

# Control of Multi-variable Processes using Self-Tuning Fuzzy

Abdolrahman Ramezani<sup>1</sup>, Abdollah Rahimi<sup>2</sup>, Alireza Roosta<sup>3</sup>

1- Student of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz,  
Email: a.ramezani@sutech.ac.ir

2- Student of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz,  
Email: ab.rahimi@sutech.ac.ir

3- Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz, Iran  
Email: roosta@sutech.ac.ir

Received: April 2014

Revised: August 2014

Accepted: February 2015

## ABSTRACT:

Many industrial processes are Multi-Input Multi-Output (MIMO) that has more than one controlled variable. Therefore, without considering the impact of these factors, it is not possible to achieve the desired performance. In this paper, two methods, adaptive controller and self-tuning fuzzy PID controller is used to control the quadruple-tank process. Although the both presented methods are able to eliminate disturbance effect and reach steady-state with acceptable performance, the fuzzy controller is preferred to the adaptive controller due to the lower computational effort. Moreover, the fuzzy controller does not need the transfer function of the system, while it has a simple design procedure and simple arithmetic. Superiority of the proposed method is automatic adjustment of multivariable fuzzy controller parameters to achieve desirable performance.

**KEYWORDS:** Self-tuning fuzzy controller, adaptive controller, multi-variable systems, recursive least square, relative gain arrays.

## 1. INTRODUCTION

Instability and undesirable interaction between variables are always common problems in industrial processes. Moreover, achieving the desired performance is very important. For this purpose, researches are concentrated on multi-variable control. Most of the time, decentralized multi-variable controllers are used because they are understandable and need less parameter for tuning than general multi-variable one. In this regards several types of control schemes such as PID controller, robust controller, adaptive controller, and fuzzy controllers are presented [1-6]. The typical PID controllers due to their simplicity in arithmetic, good robustness, ease of use, zero steady state error, stabilization and high reliability are widely used in industry [7]. However, PID controllers cannot provide a general solution to all control problems. The processes involved are in general complex and time-variant, with delays and non-linearity, and often with poorly defined dynamics. When the process becomes too complex to describe by analytical models, it is doubtful to control efficiently by conventional approaches. Since PID controllers are often not properly tuned (e.g., due to plant parameter variations or changing in operating conditions), there is a significant need to develop methods for tuning PID controllers such as hand-tuning, Ziegler-Nichols tuning, loop shaping, analytical methods, optimization methods, pole placement, and auto-tuning [3, 8].

Although the expressed methods for automatic tuning of (single-input-single-output) SISO systems are available and approximately user friendly, controller tuning of multi-variable systems are difficult and depends on various parameters of plants and operating conditions. Therefore, developed methods for the design of multi-variable control systems are needed. Most modern multi-variable control design methods require a complete model of the plant [9]. In many cases, the exact model of the system is not available or is affected by the environmental factors, which can change over time. Thus, the adaptive control methods can be developed for multi-variable systems [10]. The adaptive control methods are proposed when the parameters of the plant's dynamic model are unknown and/or time-varying. Therefore, to achieve desire performance; these techniques provide a systematic approach for automatic adjustment of controllers in real time.

First step of adaptive controller design is system identification. In some cases, system identification may require a prohibitive engineering effort with high computation that leads to an increase in the designing cost. Hence, in these cases, adaptive controllers are not suggested. Therefore, it is necessary to use simple methods of multi-variable controllers tuning. The fuzzy controller design is independent from transfer function of the system, so system identification is not a necessary step of the design procedure and it is one

benefit of this method.

In past decade, Fuzzy Logic Controllers (FLC) has been applied to handle complex nonlinear processes [11], [12]. Recent researches show that application of FLC enhances the closed loop performance of a PID controller when the operating point of nonlinear processes changes in real time [2], [13].

Three inputs are needed for the conventional fuzzy PID controllers, and the rule base of this controller has three dimensions, however, this controller can be designed with just two inputs. Performance of this controller is better than the fuzzy PD and PI controllers [14]. Design of fuzzy controller has three important stages, tuning control parameters, membership functions, and knowledge base design. These three stages of fuzzy controller design must be adjusted to achieve the prospective target for the fuzzy controller, but requisite of real time control can only be achieved by the scaling factor adjustment. Hence, in order to apply the fuzzy control, it is necessary to adjust factors of fuzzy controller. The membership function methods with the tuning scaling factor and tuning the scaling factor of the fuzzy PID controller are studied in [2, 4, 9, 13, 15-24].

One of the basic problems in the above-mentioned methods is adjustment of the controller factors for achieving the desired performance. This problem would be more complex in multi-variable systems. In this paper, to overcome this problem, a new control strategy based on self-tuning fuzzy controller is presented for control of multi-variable systems. In general the proposed method has some advantages, such as explicitly in the controller design procedure, lower size of computational effort, non-linear nature of controller, needless to identification of the system, reduction of the effect of disturbance on system, the steady-state response without error, and the appropriate settling time. The applied controller to the quadruple-tank process is decentralized. Simulation results indicate that fuzzy controller has a good performance. Also, this controller is able to reduce the disturbance effects. Therefore, presented self-tuning fuzzy controller is preferred due to simplicity in design procedure and less computational effort.

The rest of this paper is organized as follows: In section 2, a brief introduction of decentralized control structure is presented. Section 3 contains overview of a PID type fuzzy controller with self-tuning scaling factors for systems multivariable. Section 4 is related to description of the quadruple-tank process and simulation results. Subsequently, conclusions are presented in section 5.

## 2. DECENTRALIZED CONTROL STRUCTURE

In this paper a system with two inputs and two outputs is considered. In order to control the process, two

control structure, centralized and decentralized, are presented and shown in figures 1 and 2, respectively. Each proposed control methodologies has their own advantages and deficiencies.

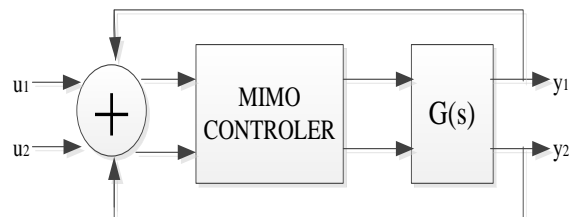


Fig. 1. Centralized control structure.

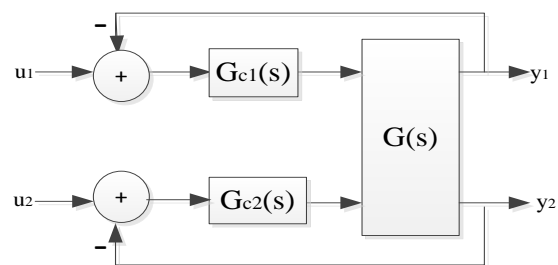


Fig. 2. Decentralized control structure.

The centralized controller has non-diagonal transfer function matrices, therefore; this type of controller can deal with highly interactive loops. Hence, the masterfully designing of all the controller transfer function matrix elements is required to achieve the desired performance. On the other hand, decentralized control has independent feedback controllers that are used to control a subset of the plant outputs by a subset of the plant inputs. In this case, there is no communication between these independent measurements and feed-back loops. Therefore, the controller transfer function matrix becomes block diagonal.

It is obvious that the computational load of control system analysis and complexity design controller for multi-variable process grows quickly when the numbers of inputs-outputs increase [17]. Hence, in order to reduce computational effort and make the design procedure simple, the multi-variable plant is decomposed into two subsystems and independent design procedures are performed for each subsystem with taking into account their dynamical behaviour interactions from other subsystems. The main reasons of the extensive use of decentralized control structures are as follows:

- ✓ Easy implementation.
- ✓ Efficient maintenance.
- ✓ Simple tuning procedures.
- ✓ Robust behaviour in the confronting of fault and model uncertainties.

The decentralized controller design in general is

included six steps as follows [25]:

- 1) *Control objectives formulation*: The objectives in time and/or frequency domain characteristics are determined.
- 2) *Process modeling*: Type of structures is selected and input-output or internal state space plant descriptions are determined according to the desired goals.
- 3) *Control structure selection*: Two key steps in a successful control structure selection are the inputs and outputs selection and the control configuration selection or the input-output pairing problem.
- 4) *Controller design*: The selection of control system design strategy is determined according to the multi-variable process characteristics and specifications of closed loop performance.
- 5) *Simulation plant*: Before hardware implementations, the design should be verified and tested via system simulation.
- 6) *Implementation. Finally*: In this step, controller is implemented practically.

In order to perform fourth step or controller design section, the first input must be paired with the output with the greatest impact of this input. Then a SISO controller for the input-output pairing is designed. Also, for the design of second controller, the closed-loop effect of first controller is considered. Stability of the system should be confirmed in each time of closing the system loop. Thus, the system stability is guaranteed. An improper input-output pairing eventuates in weak performance of closed loop or instability of closed loop. Thus, the selection of control configuration or the input-output pairing problem is the most important step. The various methods for input-output pairing are presented in the next subsection and one of these methods is reviewed briefly.

**2.1. Selection of desired pair of input-output via relative gain array**

The set of manipulated inputs and exogenous inputs such as sensor noise, disturbances, and reference inputs or the set-points can be considered as the system inputs. Also, the set of measured variables and the controlled variables are defined as the system outputs. Choosing the input-output pair is important because many plant properties such as hardware issues including cost and maintenance, reliability and complexity of control depend on it. The most used techniques to determinate the pair of input - output are Relative Gain Arrays (RGA). This section presents a calculative route way of the RGA fundamentals [26].

The linear multi-variable plant is described by the following transfer function matrix model that can be written as,

$$G(s) = [g_{ij}(s)] \quad i, j = 1, 2, \dots, m \quad (1)$$

where  $g_{ij}(s)$  is the open-loop gain from  $i^{th}$  input to  $j^{th}$  output. Input vector and output vector into Equation (2) and Equation (3) are written, respectively as follows,

$$U(s) = [u_1(s) \dots u_m(s)]^T \quad (2)$$

$$Y(s) = [y_1(s) \dots y_m(s)]^T \quad (3)$$

Assume that  $u_k = 0 \forall k \neq j$ , in other words, the loops are open and the effect of  $u_j$  on the  $i^{th}$  output  $y_i$  is considered. Thus, the steady-state response is achieved by applying input  $u = [0 \dots 0 \ u_j \ 0 \dots 0]^T$ . The steady-state gain of open-loop between the  $j^{th}$  input and the  $i^{th}$  output is given, which is shown by  $g_{ij}(0)$ . Also, in the case of closed loop regulation let  $y_k = 0 \forall k \neq i$ , namely keep all the outputs constant except the  $i^{th}$  output with this assuming that all other channels are under tight control. This steady-state gain is shown by  $h_{ij}(0)$  which is the gain between the  $i^{th}$  output to the  $j^{th}$  input, Whereas all the loops except the  $i^{th}$  output are under tight control. To obtain RGA only steady-state values are considered, therefore  $G(0)=G$ , and  $g_{ij}(0)=g_{ij}$ . The relative gain denoted by  $\lambda_{ij}$ . Also, this gain is defined as a dimensionless number and described as following,

$$\lambda_{ij} = \frac{g_{ij}}{h_{ij}} \quad (4)$$

and the RGA is defined as:

$$\Lambda = [\lambda_{ij}] \quad i, j = 1, \dots, m \quad (5)$$

The crucial points to choose the input-output pair are:

- ✓ Choose the input-output pair in a way that diagonal elements of the RGA be close to one [27].
- ✓ RGA diagonal elements to the input-output pair must be positive.
- ✓ The elements which have Large or negative RGA are not appropriate for input-output pairing.

**3. Overview of a PID type fuzzy controller with self-tuning scaling factors for systems multivariable**

A type of fuzzy PID controller that is obtained simply from the connection of the PD type and the PI type fuzzy controllers together in parallel is illustrated in figure 3.

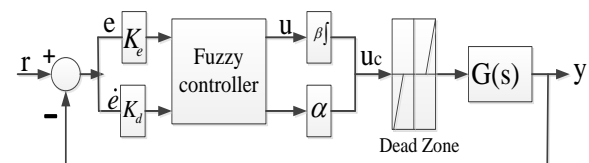


Fig. 3: A type of the fuzzy PID controller.

The fuzzy system rules can be written as:

If {e is ZR and e is ZR }, then {u is ZR }

The PID type fuzzy controller output is:

$$u_c = \alpha u + \beta \int u dt$$

$$= \alpha(A + PK_e e + DK_d \dot{e}) + \beta \int (A + PK_e e + DK_d \dot{e}) dt \quad (6)$$

$= \alpha A + \beta A T + (\alpha K_e P + \beta K_d D)e + \beta K_e P \int e dt + \alpha K_d D \dot{e}$   
 where,  $\beta K_e P$  and  $\alpha K_d D$  are the proportional, integral and derivative coefficient.

In order to adjusting self-tuning the scaling factors , the same idea in the parameter adaptive method is used and then the functions  $f(e(t))$  and  $g(e(t))$  is defined as follows [16],

$$f(e(t)) = a_1 \times abs(e(t)) + a_2 \quad (7)$$

$$g(e(t)) = b_1 \times (1 - abs(e(t))) + a_2 \quad (8)$$

where  $a_1, a_2, b_1,$  and  $b_2$  are all positive constants. Then the time variable self-tuning scaling factors are expressed as follows:

$$\beta_s(e(t)) = \beta \times f(e(t)) \quad (9)$$

$$k_{ds}(e(t)) = k_d \times g(e(t)) \quad (10)$$

where  $\beta$  and  $k_d$  are the initial values of the scaling factors. The aim of the function  $f(e(t))$  is to decrease the  $\beta_s(e(t))$  by changing the error. Namely,  $f(e(t))$  will be equal to  $a_2$  when the error will be zero. However, the function  $g(e(t))$  is the inverse objective, the  $g(e(t))$  will be equal to  $(b_1 + b_2)$  in the steady state, therefore the  $\beta_s(e(t))$  and  $k_{ds}(e(t))$  can be adjusted roughly with the error of the time. To find how  $a_1, a_2, b_1,$  and  $b_2$  are very important, we can progress in the following way:

In the beginning,

$$e = 1 : f(1) = a_1 + a_2 \quad (11)$$

$$g(1) = b_2$$

In the steady state,

$$e = 0 : f(0) = a_2 \quad (12)$$

$$g(0) = b_1 + b_2$$

The PID type fuzzy controller structure with the self-tuning scaling factors is shown in figure 4. Also, the simulation of fuzzy controller for systems multi-variable is like figure 5.

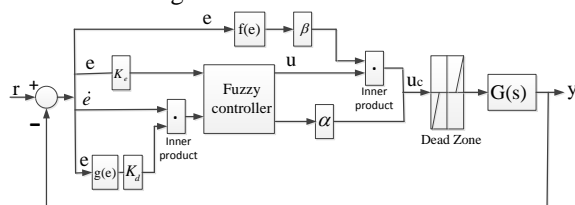


Fig. 4. The PID type fuzzy control system with function tuner.

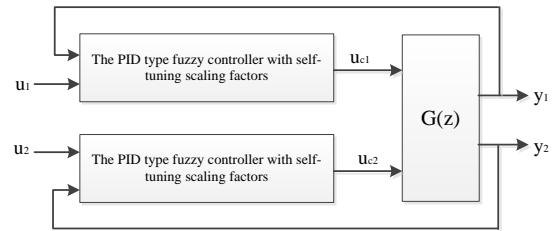


Fig. 5. The fuzzy PID controller for systems multivariable.

### 3. RESULTS AND DISCUSSION

The simulation model in this research is the quadruple-tank process [28]. This system is a suitable laboratory process to demonstrate the impact of environmental factors on the multi-variable systems. A picture of the quadruple-tank process is shown in figure 6.

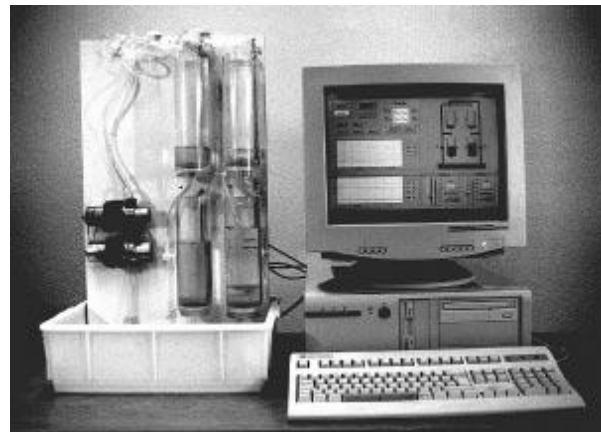


Fig. 6. The quadruple-tank laboratory process [ref].

The goal is controlling the liquid level in the bottom of two tanks by two pumps. The process inputs are the input voltage of pumps. The voltages from level measurement sensors are the process outputs. In order to distribute the flows to the tanks, two valves have been considered.

The positions of the valves can be expressed with two parameters  $\gamma_1, \gamma_2 \in [0, 1]$ . When the flow goes only to the lower left tank  $\gamma_1=1$ , and when it goes only to the upper right tank  $\gamma_1=0$ . The parameter  $\gamma_2$  is defined similarly. Four nonlinear differential equations obtain from plant. Linearization of these gives the transfer function matrix,

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_{11}}{1 + sT_1} & \frac{(1 - \gamma_2)c_{12}}{(1 + sT_3)(1 + sT_1)} \\ \frac{(1 - \gamma_1)c_{21}}{(1 + sT_4)(1 + sT_2)} & \frac{\gamma_2 c_{22}}{1 + sT_2} \end{bmatrix}$$

where  $c_{ij}$  and  $T_i$  are positive constants that depend on the amplification in the actuators, the cross-section areas of the tanks and the outlets, measurement sensors, and the operating point. For example, the operation of one of the tanks is illustrated in figure 7.

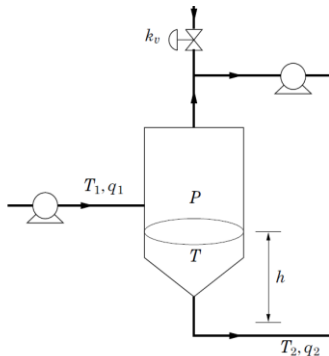


Fig. 7. The operation of a tank.

In this simulation,  $\gamma_1$  and  $\gamma_2$  are initialized as follows:

$$(\gamma_1, \gamma_2) = (0.7, 0.6)$$

The transfer function system by putting above values is obtained as following:

$$G(s) = \begin{bmatrix} \frac{2.6}{1 + 62s} & \frac{1.5}{(1 + 23s)(1 + 62s)} \\ \frac{1.4}{(1 + 30s)(1 + 90s)} & \frac{2.8}{1 + 90s} \end{bmatrix}$$

**4.1. The self-tuning fuzzy PID controller implementation and simulation results**

In order to, implement a fuzzy controller, the first step is determination input - output and membership function (MF) of fuzzy system. In this paper, a fuzzy controller with one output and two inputs, error and error changes, is presented. Figure 8 shows the membership functions of error, derivative of the error and output. TABLE I shows the rules base of self-tuning fuzzy PID controller.

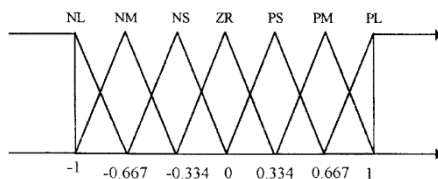


Fig. 8. The MF of e, e-dot and output.

Table 1. The Rules Base Of the Self-Tuning Fuzzy PID Controller.

E/CE	NL	NM	NS	ZR	PS	PM	PL
PL	ZR	PS	PM	PL	PL	PL	PL
PM	NS	ZR	PS	PM	PL	PL	PL
PS	NM	NS	ZR	PS	PM	PL	PL
ZR	NL	NM	NS	ZR	PS	PM	PL
NS	NL	NL	NM	NS	ZR	PS	PM
NM	NL	NL	NL	NM	NS	ZR	PS
NL	NL	NL	NL	NL	NM	NS	ZR

The fuzzy inference system (FIS) settings are based on design choices described in:

- Use Mamdani style fuzzy inference system.
- Use algebraic product for AND connective.
- The ranges of both inputs are normalized to [-1 1].
- The input sets are triangular and cross neighbor sets at membership value of 0.5.
- The output range is [-1 1].
- Use singletons as output, determined by the sum of the peak positions of the input sets.
- Use the center of gravity method (COG) for defuzzification.

According to the rules of the fuzzy and the combination manner of these rules, a three-dimensional surface, which shows the relationship between inputs and output fuzzy system, would be created. This surface is known as the controller surface and it is shown in figure 9.

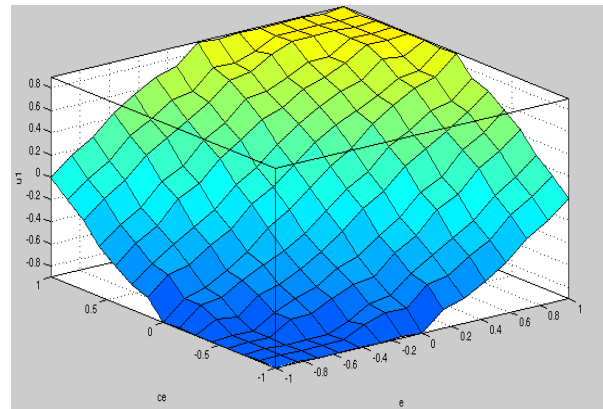


Fig. 9. Nonlinear controller surface.

The parameter values  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2$  that are given in Equations (22, 23) as follows:

$$a_1=1.3; a_2=0.25; b_1=4.3; b_2=0.8$$

The outputs of closed-loop system with using the self-tuning fuzzy PID controller are shown in figure 10. It is obvious that, the self-tuning fuzzy PID controller has an excellent performance, disturbances are rejected and finally the system is in a steady state without error. The applied control signals are shown in figure 11.

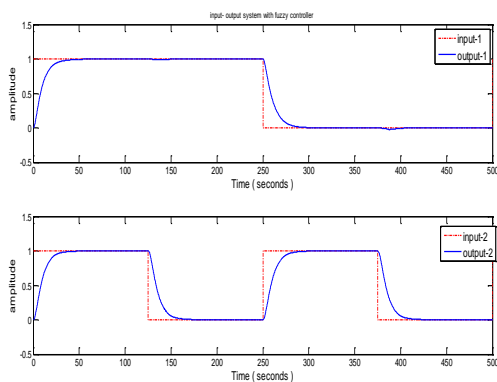


Fig. 10. The outputs of the closed-loop system with using the self-tuning fuzzy PID controller.

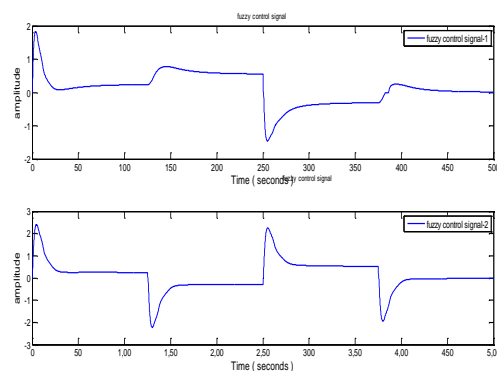


Fig. 11. The applied fuzzy control signals to system.

## 5. CONCLUSION

Simulation results show that the system is well controlled with both adaptive and fuzzy controllers. Both controllers are able to reduce the disturbance effects. However, it is worth mentioning that the fuzzy controller is superior to the adaptive controller. Because in practice the adaptive controller has many shortcomings such as high calculations (Due to a necessary step for identifying the system parameters), necessity of knowing the basic information about the system as a background (such as estimation of the degree of numerator and denominator), and design complexity. Moreover, the system transfer function is not required for fuzzy controller design. As a result, the proposed design procedure has simpler controllers and it also reduces the computational effort. Considering nonlinearity is another advantage of fuzzy controller in comparison to the adaptive controller.

## REFERENCES

- [1] F. Zheng, et al., "On the design of multivariable PID controllers via LMI approach," *Automatica*, Vol. 38, pp. 517-526, 2002.
- [2] R. K. Mudi and N. R. Pal, "A robust self-tuning scheme for PI-and PD-type fuzzy controllers,"

*Fuzzy Systems, IEEE Transactions on*, Vol. 7, pp. 2-16, 1999.

- [3] S. P. Bhattacharyya, et al., *Robust control*: Prentice-Hall Upper Saddle River, New Jersey, 1995.
- [4] C. Batur and V. Kasparian, "Adaptive expert control," *International Journal of Control*, Vol. 54, pp. 867-881, 1991.
- [5] E. Davison, "Multivariable tuning regulators: the feedforward and robust control of a general servomechanism problem," *Automatic Control, IEEE Transactions on*, Vol. 21, pp. 35-47, 1976.
- [6] G. H. Ellis, *Control system design guide: a practical guide*: Academic Press, 2004.
- [7] W. Jian and C. Wenjian, "Development of an adaptive neuro-fuzzy method for supply air pressure control in HVAC system," Vol. 5, pp. 3806-3809, 2000.
- [8] K. J. Astrom, "Oscillations in systems with relay feedback," *The IMA Volumes in Mathematics and its Applications: Adaptive Control, Filtering and Signal Processing*, Vol. 74, pp. 1-25, 1995.
- [9] J. M. Maciejowski, "Multivariable feedback design," *Electronic Systems Engineering Series, Wokingham, England: Addison-Wesley, c1989*, Vol. 1, 1989.
- [10] G. C. Shen and W. K. Lee, "Multivariable adaptive inferential control," *Industrial & Engineering Chemistry Research*, Vol. 27, pp. 1863-1872, 1988.
- [11] K. M. Passino and S. Yurkovich, *Fuzzy control*: Citeseer, 1998.
- [12] D. Driankov, et al., *An introduction to fuzzy control*: Springer Verlag, 1996.
- [13] R. K. Mudi and N. R. Pal, "A self-tuning fuzzy PI controller," *Fuzzy sets and systems*, Vol. 115, pp. 327-338, 2000.
- [14] H. X. Li and H. Gatland, "Conventional fuzzy control and its enhancement," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, Vol. 26, pp. 791-797, 1996.
- [15] M. Cheng, et al., "New self-tuning fuzzy PI control of a novel doubly salient permanent-magnet motor drive," *Industrial Electronics, IEEE Transactions on*, Vol. 53, pp. 814-821, 2006.
- [16] Z. W. Woo, et al., "A PID type fuzzy controller with self-tuning scaling factors," *Fuzzy sets and systems*, Vol. 115, pp. 321-326, 2000.
- [17] D. Siljak, "Decentralized control and computations: status and prospects," *Annual Reviews in Control*, Vol. 20, pp. 131-141, 1996.
- [18] C. H. Chou and H. C. Lu, "A heuristic self-tuning fuzzy controller," *Fuzzy sets and systems*, Vol. 61, pp. 249-264, 1994.
- [19] L. Zheng, "A practical guide to tune of proportional and integral (PI) like fuzzy controllers," pp. 633-640, 1992.
- [20] J. Buckley and H. Ying, "Expert fuzzy controller," *Fuzzy sets and systems*, Vol. 44, pp. 373-390, 1991.
- [21] C. Batur and V. Kasparian, "Predictive fuzzy expert controllers," *Computers & industrial engineering*, Vol. 20, pp. 199-209, 1991.

- [22] P. Xian-Tu, "Generating rules for fuzzy logic controllers by functions," *Fuzzy sets and systems*, Vol. 36, pp. 83-89, 1990.
- [23] M. Braae and D. Rutherford, "Selection of parameters for a fuzzy logic controller," *Fuzzy sets and systems*, Vol. 2, pp. 185-199, 1979.
- [24] X. G. Duan, et al., "Effective tuning method for fuzzy PID with internal model control," *Industrial & Engineering Chemistry Research*, Vol. 47, pp. 8317-8323, 2008.
- [25] M. van de Wal and B. de Jager, "A review of methods for input/output selection," *Automatica*, Vol. 37, pp. 487-510, 2001.
- [26] A. Khaki-Sedigh and B. Moaveni, "Control configuration selection for multivariable plants," *Springer Verlag*, Vol. 391, 2009.
- [27] C. PFEIFFER, "On a new measure of interaction for multivariable process control," 1966.
- [28] K. H. Johansson, "Relay feedback and multivariable control," *PhD thesis*, Department of Automatic Control, Lund Institute of Technology, Sweden, 1997.