

Improvement of Channel Estimation in MIMO-OFDM Using Improved LS Algorithm on Multipath Channels

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ABSTRACT:

In high capacity systems, as the bit transmission rate increases, Inter Symbol Interference (ISI) caused by the multi-path channel reduces the system efficiency. Orthogonal Frequency Division Multiplexing (OFDM) technique acts very well against this phenomenon. Moreover, an accurate estimation of the communication channel coefficients improves the performance of communication systems effectively. In this paper, for Multiple Input Multiple output (MIMO) systems using OFDM technique that is based on the least Squares (LS) algorithm, an improved channel estimation algorithm is presented. Accordingly, investigating the channel estimation method, we can design optimum training courses for these systems based on LS algorithm. Simulation shows the efficiency of suggested LS algorithm. We also provided the results of the communication channel estimation.

KEYWORDS: Multiple Input Multiple Output (MIMO) Systems, Least Squares (LS) Algorithm, Enhanced Least Squares (ELS) Algorithm, Improved Least Squares (ILS) Algorithm.

1. INTRODUCTION

Propagation environment phenomena such as fading and ISI reduce the performance of mobile wireless networks. Communication systems should be designed in such a way that they dominate these problems reliably. One of the modern methods to overcome the problems and also increase the transmission rate is to use several antennas in the transmitter and receiver or the so called MIMO system [1]. Using spatial-temporal processing, problems of a wireless channel can be overcome in these systems. Since most of MIMO algorithms are used for narrow band channels, the technique of OFDM can be used in the MIMO system to confront with the selective nature of wideband wireless channels frequencies. OFDM converts effectively a wideband channel to a few parallel flat sub-channels. Given that in detecting the data vectors transmitted in MIMO-OFDM systems, it is assumed that the channel coefficients matrix is

quite known for the receiver, it is necessary to estimate the channel matrix in all sub carriers and at every time period to correctly detect the transmitted data.

In order to reveal the coherent of received signals, digital communication systems must have an exact estimation of the situation of exchange channel between transmitter and receiver. Since increasing the number of transmitter and receiver antennas causes an increase in the number of unknowns (coefficients of the channel between both antennas of transmitter and receiver), the estimation of channels in multi-antenna systems is a lot more challenging than in one-antenna ones [2].

Many references have so far been published on channel estimation in OFDM based single antenna systems [3]-[8]. However, since the signal received by each sub carrier of MIMO-OFDM is sum of the faded signals transmitted by different transmitter antennas plus noise, the channel estimation techniques commonly used in SISO-OFDM systems are not applicable to MIMO-OFDM systems [9].

As the channel estimation is important in the MIMO-OFDM systems, this issue has been studied by many researchers. In [10] based on a least squares (LS) type algorithm with considering some significant taps, a channel estimation algorithm has been proposed. By using optimum training sequences and cancelling co-channel interferences, a simplified version of the algorithm derived in [10] has been proposed in [11].

MIMO-OFDM channel estimation is enhanced in [9], in order for all sub-channels in MIMO system to have the same delay profile and to approximate it by averaging over the power of the initial estimated sub-channel impulse responses.

Another algorithm has been introduced in [12] based on decoupling sub-channel impulse responses corresponding to different transmit antennas and adaptively finding the number of significant channel taps, in order to reduce the complexity of the proposed algorithm in [10] that is associated with the inverse matrix. An iterative least squares algorithm has been proposed in [13] for MIMO-OFDM channel estimation, by using the channel correlation in frequency domain and exploiting pilot tones. Here in this research we present an improved MIMO-OFDM channel estimation algorithm which significantly performs better than the regular LS algorithm.

2. SYSTEM DESCRIPTION

The block diagram of a MIMO-OFDM system is shown in Figure 1. Basically, the MIMO-OFDM transmitter has N_t parallel transmission paths which are very similar to the single antenna OFDM system, each branch performing serial-to-parallel conversion, pilot insertion, N -point IFFT and cyclic extension before the final TX signals are up-converted to RF and transmitted. It is worth noting that the channel encoder and the digital modulation, in some spatial multiplexing systems [14], can also be done per branch, not necessarily implemented jointly over all the N_t branches. The receiver first must estimate and correct the possible symbol timing error and frequency offsets, e.g., by using some training symbols in the preamble. Subsequently, the Cyclic Prefix (CP) is removed and N -point FFT is performed per receiver branch.

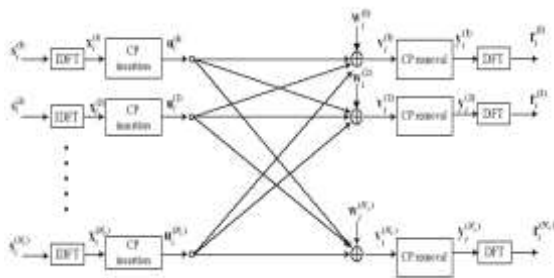


Fig.1.Block diagram of transmitter and receiver in a MIMO-OFDM system [15]

3. LS ALGORITHM

A MIMO-OFDM system is supposed, with M transmitter antennas and N receiver antennas and K sub carrier:

$$Y_i[n] = S[n]H_i[n] + z_i[n] \quad i = 1, 2, \dots, N \quad (1)$$

$$Y_i[n] = [Y_i[n]^T, Y_i[n-1]^T, \dots, Y_i[n-M+1]^T]^T$$

$$S[n] = [X[n]^T, X[n-1]^T, \dots, X[n-M+1]^T]^T$$

$$Z_i[n] = [V_i[n]^T, V_i[n-1]^T, \dots, V_i[n-M+1]^T]^T$$

The LS estimation of the $H_i[n]$ channel at the n -th symbol is obtained as follows.

$$\hat{H}_i[n] = (S[n]^H S[n])^{-1} S[n]^H Y_i[n] \quad i = 1, 2, \dots, N \quad (2)$$

By substituting relation (2) in relation (1) we have:

$$\hat{H}[n] = H + U[n] \quad (3)$$

Where, $U[n]$ is the LS estimator error.

$$U[n] = (S[n]^H S[n])^{-1} S[n]^H Z[n] \quad (4)$$

By obtaining FFT transformation of $\hat{H}[n]$, the time estimate of H is obtained as follows.

$$\hat{h}[n] = W^{-1} \hat{H}[n] = \check{I}_L^T h[n] + e[n] \quad (5)$$

Where, $e[n] = W^{-1} U[n]$ is the channel impulse response estimation error and $h_{ML \times 1}$ is channel impulse response vector.

In addition, $W = \text{diag}(\overbrace{W, W, \dots, W}^{M \text{ times}})$, where W is the Fourier transform matrix.

$$[W]_{pq} = \frac{1}{\sqrt{K}} e^{j \frac{2\pi}{K} pq} \quad p, q = 0, 1, 2, \dots, K-1 \quad (6)$$

\check{I}_L is a $ML \times MK$ matrix defined as follows.

$$\ddot{I}_L = \begin{bmatrix} I_L & 0_{L \times (K-L)} & & 0 \\ 0_{L \times K} & I_L & 0_{L \times (K-L)} & \\ & 0_{L \times K} & I_L & 0_{L \times (K-L)} \\ & & & \ddots \\ 0 & & 0_{L \times K} & I_L & 0_{L \times (K-L)} \end{bmatrix} \quad (7)$$

Where, I_L is the diagonal $L \times L$ matrix, in which the elements on the main diagonal are one.

Figure 2 presents results of simulation with the LS estimation algorithm for a MIMO-OFDM system with $M=2$ transmitting antennas and $N=2$ receiver antennas. The total channel bandwidth is assumed to be $B=5$ MHz, which is divided into $K=256$ sub-channels. The number of paths in the multipath channel is 6 and the channel's delay spread is assumed to be $8 \mu s$.

f_d is the maximum Doppler frequency of the channel ($f_d = \frac{Bd}{2}$). Figure 2 depicts performance of LS estimator as the estimator NMSE in terms of average SNR for different values of normalized Doppler frequency $f_d T$. In this figure, the normalized Doppler frequency shows the maximum channel variation cycle in relation to time based on the symbol sent.

As seen, for channels with slow fading ($f_d T < 0.1$), NMSE of the LS algorithm decreases linearly in terms of SNR. However, with an increase in the normalized Doppler frequency, rate of channel variations increases in terms of time, and as a result the hypothesis regarding equality of channel impulse responses (and the channel frequency response) of two consecutive symbols is rejected partly due to the existing lack of correlation. Hence, as seen in this figure, performance of the LS estimator declines with an increase in the normalized Doppler frequency (NMSE). This decrease is extremely significant in channels with fast fading ($f_d T \geq 0.1$), where ICI occurs.

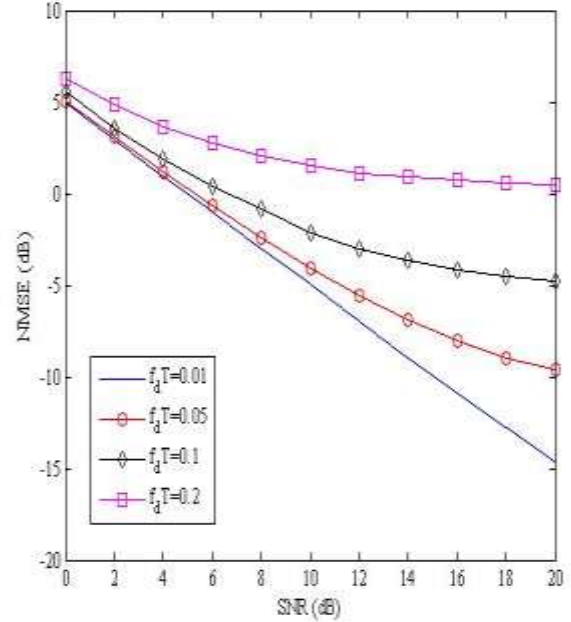


Fig.2. NMSE of the LS channel estimation algorithm based on the average SNR

4. ENHANCED LEAST SQUARES (ELS) ALGORITHM

The ELS estimate of the channel impulse response vector is obtained as follows.

$$\check{h}[n] = \check{I}_L \hat{h}[n] \quad (8)$$

And the ELS estimate of the channel frequency response vector is as follows.

$$\check{H}[n] = \mathbf{W} \check{I}_L^T \check{I}_L \check{h}[n] = \mathbf{W} \check{I}_L^T \check{I}_L \mathbf{W}^{-1} \hat{H}[n] \quad (9)$$

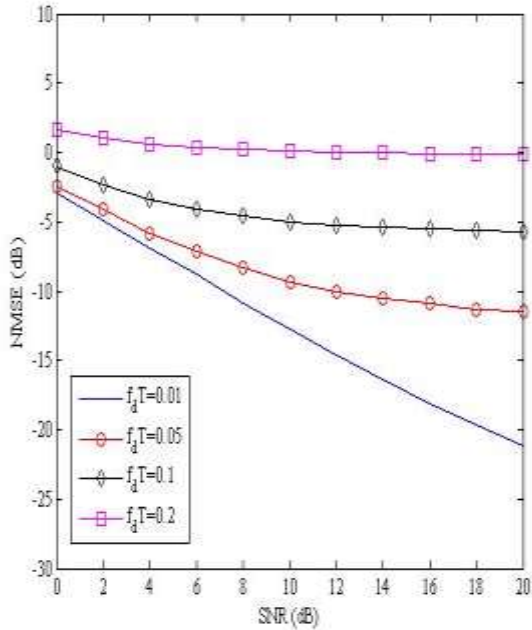


Fig.3. NMSE of the ELS channel estimation algorithm in terms of average SNR

Figure 3 depicts the NSME diagram of the ELS channel estimation algorithm in terms of average SNR for the MIMO-OFDM system.

5. IMPROVED LEAST SQUARES (ILS) ALGORITHM

For the channel impulse response estimation error vector we have:

$$\mathbf{e}[n] = \mathbf{W}^{-1}(\mathbf{S}[n]^H \mathbf{S}[n])^{-1} \mathbf{S}[n]^H \mathbf{Z}[n] \quad (10)$$

The ILS algorithm improves the channel estimation using this correlation and estimation of a part of the noise components in the channel's delay spread domain based on components outside of this domain, which were omitted from the ELS algorithm. The $\mathbf{e}[n]$ vector is divided into the following two sections: channel estimation noise in the channel delay spread domain $\mathbf{e}_1[n]$ and channel estimation noise outside of the domain, $\mathbf{e}_2[n]$.

$$\begin{aligned} \mathbf{e}_1[n] &= \mathbf{I}_L \mathbf{e}[n] \\ \mathbf{e}_2[n] &= \mathbf{I}_{(K-L)} \mathbf{e}[n] \end{aligned} \quad (11)$$

Where, $\mathbf{I}_{(K-L)}$ is a $M(K-L) \times MK$ matrix defined as follows.

$$\mathbf{I}_{(K-L)} = \begin{bmatrix} 0_{(K-L) \times L} & \mathbf{I}_{(K-L)} & 0 \\ 0_{(K-L) \times K} & 0_{(K-L) \times L} & \mathbf{I}_{(K-L)} \\ 0_{(K-L) \times K} & 0_{(K-L) \times L} & \mathbf{I}_{(K-L)} \\ \vdots & \vdots & \vdots \\ 0 & 0_{(K-L) \times K} & 0_{(K-L) \times L} & \mathbf{I}_{(K-L)} \end{bmatrix} \quad (13)$$

Since components outside of the channel delay spread domain only consist of noise in LS estimation, which is in fact $\mathbf{e}_2[n]$, we have:

$$\mathbf{e}_2[n] = \mathbf{I}_{(K-L)} \hat{\mathbf{h}}[n] = \mathbf{I}_{(K-L)} \mathbf{W}^{-1} \hat{\mathbf{H}}[n] \quad (14)$$

To estimate $\mathbf{e}_1[n]$ using $\mathbf{e}_2[n]$ and the Maximum A Posterior criterion we have:

$$\hat{\mathbf{e}}_1[n] = \arg \max_{\mathbf{e}_1} \{p(\mathbf{e}_1[n] | \mathbf{e}_2[n])\} \quad (15)$$

Based on Gaussian distribution of $\mathbf{e}[n]$ we have:

$$\hat{\mathbf{e}}_1[n] = \arg \min_{\mathbf{e}_1} \{C(\mathbf{e}_1)\} \quad (16)$$

Where, $C(\mathbf{e}_1) = [\mathbf{e}_1[n]^H, \mathbf{e}_2[n]^H] \mathbf{P} \begin{bmatrix} \mathbf{e}_1[n] \\ \mathbf{e}_2[n] \end{bmatrix}$, and the $\mathbf{P}_{MK \times MK}$ matrix is defined as follows.

$$\mathbf{P} = E \left\{ \begin{bmatrix} \mathbf{e}_1[n] \\ \mathbf{e}_2[n] \end{bmatrix} \begin{bmatrix} \mathbf{e}_1[n]^H & \mathbf{e}_2[n]^H \end{bmatrix} \right\}^{-1} \quad (17)$$

After substituting (11) and (12) in the above relation, the following relation is obtained for P.

$$\mathbf{P} = E \{ \mathbf{I} \mathbf{e}[n] \mathbf{e}[n]^H \mathbf{I}^T \}^{-1} = \mathbf{I} \mathbf{Q}_e^{-1} \mathbf{I}^T \quad (18)$$

Where, $\mathbf{I}_{MK \times MK} = \begin{bmatrix} \mathbf{I}_L \\ \mathbf{I}_{(K-L)} \end{bmatrix}$. Considering relation (10) it is indicated that [16]:

$$\mathbf{Q}_e = E \{ \mathbf{e}[n] \mathbf{e}[n]^H \} = \sigma_n^2 \mathbf{W}^H (\mathbf{S}[n]^H \mathbf{S}[n])^{-1} \mathbf{W} \quad (19)$$

Since, $\mathbf{W}^H = \mathbf{W}^{-1}$ and $\mathbf{I}^T = \mathbf{I}^{-1}$, by substituting relation (19) in (18) the following result is obtained for the P matrix.

$$\mathbf{P} = \frac{1}{\sigma_n^2} \mathbf{I} \mathbf{W}^H \mathbf{S}[n]^H \mathbf{S}[n] \mathbf{W} \mathbf{I}^T \quad (20)$$

The P matrix is the same for all of the receiver antennae. Hence, it should be calculated for the receiver only once. The P matrix is classified as follows [16]:

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 \\ \mathbf{P}_3 & \mathbf{P}_4 \end{bmatrix} \quad (21)$$

$$\mathbf{P}_1 \triangleq \mathbf{P}(1:ML, 1:ML)$$

$$\mathbf{P}_2 \triangleq \mathbf{P}(1:ML, ML+1:MK)$$

$$\mathbf{P}_3 \triangleq \mathbf{P}(ML+1:MK, 1:ML)$$

$$\mathbf{P}_4 \triangleq \mathbf{P}(ML+1:MK, ML+1:MK)$$

By substituting the above relation in relation (16) and differentiation $C(e_1)$ in relation to e_1 we have [16]:

$$\hat{e}_1[n] = -\mathbf{P}_1^{-1}\mathbf{P}_2e_2[n] \quad (22)$$

Where, according to relations (20) and (21) we have:

$$\mathbf{P}_1 = \frac{1}{\sigma_n^2} \mathbf{I}_L \mathbf{W}^H \mathbf{S}[n]^H \mathbf{S}[n] \mathbf{W} \mathbf{I}_L^T \quad (23)$$

$$\mathbf{P}_2 = \frac{1}{\sigma_n^2} \mathbf{I}_L \mathbf{W}^H \mathbf{S}[n]^H \mathbf{S}[n] \mathbf{W} \mathbf{I}_{(K-L)}^T \quad (24)$$

In addition, $e_2[n]$ is obtained from relation (14). By subtracting $\hat{e}_1[n]$ from the ELS channel estimation ($\check{h}[n]$), the ILS channel estimation in relation to time is obtained as follows [16].

$$\tilde{h}[n] = \check{h}[n] - \hat{e}_1[n] \quad (25)$$

Moreover, the ILS channel estimation in terms of frequency is as follows.

$$\tilde{H}[n] = \mathbf{W} \mathbf{I}_L^T \tilde{h}[n] \quad (26)$$

By substituting relation (25) in (26) and using relations (8), (14) and (22), the ILS channel estimation in terms of frequency for the i-th receiver antenna is as follows.

$$\tilde{H}_i[n] = \mathbf{G} \mathbf{Y}_i[n] \quad (27)$$

G

$$\begin{aligned} &\triangleq \mathbf{W}(\mathbf{I}_L^T \mathbf{I}_L \\ &+ \mathbf{I}_L^T \mathbf{P}_1^{-1} \mathbf{P}_2 \mathbf{I}_{(K-L)}) \mathbf{W}^H (\mathbf{S}[n]^H \mathbf{S}[n])^{-1} \mathbf{S}[n]^H \end{aligned} \quad (28)$$

According to the above relation, regardless of the number of receiver antennae, G should only be calculated once in the receiver, and there is no need for knowing noise variance in the receiver. Moreover, ILS channel estimation in terms of time in the i-th receiver antenna is as follows.

$$\tilde{h}_i[n] = \mathbf{I}_L \mathbf{W}^H \mathbf{G} \mathbf{Y}_i[n] = \mathbf{h}_i[n] + (e_1[n] - \hat{e}_1[n]) \quad (29)$$

Figure 4 depicts structure of the ILS algorithm. The NMSE diagram of the ILS channel estimation algorithm is shown in Figure 4 in terms of the average SNR rate for the MIMO-OFDM system. As seen, similar to the two previously mentioned algorithms, the ILS algorithm's performance declines with an increase in the normalized Doppler frequency.

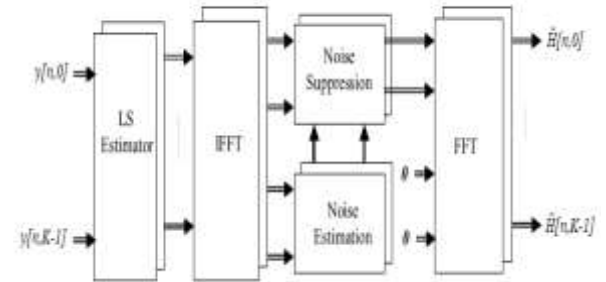


Fig.4. Structure of the ILS channel estimation algorithm [16]

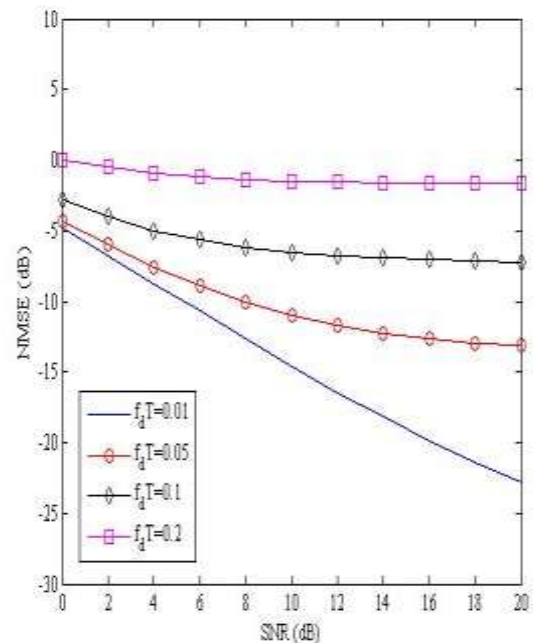


Fig.5. NMSE of the ILS channel estimation algorithm in terms of average SNR

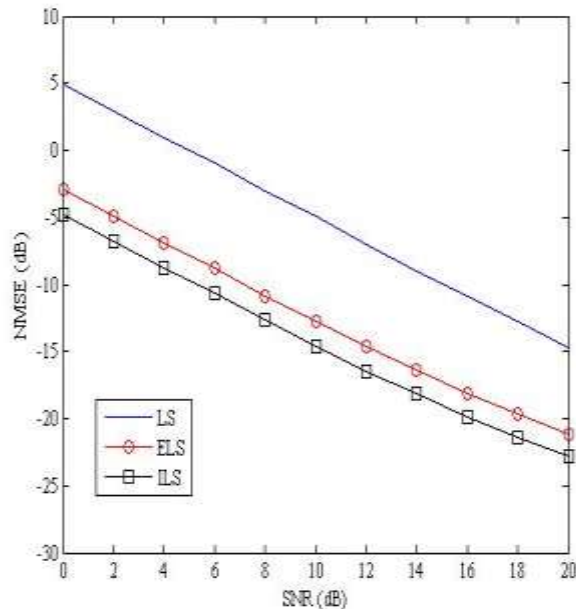


Fig.6. Comparison of NMSEs of the LS, ELS, and ILS channel estimation algorithms in terms of average SNR for $f_d T = 0.01$

6. CONCLUSION

It could be stated that the ELS channel estimation algorithm performs better than the LS channel estimation algorithm, while the ILS estimation algorithm performs better than the two. Unlike the other two algorithms, in which performance declines with an increase in the number of transmitter and receiver antennas, performance of the ILS algorithm does not change. Since the ELS and ILS algorithms require the channel length (L) information, this value is assumed to be equal to the cyclic prefix length in practice. It should be noted that the computational complexity of the ELS algorithm is more than the LS algorithm, while the computational complexity of the ILS algorithm is more than the two. As simulations indicated, with an increase in Doppler frequency, channel estimation error increased with all of the three algorithms and their performances declined. The decrease in performance is considerable especially in channels with fast fading.

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