat{x}_k \notin \bar{\Omega}_i \end{cases}\ , \ \mathrm{i} \in \bar{M} \tag{3}$$

that is, if and only if $\sigma = \sigma(\hat{x}_k) = i$, $\mu_i(\hat{x}_k) = 1$. Thus the system (1) is described by [17]:

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{m}\sum_{l=1}^{N_i} \mu_i(\hat{x}_k)\eta_{il}(\xi_k) \begin{bmatrix} (A_{il} + \Delta A_{il})x_k + (A_{dil} + \Delta A_{dil})x_{k-d} \\ +(\mathrm{B}_{il} + \Delta B_{il})\mathrm{u}_k + \mathrm{G}_{il}\ f_k + E_{il}d_k \end{bmatrix} \\ y_k &= \sum_{i=1}^{m}\sum_{l=1}^{N_i} \mu_i(\hat{x}_k)\eta_{il}(\xi_k) \begin{bmatrix} C_{il}x_k + C_{dil}x_{k-d} + Q_{il}u_k \\ +D_{il}d_k + J_{il} f_k \end{bmatrix} \end{aligned} \tag{4}$$

Consider the discrete-time switched fuzzy system that is described by (4) in which ΔA_{il}, ΔA_{dil}, ΔB_{il} satisfying

$$\left[\Delta A_{il}(k)\ \ \Delta A_{dil}(k)\ \ \Delta B_{il}(k)\right] = H_{il}\bar{F}_k \left[\bar{C}_{1il}\ \ \bar{C}_{2il}\ \ \bar{C}_{3il}\right] \tag{5}$$

where $\bar{H}_{il}$, $\bar{C}_{1il}$, $\bar{C}_{2il}$, $\bar{C}_{3il}$ are constant matrices and $\bar{F}_k$ is an unknown time-varying matrix satisfying $\bar{F}_k^T \bar{F}_k \le I$. The considered observer is as:

$$\begin{cases} \hat{x}_{k+1} = \sum_{i=1}^{m}\sum_{l=1}^{N_i} \mu_i(\hat{x}_k)\eta_{il}(\xi_k)(A_{fil}\hat{x}_k + B_{fil}y_k) \\ r_k = \sum_{i=1}^{m}\sum_{l=1}^{N_i} \mu_i(\hat{x}_k)\eta_{il}(\xi_k)(\mathrm{C}_{fil}\ \hat{x}_k + D_{fil}y_k) \end{cases} \tag{6}$$

Where $\hat{x}_k$ is the estimated state and r_k is the residual signal, A_{fil}, B_{fil}, C_{fil}, D_{fil} are the filter parameters and $\hat{f}(z) = W_f(z)f(z)$ is the weighted fault with the following minimal realization:

$$\begin{cases} \bar{x}_{k+1} = A_\omega \bar{x}_k + B_\omega f_k \\ \hat{\mathrm{f}}_k = \mathrm{C}_\omega \bar{x}_k + D_\omega f_k \end{cases} \tag{7}$$

where $\bar{x}_k$ is the state of the weighted fault and A_ω, B_ω, C_ω, D_ω are known constant matrices. Denoting $e_k = r_k - \hat{f}_k$, which r_k is an estimate of the $\hat{f}_k$. The augmented system can be written as:

$$\begin{cases} \tilde{x}_{k+1} = \sum_{i=1}^{m}\sum_{l=1}^{N_i} \mu_i(\hat{x}_k)\eta_{il}(\xi_k)\left(\tilde{A}_{il}\tilde{x}_k + \tilde{A}_{dil}\tilde{x}_{k-d} + \tilde{B}_{il}\omega_k\right) \\ e_k = \sum_{i=1}^{m}\sum_{l=1}^{N_i} \mu_i(\hat{x}_k)\eta_{il}(\xi_k)\left(\tilde{C}_{il}\tilde{x}_k + \tilde{C}_{dil}\tilde{x}_{k-d} + \tilde{D}_{il}\omega_k\right) \end{cases} \tag{8}$$

$$\tilde{x}_k = \begin{pmatrix} x_k \\ \hat{x}_k \\ \bar{x}_k \end{pmatrix} \quad \tilde{x}_{k+1} = \begin{pmatrix} x_{k+1} \\ \hat{x}_{k+1} \\ \bar{x}_{k+1} \end{pmatrix} \quad \tilde{A}_{il} = \begin{pmatrix} A_{il}+\Delta A_{il} & 0 & 0 \\ B_{fil}C_{il} & A_{fil} & 0 \\ 0 & 0 & A_\omega \end{pmatrix}$$

$$\tilde{x}_{k-d} = \begin{pmatrix} x_{k-d} \\ \hat{x}_{k-d} \\ \bar{x}_{k-d} \end{pmatrix} \qquad \tilde{A}_{dil} = \begin{pmatrix} A_{dil}+\Delta A_{dil} & 0 & 0 \\ B_{fil}C_{dil} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\omega_k = \begin{pmatrix} u_k \\ d_k \\ f_k \end{pmatrix} \qquad \tilde{B}_{il} = \begin{pmatrix} B_{il}+\Delta B_{il} & E_{il}+\Delta E_{il} & G_{il} \\ B_{fil}Q_{il} & B_{fil}D_{il} & B_{fil}J_{il} \\ 0 & 0 & B_\omega \end{pmatrix}$$

$$\tilde{C}_{il} = (D_{fil}C_{il} \quad C_{fil} \quad -C_\omega) \quad \tilde{C}_{dil} = (D_{fil}C_{dil} \quad 0 \quad 0)$$

$$\tilde{D}_{il} = (D_{fil}Q_{il} \quad D_{fil}D_{il} \quad D_{fil}J_{il} - D_\omega)$$

Remark 2 [19]: For predetermined scalars $\gamma > 0$, $0 < \alpha < 1$, system (8) is exponentially stable with an exponential H_∞ performance γ, if it is exponentially stable and under zero initial conditions the estimated error e_k satisfies:

$$\sum_{s=k_0}^{\infty} (1-\alpha)^s e^T(s)e(s) \le \sum_{s=k_0}^{\infty} \gamma^2 \omega^T(s)\omega(s) \tag{9}$$

Lemma 1 [20]: For any matrix $W \in \mathbb{R}^{n\times n}$, $W = W^T \ge 0$ and two positive integers r, r_0, which $r \ge r_0 \ge 1$, the following inequality holds

$$\left(\sum_{i=r_0}^{r} x(i)\right)^T W \left(\sum_{i=r_0}^{r} x(i)\right) \le \tilde{r} \sum_{i=r_0}^{r} x^T(i)Wx(i) \tag{10}$$

where $\tilde{r} = r - r_0 + 1$.

Lemma 2 [21]: For any matrices A, $Q = Q^T$ if there exist a matrix T, following inequalities are equal:

a. $A^T PA - Q < 0$

b. $$\begin{bmatrix} -Q & A^T T \\ * & P - T - T^T \end{bmatrix} < 0 \tag{11}$$

2.1. Filter Synthesis

In this section a delay-dependent sufficient condition on the existence of the robust filters would be given.

Theorem 1: For given scalars $\alpha > 0$, $\mu > 1$ and any delay, $d(k)$ satisfying $d_m \le d(k) \le d_M$, if there exist the positive definite matrices P_{il}, Q_{1il}, Q_{2il}, R_{1il}, R_{2il} such that the following inequalities holds:

$$\begin{bmatrix} \psi & d_m\phi_2^T R_{1il} & \bar{d}\phi_2^T R_{2il} & \phi_3^T & \phi_1^T P_{il} \\ * & -R_{1il} & 0 & 0 & 0 \\ * & * & -R_{2il} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P_{il} \end{bmatrix} < 0 \tag{12}$$

$$\begin{gathered} P_{il} \le \mu P_{jl},\ Q_{1il} \le \mu Q_{1jl},\ Q_{2il} \le \mu Q_{2jl}, \\ R_{1il} \le \mu R_{1jl},\ R_{2il} \le \mu R_{2jl} \end{gathered} \tag{13}$$

$$T_a > T_a^* = -\frac{\ln u}{\ln(1-\alpha)} \tag{14}$$

then the system (8) is exponentially stable with decay rate $\beta = \sqrt{(1-\alpha)\mu^{1/T_a}}$ and under any switching signal with the average dwell time T_a satisfying (14) where:

$$\bar{d} = d_M - d_m + 1$$

$$\psi = \begin{bmatrix} \psi_{11} & 0 & \psi_{13} & 0 & 0 \\ * & \psi_{22} & \psi_{23} & \psi_{24} & 0 \\ * & * & \psi_{33} & 0 & 0 \\ * & * & * & \psi_{44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\phi_1 = \begin{bmatrix} \tilde{A}_{il} & \tilde{A}_{dil} & 0 & 0 & \tilde{B}_{il} \end{bmatrix}$$

$$\phi_2 = \begin{bmatrix} (A_{il} - I)H & A_{dil} & 0 & 0 & \bar{B}_{il} \end{bmatrix}$$

$$\phi_1 = \begin{bmatrix} \tilde{C}_{il} & \tilde{C}_{dil} & 0 & 0 & \tilde{D}_{il} \end{bmatrix}$$

where

$$\bar{B}_{il} = \begin{bmatrix} B_{il} & G_{il} & E_{il} \end{bmatrix}$$

$$\psi_{11} = -(1-\alpha)P_{il} + H^T Q_{1il} H + H^T Q_{2il} H - (1-\alpha)^{d_m} H^T R_{1il} H,$$

$$\psi_{13} = (1-\alpha)^{d_m} H^T R_{1il}, \quad \psi_{22} = -2(1-\alpha)^{d_M} R_{2il},$$

$$\psi_{23} = (1-\alpha)^{d_M} R_{2il}, \qquad \psi_{24} = (1-\alpha)^{d_M} R_{2il},$$

$$\psi_{33} = -(1-\alpha)^{d_m} R_{1il} - (1-\alpha)^{d_M} R_{2il} - (1-\alpha)^{d_m} Q_{1il},$$

$$\psi_{44} = -(1-\alpha)^{d_M} Q_{2il} - (1-\alpha)^{d_M} R_{2il}.$$

Proof: First, exponential stability of the system (8) with $\omega_k = 0$ is considered. Choose the following switched Lyapunov functional

$$V_{il}(k)=V_{1il}(k)+V_{2il}(k)+V_{3il}(k), \qquad k\in[k_l,k_{l+1}]$$
$$V_{1il}(k)=\tilde{x}^T(k)\left(\sum_{i=1}^{m}\xi_{il}(k)P_{il}\right)\tilde{x}(k)$$
$$V_{2il}(k)=\sum_{s=k-d_m}^{k-1}x^T(s)(1-\alpha)^{k-s-1}\left(\sum_{i=1}^{m}\xi_{il}(k)Q_{1il}\right)x(s)+\sum_{s=k-d_M}^{k-1}x^T(s)(1-\alpha)^{k-s-1}\left(\sum_{i=1}^{m}\xi_{il}(k)Q_{2il}\right)x(s)$$
$$V_{3il}(k)=d_m\sum_{s=-d_m}^{-1}\sum_{n=k+s}^{k-1}\left[\begin{array}{l}\eta^T(n)(1-\alpha)^{k-n-1}\left(\sum_{i=1}^{m}\xi_{il}(k)R_{1il}\right)\eta(n)\\ +\bar{d}\sum_{s=-d_M}^{-d_m-1}\sum_{n=k+s}^{k-1}\left(\begin{array}{l}\eta^T(n)(1-\alpha)^{k-m-1}\\ \left(\sum_{i=1}^{m}\xi_{il}(k)R_{2il}\right)\eta(n)\end{array}\right)\end{array}\right] \quad (15)$$

Where $\eta(n)=x(n+1)-x(n).$ by taking the difference between the considered Lyapunov function for two consecutive time instants and using lemma 1 one has:

$$\Delta V_{il}+\alpha V_{il}\le \varsigma_1^T(k)(\bar{\psi}+d_m^2\bar{\phi}_2^T R_{1il}\bar{\phi}_2+\bar{d}^2\bar{\phi}_2^T R_{2il}\bar{\phi}_2+\bar{\phi}_1^T P_{il}\bar{\phi}_1)\varsigma_1(k) \quad (16)$$

where

$$\bar{\psi}=\begin{bmatrix}\psi_{11} & 0 & \psi_{13} & 0\\ * & \psi_{22} & \psi_{23} & \psi_{24}\\ * & * & \psi_{33} & 0\\ * & * & * & \psi_{44}\end{bmatrix}$$
$$\bar{\phi}_1=\begin{bmatrix}\tilde{A}_{il} & \tilde{A}_{dil} & 0 & 0\end{bmatrix},\ \phi_3=\begin{bmatrix}\tilde{C}_{il} & \tilde{C}_{dil} & 0 & 0\end{bmatrix} \quad (17)$$
$$\bar{\phi}_2=\begin{bmatrix}(A_{il}-I)H & A_{dil} & 0 & 0\end{bmatrix}$$
$$\varsigma_{1k}=\begin{bmatrix}\tilde{x}_k^T & \tilde{x}^T(k-d(k)) & \tilde{x}_{k-d_m}^T & \tilde{x}_{k-d_M}^T\end{bmatrix}$$

Using Schur complement we have

$$\Delta V_{il}+\alpha V_{il}(k)\le 0\rightarrow V_{\sigma(k)}(k)\le(1-\alpha)^{k-k_l}V_{\sigma(k_l)}(k_l) \quad (18)$$

Using (13) and (18) we have

$$V_{\sigma(k)}(k)\le\ldots\le\left((1-\alpha)\mu^{1/T_a}\right)^{(k-k_0)}V_{\sigma(k_0)}(k_0) \quad (19)$$

Using (19) and for considered Lyapunov functional we have

$$\beta_1\|\tilde{x}(k)\|^2\le V_{\sigma(k)}(k)\le\left((1-\alpha)\mu^{1/T_a}\right)^{(k-k_0)}V_{\sigma(k_0)}(k_0)\le\left((1-\alpha)\mu^{1/T_a}\right)^{(k-k_0)}\beta_2\|\tilde{\varphi}(k)\|_L^2 \quad (20)$$

Define $\beta=\sqrt{(1-\alpha)\mu^{1/T_a}}$ then we can obtain that $\|\tilde{x}(k)\|\le\sqrt{\beta_2/\beta_1}\beta^{(k-k_0)}\|\tilde{\varphi}\|_L$, where

$$\beta_1=\min_{i\in M}\lambda_{\min}(P_{il}),$$
$$\beta_2=\max_{i\in M}\lambda_{\max}(P_{il})+d_1\max_{i\in M}\lambda_{\max}(H^TQ_{1il}H)+d_2\max_{i\in M}\lambda_{\max}(H^TQ_{2il}H)+2d_1(d_1+1)\max_{i\in M}\lambda_{\max}(H^TZ_{1il}H)+2d_2(\bar{d}+1)\max_{i\in M}\lambda_{\max}(H^TZ_{2il}H)$$

From (14) we have

$$(1-\alpha)\mu^{1/T_a}\le(1-\alpha)\mu^{-\ln(1-\alpha)/\ln\mu}\le\frac{1-\alpha}{1-\alpha}=1\rightarrow\beta<1$$

therefore, using Remark 2, the augmented system with $\omega(k)=0$ is exponentially stable. Like the previous steps we have:

$$\Delta V_{il}(k)+\alpha V_{il}(k)+\Gamma(k)\le \varsigma_2^T(k)\begin{pmatrix}\psi+d_m^2\phi_2^TR_{1il}\phi_2+\bar{d}^2\phi_2^TR_{2il}\phi_2\\ +\phi_1^TP_{il}\phi_1+\phi_3^T\phi_3\end{pmatrix}\varsigma_2(k) \quad (21)$$

Where

$$\varsigma_2=\begin{bmatrix}\tilde{x}_k^T & x^T(k-d(k)) & x^T(k-d_m) & x^T(k-d_M) & \omega^T(k)\end{bmatrix}^T$$
$$\Gamma(k)=e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)$$

Using Schur complement from (12), we conclude:

$$\Delta V_{il}(k)+\alpha V_{il}(k)+\Gamma(k)<0 \quad (22)$$

Using (22) recursively gives:

$$V_{il}(k)\le(1-\alpha)^{k-k_0}V_{il}(k_0)-\sum_{s=k_0}^{k-1}(1-\alpha)^{k-s-1}\Gamma(s) \quad (23)$$

Now consider following performance index

$$J_{per}\triangleq\sum_{s=k_0}^{\infty}(1-\alpha)^s e^T(s)e(s)-\gamma^2\omega^T(s)\omega(s) \quad (24)$$

Using (13) and (23) we can obtain:

$$V_{\sigma(k)}(k)\le(1-\alpha)^{k-k_0}\mu^{N(k_0,k)}V_{\sigma(k_0)}(k_0)-\sum_{s=k_0}^{k-1}\mu^{N(s,k)}(1-\alpha)^{k-s-1}\Gamma(s) \quad (25)$$

Under zero initial condition, from (30) one can obtain

$$\sum_{s=k_0}^{k-1}\mu^{N(s,k)}(1-\alpha)^{k-s-1}\Gamma(s)\le 0 \quad (26)$$

Using $N_\sigma(0,s)\le\frac{s}{T_a}\le\frac{-s\ln(1-\alpha)}{\ln\mu}$ from (26) we have:

$$\sum_{s=k_0}^{k-1} \mu^{-N_\sigma(0,s)} (1-\alpha)^{k-s-1} e^T(s) e(s) \le \sum_{s=k_0}^{k-1} (1-\alpha)^{k-s-1} \gamma^2 \omega^T(s) \omega(s) \tag{27}$$

Considering $k \to \infty$ gives:

$$\sum_{s=k_0}^{\infty} (1-\alpha)^s e^T(s) e(s) \le \sum_{s=k_0}^{\infty} \gamma^2 \omega^T(s) \omega(s) \to J_{per} < 0 \tag{28}$$

therefore, for any nonzero $\omega_k \in l_2[0,\infty)$

$$\sum_{s=k_0}^{\infty} (1-\alpha)^s e^T(s) e(s) \le \sum_{s=k_0}^{\infty} \gamma^2 \omega^T(s) \omega(s)$$

which completes the proof.

Theorem 2: For given scalars $\alpha > 0,\ \mu > 1$ and any delay, $d(k)$ satisfying $d_m \le d(k) \le d_M$, if there exist the positive definite matrices $P_{il},\ Q_{1il},\ Q_{2il},\ R_{1il},\ R_{2il}$ and matrices $T_{il},\ M_{1il},\ M_{2il}$ such that the following inequality holds

$$\begin{bmatrix} \Omega & \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 \\ * & Z_{1il} - M_{1il} - M_{1il}^T & 0 & 0 & 0 \\ * & * & Z_{2il} - M_{2il} - M_{2il}^T & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Xi_5 \end{bmatrix} < 0 \tag{29}$$

where

$$P_{il} = \begin{bmatrix} P_{11il} & P_{12il} & P_{13il} \\ * & P_{22il} & P_{23il} \\ * & * & P_{33il} \end{bmatrix},\ T_{il} = \begin{bmatrix} T_{11il} & T_{12il} & T_{13il} \\ T_{22il} & T_{22il} & 0 \\ T_{31il} & T_{32il} & T_{33il} \end{bmatrix}$$

and

$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & \Omega_{13} & 0 & 0 \\ * & \psi_{22} & \psi_{23} & \psi_{24} & 0 \\ * & * & \psi_{33} & 0 & 0 \\ * & * & * & \psi_{44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix},\ \Xi_1 = \begin{bmatrix} \Xi_{11} \\ d_m A_{dil}^T M_{1il} \\ 0 \\ 0 \\ \Xi_{15} \end{bmatrix}$$

$$\Xi_2 = \begin{bmatrix} \Xi_{21} \\ \bar{d} A_{dil}^T M_{2il} \\ 0 \\ 0 \\ \Xi_{25} \end{bmatrix},\ \Xi_3 = \begin{bmatrix} \Xi_{31} \\ C_{dil}^T D_{Fil}^T \\ 0 \\ 0 \\ \Xi_{35} \end{bmatrix},\ \Xi_4 = \begin{bmatrix} \Xi_{41} \\ \Xi_{42} \\ 0 \\ 0 \\ \Xi_{45} \end{bmatrix}$$

with

$$\Omega_{11} = \begin{bmatrix} \Omega_{111} & -(1-\alpha)P_{12il} & -(1-\alpha)P_{13il} \\ * & -(1-\alpha)P_{22il} & -(1-\alpha)P_{23il} \\ * & * & -(1-\alpha)P_{33il} \end{bmatrix},$$

$$\Omega_{111} = -(1-\alpha)P_{11il} + Q_{1il} + Q_{2il} - (1-\alpha)^{d_m} R_{1il}$$

$$\Omega_{13} = \begin{bmatrix} (1-\alpha)^{d_m} R_{1il} \\ 0 \\ 0 \end{bmatrix},\ \Xi_{11} = \begin{bmatrix} d_m [A_{il} - I]^T M_{1il} \\ 0 \\ 0 \end{bmatrix},$$

$$\Xi_{15} = \begin{bmatrix} d_m E_{il}^T M_{1il} \\ d_m G_{il}^T M_{1il} \\ d_m B_{il}^T M_{1il} \end{bmatrix},\ \Xi_{21} = \begin{bmatrix} \bar{d} [A_{il} - I]^T M_{2il} \\ 0 \\ 0 \end{bmatrix},$$

$$\Xi_{25} = \begin{bmatrix} \bar{d}\ E_{il}^T M_{1il} \\ \bar{d}\ G_{il}^T M_{1il} \\ \bar{d}\ B_{il}^T M_{1il} \end{bmatrix},\ \Xi_{35} = \begin{bmatrix} D_{il}^T D_{Fil}^T \\ J_{il}^T D_{Fil}^T - D_w^T \\ Q_{il}^T D_{Fil}^T \end{bmatrix},\ \Xi_{31} = \begin{bmatrix} C_i^T D_{Fil}^T \\ C_{Fil}^T \\ -C_w^T \end{bmatrix}$$

$$\Xi_{41} = \begin{bmatrix} A_{il}^T T_{11il} + C_{il}^T B_{Fil}^T & A_{il}^T T_{12il} + C_{il}^T B_{Fil}^T & A_{il}^T T_{13il} \\ A_{Fil}^T & A_{Fil}^T & 0 \\ A_w^T T_{31il} & A_w^T T_{32il} & A_w^T T_{33il} \end{bmatrix}$$

$$\Xi_{42} = \begin{bmatrix} A_{dil}^T T_{11il} + C_{dil}^T B_{Fil}^T & A_{dil}^T T_{12il} + C_{dil}^T B_{Fil}^T & A_{dil}^T T_{13il} \end{bmatrix},$$

$$\Xi_{45} = \begin{bmatrix} B_{il}^T T_{11il} + D_{il}^T B_{Fil}^T & B_{il}^T T_{12il} + D_{il}^T B_{Fil}^T & B_{il}^T T_{13il} \\ \Xi_{451} & \Xi_{452} & G_{il}^T T_{13il} + B_w^T T_{33il} \\ E_{il}^T T_{11il} + Q_{il}^T B_{Fil}^T & E_{il}^T T_{12il} + Q_{il}^T B_{Fil}^T & E_{il}^T T_{13il} \end{bmatrix},$$

$$\Xi_{451} = G_i^T T_{11il} + J_{il}^T B_{Fil}^T + B_w^T T_{31il},$$

$$\Xi_{452} = G_{il}^T T_{12il} + J_{il}^T B_{Fil}^T + B_w^T T_{32il}$$

$$\Xi_5 = \begin{bmatrix} P_{11il} - T_{11il} - T_{11il}^T & P_{12il} - T_{12il} - T_{22il}^T & P_{13il} - T_{13il} - T_{31il}^T \\ * & P_{22il} - T_{22il} - T_{22il}^T & P_{23il} - T_{32il}^T \\ * & * & P_{33il} - T_{33il} - T_{33il}^T \end{bmatrix}$$

and desired filter can be constructed by:

$$\begin{bmatrix} A_{fil} & B_{fil} \\ C_{fil} & D_{fil} \end{bmatrix} = \begin{bmatrix} T_{22il}^{-T} A_{Fil} & T_{22il}^{-T} B_{Fil} \\ C_{Fil} & D_{Fil} \end{bmatrix} \tag{30}$$

Proof: By using lemma 2 and introducing matrices $M_{1i},\ M_{2i},\ T_i$ inequality (29) is equivalent to

$$\begin{bmatrix} \Xi & d_m \phi_2^T M_{1il} & \bar{d} \phi_2^T M_{2il} & \phi_3^T & \phi_1^T T_{il} \\ * & Z_{1il} - M_{1il} - M_{1il}^T & 0 & 0 & 0 \\ * & * & Z_{2il} - M_{2il} - M_{2il}^T & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & P_{il} - T_{il} - T_{il}^T \end{bmatrix}$$

considering

$$P_{il}=\begin{bmatrix}P_{11il} & P_{12il} & P_{13il}\\ * & P_{22il} & P_{23il}\\ * & * & P_{33il}\end{bmatrix},\ T_i=\begin{bmatrix}T_{11il} & T_{12il} & T_{13il}\\ T_{22il} & T_{22il} & 0\\ T_{31il} & T_{32il} & T_{33il}\end{bmatrix},$$ one can obtain inequality (29). The proof is complete.

3. SIMULATION RESULTS

Consider following discrete-time switched fuzzy system consisting of two subsystems:

$$A_{11}=\begin{bmatrix}0.2 & -0.1\\ 0 & 0.4\end{bmatrix}\ A_{d11}=\begin{bmatrix}0.1 & 0\\ 0.1 & 0.3\end{bmatrix}\ B_{11}=\begin{bmatrix}0.2\\ 0.1\end{bmatrix}$$

$$E_{11}=\begin{bmatrix}0.1\\ 0.3\end{bmatrix}\ G_{11}=\begin{bmatrix}1.3\\ 1.6\end{bmatrix} C_{11}=\begin{bmatrix}0.1 & 0\end{bmatrix}\ C_{d11}=\begin{bmatrix}0 & 0.1\end{bmatrix}$$

$$\bar{H}_{11}=\begin{bmatrix}0.01\\ 0.1\end{bmatrix}\ \bar{C}_{111}=\begin{bmatrix}0.1 & 0.01\end{bmatrix}\ \bar{C}_{121}=\begin{bmatrix}0.01 & 0.01\end{bmatrix}$$

$$\bar{C}_{131}=0.01\ D_{11}=1.1\ J_{11}=1.14\ Q_{11}=1\ \xi_1=0.5$$

$$A_{12}=\begin{bmatrix}0.4 & 0.1\\ 0.1 & 0.3\end{bmatrix}\ A_{d12}=\begin{bmatrix}0.1 & 0\\ 0.2 & 0.1\end{bmatrix}\ B_{12}=\begin{bmatrix}0.2\\ 0.6\end{bmatrix}$$

$$E_{12}=\begin{bmatrix}0.3\\ 0.2\end{bmatrix}\ G_{12}=\begin{bmatrix}1.5\\ 1.2\end{bmatrix} C_{12}=\begin{bmatrix}0 & 0.1\end{bmatrix}\ C_{d12}=\begin{bmatrix}0.1 & 0\end{bmatrix}$$

$$\bar{H}_{12}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix}\ \bar{C}_{112}=\begin{bmatrix}0.1 & 0.1\end{bmatrix}\ \bar{C}_{122}=\begin{bmatrix}0.1 & 0.1\end{bmatrix}$$

$$\bar{C}_{132}=0.1\ D_{12}=0.1\ J_{12}=1.5\ Q_{12}=1.1\ \xi_2=0.5$$

$$A_{21}=\begin{bmatrix}0.1 & -0.1\\ 0 & 0.4\end{bmatrix}\ A_{d21}=\begin{bmatrix}0.15 & 0\\ 0.1 & 0.3\end{bmatrix}\ B_{21}=\begin{bmatrix}0.2\\ 0.05\end{bmatrix}$$

$$E_{21}=\begin{bmatrix}0.1\\ 0.4\end{bmatrix}\ G_{21}=\begin{bmatrix}1.3\\ 1.5\end{bmatrix}\ C_{21}=\begin{bmatrix}0.2 & 0\end{bmatrix}\ C_{d21}=\begin{bmatrix}0 & 0.2\end{bmatrix}$$

$$\bar{H}_{21}=\begin{bmatrix}0.03\\ 0.2\end{bmatrix}\ \bar{C}_{211}=\begin{bmatrix}0.2 & 0.04\end{bmatrix}\ \bar{C}_{221}=\begin{bmatrix}0.03 & 0.03\end{bmatrix}$$

$$\bar{C}_{231}=0.02\ D_{21}=0.9\ J_{21}=1.6\ Q_{21}=0.9\ \xi_3=0.5$$

$$A_{22}=\begin{bmatrix}0.3 & 0.1\\ 0.1 & 0.3\end{bmatrix}\ A_{d22}=\begin{bmatrix}0.1 & 0\\ 0.3 & 0.1\end{bmatrix}\ B_{22}=\begin{bmatrix}0.2\\ 0.7\end{bmatrix}$$

$$E_{22}=\begin{bmatrix}0.2\\ 0.2\end{bmatrix}\ G_{22}=\begin{bmatrix}1.4\\ 1.2\end{bmatrix}\ C_{22}=\begin{bmatrix}0.1 & 0.1\end{bmatrix}\ C_{d22}=\begin{bmatrix}0.2 & 0\end{bmatrix}$$

$$\bar{H}_{22}=\begin{bmatrix}0.2\\ 0.1\end{bmatrix}\ \bar{C}_{212}=\begin{bmatrix}0.1 & 0.1\end{bmatrix}\ \bar{C}_{222}=\begin{bmatrix}0.1 & 0.1\end{bmatrix}\ \bar{C}_{232}=0.1$$

$$D_{22}=1\ J_{22}=1.5\ Q_{22}=1.3\ \xi_4=0.5$$

The weighted matrix of the faults is considered as $W_f(z)=0.5z/(z-0.5)$ with the minimal realization $A_\omega=0.5,\ B_\omega=0.25,\ C_\omega=1,\ D_\omega=0.5$ and the time-varying delay satisfying $2\le d(k)\le 4$. Then we have $\bar{d}=3$. Assuming $\alpha=0.05,\ \mu=1.05$ yields $T_a^*=-(\ln\mu/\ln(1-\alpha))=0.9512$. Choosing $T_a=2$, then $\beta=0.9866<1$. For $\gamma=2.4$ by solving (35) filter matrices are as follows:

$$A_{f1}=\begin{bmatrix}0.0042 & -0.0077\\ -0.008 & 0.0101\end{bmatrix},\ \mathrm{B}_{f1}=\begin{bmatrix}0.00008\\ -0.00075\end{bmatrix},$$

$$\mathrm{C}_{f1}=\begin{bmatrix}-0.039 & 0.0087\end{bmatrix},\ \mathrm{D}_{f1}=0.1609$$

$$A_{f2}=\begin{bmatrix}0.0033 & -0.0013\\ -0.0013 & 0.0009\end{bmatrix},\ \mathrm{B}_{f2}=\begin{bmatrix}-0.0089\\ 0.0042\end{bmatrix},$$

$$\mathrm{C}_{f2}=\begin{bmatrix}-0.0757 & 0.0156\end{bmatrix},\ \mathrm{D}_{f2}=0.2179$$

$$A_{f3}=\begin{bmatrix}0.0023 & -0.055\\ -0.0059 & 0.0086\end{bmatrix},\ \mathrm{B}_{f3}=\begin{bmatrix}-0.003\\ -0.009\end{bmatrix},$$

$$\mathrm{C}_{f3}=\begin{bmatrix}-0.0747 & 0.0256\end{bmatrix},\ \mathrm{D}_{f3}=0.1858$$

$$A_{f4}=\begin{bmatrix}0.0014 & -0.001\\ -0.001 & 0.0021\end{bmatrix},\ \mathrm{B}_{f4}=\begin{bmatrix}-0.0001\\ -0.0129\end{bmatrix},$$

$$\mathrm{C}_{f4}=\begin{bmatrix}-0.1392 & 0.1001\end{bmatrix},\ \mathrm{D}_{f4}=0.1474$$

To illustrate the effectiveness of the design, an unknown input is assumed to be $d_k=0.01\exp(-0.04k)\cos(0.03\pi k)$. The control input u_k is the unit step signal. It is assumed that two faults in different times affects each subsystems as shown in Figure 1. The switching signal is demonstrated in Figure 2. States of first subsystem, second subsystem and overall system are shown respectively in Figures 3, 4 and 5.

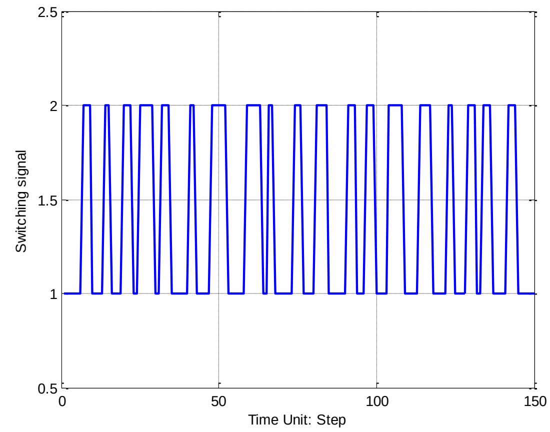

Fig. 1. The Switching signal

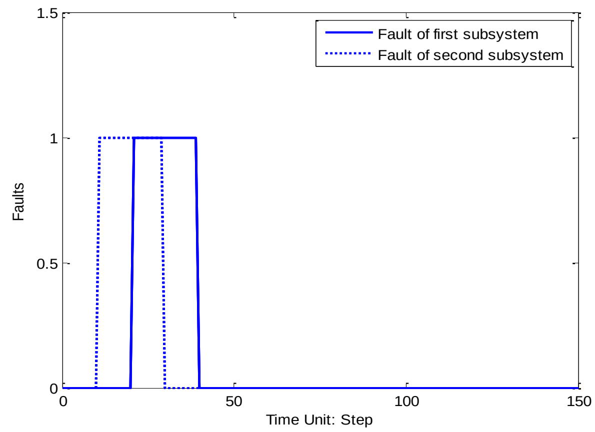

Fig. 2. The Fault signals

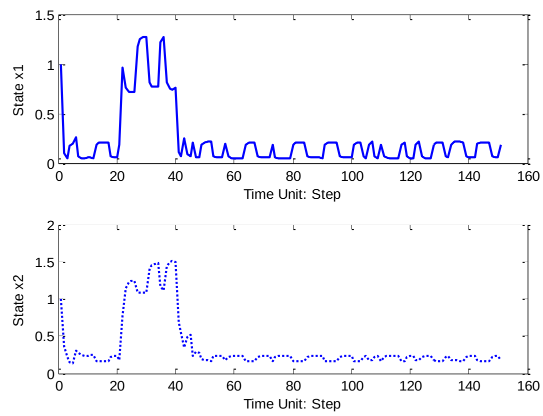

Fig. 3. States of the first subsystem

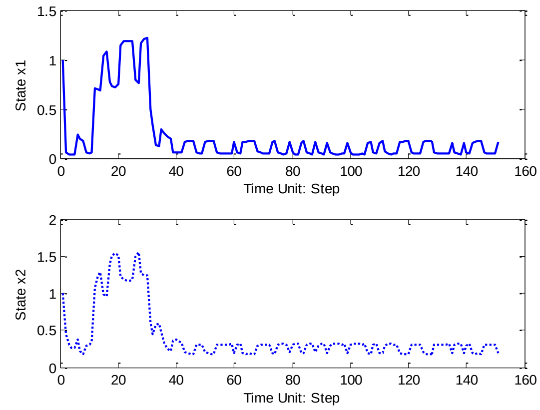

Fig. 4. States of the second subsystem

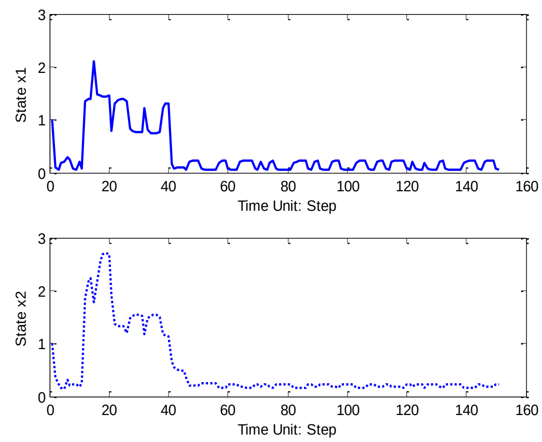

Fig. 5. States of the overall system

It is easy to verify that in the interval of fault occurrence, state variables are deviated and with the end of interval, they return to their initial values. Also, disturbance is eliminated in a large extent.

4. CONCLUSION

In this paper, the robust state estimation filter design problem for uncertain switched fuzzy systems with time-varying state-delays has been studied. Then using H_∞ filtering, switched Lyapunov functional and an average dwell time approach a delay dependent sufficient condition for solvability of this problem has been obtained in terms of LMIs, and filter matrices has been obtained. An illustrative example verified the effectiveness of the method to preserve stability of the system even in the presence of the faults.

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