Dynamic Voltage Stability Evaluation of Power Systems Considering Industrial Electrical Loads

Mohammad Reza Ansari
Department of Electrical Engineering, University of Shahreza, Shahreza, Esfahan, Iran.
Email: mr.ansari.sh@gmail.com

ABSTRACT:
This paper presents a simulation of the dynamic voltage stability in power system. In application of modern power system, dynamic assessment of voltage stability is known as basic concept. In order to study dynamic voltage stability in a power system, different dynamic boundaries are defined such as, Hopf bifurcation (HB) boundary. HB point is an oscillatory boundary in power system. For recognition of the bifurcations, it is unavoidable to study the eigenvalues of power system. In spite of this, determination of these eigenvalues needs to dynamic Jacobian matrix of power system and modal analysis that is very time consuming and complex in large systems. Also, different industrial loads (static and dynamic) e.g. induction motors can effect on dynamic voltage stability boundaries. In this paper, we proposed a solution method based on analyzing the eigenvalues of reduced Jacobian matrix and time domain simulation for assessment of dynamic voltage stability. In addition, effects of industrial electrical loads on the small disturbance voltage stability are evaluated by the proposed method. To show the effectiveness of the proposed solution method, it is tested on IEEE 14 bus and New England test systems.

KEYWORDS: Voltage stability, Dynamic Simulation, Hopf Bifurcation, Industrial Electrical Loads.

1. INTRODUCTION

Evaluation of dynamic stability is a key requirement for today power systems. Power system is subjected to a wide range of small and large disturbances such as, network variations, generation and Loads changes, faults and outages of equipment. These disturbances can lead to instability of power system. Study of power system stability is an important parameter in the operation, planning, economic, and reliability of secure electric power system.

Stability of power system can be classified to voltage stability, rotor angle stability and frequency stability. Voltage stability refers to ability of power system for maintain acceptable stable voltage for all buses after a disturbance relative to primary condition [1-5]. After a number of voltage instability events which occurred around the world, voltage stability has become a major research area in the field of power systems in recent years.

In order to examine voltage stability in a power system, different static and dynamic boundaries are defined. Static boundary such as static load margin (SLM) is obtainable by load flow equations [6], [7] and dynamic boundaries are Saddle Node Bifurcation (SNB), Hopf Bifurcation (HB) and Limit Induced Bifurcation (LIB) that are studied by eigenvalues of dynamic algebraic Jacobian matrix of power system [1], [8-11]. In dynamic study, system is stable when damping ratio of critical mode is positive and system oscillations is damped (all eigenvalues of power system is located in the left part of the complex plane). HB point is an oscillatory boundary in power system that corresponds to locating a pair of complex conjugate eigenvalue of dynamic algebraic Jacobian matrix on imaginary axis. Finally, because of presence of a single real one or a pair of complex conjugate eigenvalues of power system in the right part of the complex plane, damping ratio is negative and so power system involves in undamped oscillations with ascending or fixed scope [1, 8]. Also, SNB and LIB basically consist of loss of system equilibrium point, which is typically correlated with the lack of load flow solutions. In SNB point, Jacobian matrix has a zero eigenvalue and this matrix is singular.

Determination of dynamic boundaries of voltage stability and also maintain and increase of these, are important issues in dynamic study of voltage stability. Recently, different methods that most of them are known have been proposed to improve dynamic stability of power system and decrease system oscillation [12-15]. Also in [16-17], demand response programs and line switching, are used to increase the voltage stability margin of the power systems.

Therefore, this paper focuses on the dynamic voltage stability, which is a major concern of today power systems [2], [3], [18]. Linearized system equations can be used for bifurcation analysis and investigation of dynamic voltage stability in power system. Therefore, in this work a combination of static and dynamic analysis and also dynamic simulations are used in a complementary manner. In a power system, some reasons such as loads model, generators control system parameters e.g. AVR and governor loops, reactive power limit and terminal voltage regulations of generators and contingency conditions, such as system load variation and loss of transmission lines, have considerable effect on dynamic study of power system [1-5]. In other words, dynamic stability is influenced by the dynamic characteristics of generators, controllers and loads. Therefore, effect of industrial electrical loads (static and dynamic) e.g. constant power and induction motors, on the dynamic voltage stability are evaluated by the proposed method.

The remaining parts of the paper are organized as follows. In the second section, details of purposed method are described. In this section, static and dynamic analysis tools to evaluate dynamic voltage stability are presented. Obtained numerical results of various test systems are presented and discussed in section three. Section four concludes the paper.

2. THE PROPOSED METHOD

Limitations of steady state power system studies (load flow equations) are associated with nonlinear dynamic characteristics of power systems. The small signal voltage stability is a dynamic phenomenon, but the use of steady state analysis tools can provide useful information about it. In steady state studies of voltage stability based on the power flow equations, all dynamics are died out and all controllers have done their duty. Also, long term voltage stability is studied by steady state analysis tools [2]. The advantage of using algebraic load flow equations compared to differential equations of dynamic studies is the computation speed. However, the power system stability cannot be fully guaranteed with steady state studies. Especially, when the voltage instability occurs in the transient period of disturbance, i.e. HB point, the voltage stability margin obtained by the dynamic analysis tools (dynamic load margin (DLM)) can be considerably less than static load margin (SLM). The time domain simulations capture the events and chronology leading to voltage instability. The most accurate response of the actual dynamics of voltage instability is provided by this method [2].

Voltage stability is a nonlinear occurrence. However, valuable sensitivity information in identifying factors influencing dynamic voltage stability can be obtained by linearized equations of the power system at the equilibrium point [4]. The obtained Jacobian matrix

describes the linearized system which best approximates the nonlinear equations close to the equilibrium point. In this case the stability of the nonlinear system can be studied like the stability of linear systems in the neighborhood of operating equilibrium point [2]. Also, bifurcation theory is used to study the real behavior of the power system. Bifurcation theory is one of theories that have expansive application in the study of voltage stability. This theory is an appropriate instrument for classification, study and obtain quality and quantity information about operation of a multi-dimensional system around equilibrium point. On the other hand, exact modeling of equipment in the system and use of time simulation are necessary to perform dynamic study in a power system. This theory deals with the problem of loss stability of a nonlinear dynamic system under changing parameter values [19]. This method assumes that power system parameters vary slowly and predicts how a power system will become unstable. Changing the parameters moves the system slowly from one equilibrium to another until it reaches the collapse point [5]. Bifurcation points where change from stable to unstable, from stationary to oscillatory, or from order to chaos, are the most interesting points in voltage stability studies. In [5] it has been presented that normally one parameter, e.g. load demand, is changed (which is also the case considered here), in which case there is a possibility to reach either SNB or HB, e.g. Fig. 1.
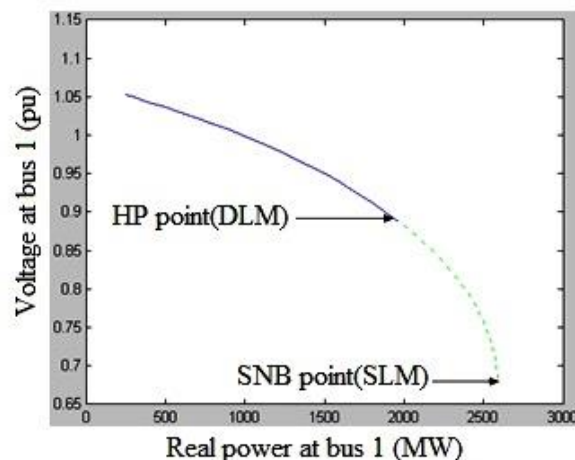

Fig. 1. P-V curve for constant power load in the 68 bus power system.

2.1. P-V AND BIFURCATION CURVES

P-V curve and bifurcation curve are shown in Fig. 2. Bifurcation curve is constructed by tracing equilibrium points of the successive dynamic responses while, P-V curve is usually constructed by successive load flow solutions [20]. In Bifurcation curve, voltage collapse is associated with saddle-node bifurcation (SNB) where, system equilibrium point disappears.
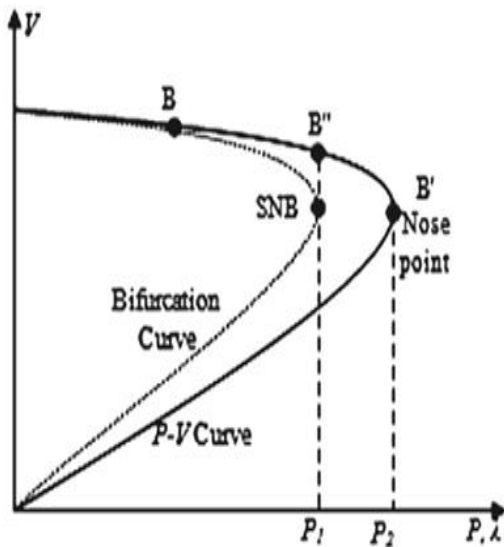
Fig. 2. P-V and Bifurcation curves.

In this point, dynamic Jacobian matrix has a zero eigenvalue and this matrix is singular. Some earlier works have been presented that the nose point of the P-V curve is the voltage collapse point. Nose point is the maximum loading level beyond which steady state solutions (power flow solutions) cannot be obtained for the system. However, the actual voltage collapse point is the SNB of the bifurcation curve and the SNB point coincides with the nose point only if the load is constant power (Fig. 1). Otherwise, on other load models, the SNB point will not coincide with the nose point of the P-V curve, but to the other part of the P-V curve (such as B" on the P-V curve in Fig. 2). In that case, the voltage collapse is no longer a nose point but at point B" on the P-V curve.

2.2. DYNAMIC VOLTAGE STABILITY

In this paper, linearized algebraic-dynamic equations of system are used for bifurcation analysis and investigation of dynamic voltage stability in power system. These algebraic-dynamic equations in power system can be formulated as follows:

$$\dot{x} = f(x, y, \lambda, p) \tag{1}$$

$$0 = g(x, y, \lambda, p) \tag{2}$$

The state variables x, in (1), describe the dynamics of power systems, such as generation dynamic, exciter control systems, dynamic loads and other control devices. p and λ define controlled and uncontrolled parameters of the power system respectively, such as loads, generation, voltage setting points and etc. The output variables y, describe algebraic constraints (2), such as power flow equations [1,8,21]. In a power system, to determine the stability by means of linearization and also for fixed values of p and λ parameters, Jacobian matrix can be represented as:

$$\begin{bmatrix} \Delta\dot{x} \\ 0 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = [J] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \tag{3}$$

$$J_{11} = \frac{\partial f}{\partial x}, J_{12} = \frac{\partial f}{\partial y}, J_{21} = \frac{\partial g}{\partial x}, J_{22} = \frac{\partial g}{\partial y} \tag{4}$$

$$\Delta\dot{x} = (J_{11} - J_{12} J_{22}^{-1} J_{12}) \Delta x = A \Delta x \tag{5}$$

However, the general structure of the Jacobian matrix J in a power system is shown in Fig. 3. In Fig. 3, k presents state variables of power system except δ and ω (e.g. E', E'_d, E'_q, E''_d, E''_q of generators dynamic and state variables of loads dynamic)

Fig. 3. General structure of the Jacobian matrix in a power system.

In equations (3-5), J is called the unreduced Jacobian matrix and A is called the reduced Jacobian matrix [22].

The dynamic stability of a power system depends on the eigenvalues of the reduced Jacobian matrix A [23]. Through tracing the eigenvalues of matrix A, the local dynamic stability of the power system can be studied. [6], [23]. Based on A Jacobins matrix, a power system has voltage stability when all eigenvalue of A, that are obtained by model analysis, are located on the left part of imaginary axis [1,8]. Also, based on A matrix, HB

point corresponds to a condition that a pair of complex conjugate eigenvalue of this matrix are located on imaginary axis. HB point is where there is an emergence of oscillatory instability. Numerous reasons lead to such oscillatory instability in power system, but the most important reason in HB is low damping ratio of system critical mode. In a power system, damping ratio of eigenvalue is shown as follows:

$$\zeta = \frac{-\alpha}{\sqrt{\alpha^2 + \beta^2}} \qquad (6)$$

In (6), α and β are real and imaginary parts of a eigenvalue respectively. According to relation (6), power system is stable when damping ratio of critical mode is a positive value. So in this condition, critical eigenvalue of system is located in the left part of imaginary axis. Fig. 4 has shown critical eigenvalue of power system for increase load of power system. Regarding to the figure, increase in load from λ_1 to λ_5 leads to HB bifurcation in loading coefficient of λ_4.
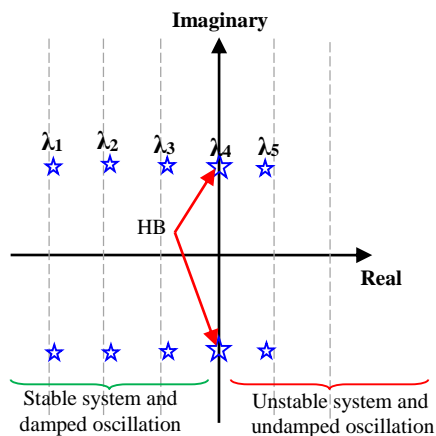

Fig. 4. Critical eigenvalues conditions.

In the following, the proposed method can be summarized by step by step algorithm:

1) Define the set of power system parameters e.g. power system structure, load pattern (contain of load quantity and type), and generation pattern. For study effect of industrial electrical loads on the dynamic voltage stability, different types of loads are considered.

2) Solve the power flow equations by static analysis tools and determine equilibrium point. Consider controller limits (e.g. generators reactive power limits) and on-load tap changers (OLTC).

3) Obtain reduced Jacobian matrix A at the equilibrium point and determine its eigenvalues. Evaluate occurrence of HB or SNB points based on the condition of eigenvalues.

4) In this step, the load pattern of the power system

is changed by one step (apply the small perturbation) according to the scheduled plan. For example, changing the load patterns may be simply in the form of linearly increasing consumption of a load bus [24].

5) Run time domain simulation of power system. In [25], dynamic models of the power system components are described.

6) If occurrence oscillatory instability or HB point in the time domain simulation is detected, voltage stability is dynamically lost at this point. HB point is determined by the eigenvalue analysis of reduced Jacobian matrix A (step 3) and time domain simulation of power system.

7) Determine post-disturbance equilibrium point (with changed load) by static analysis tools and power flow equations as described in step 2. If no solution can be found, SNB or LIB has been occurred. Occurrence of SNB point should be also determined by the eigenvalue analysis.

3. NUMERICAL RESULTS

We tested the proposed method on the IEEE 14 bus and New England test systems, frequently considered in the voltage stability studies [1], [8], [9], [26-27], where their data can be found in [28], [29]. Characteristics of these test systems are briefly shown in Table 1. Single line diagram of the IEEE 14 and New England test systems are shown in Fig. 5 and Fig. 6. In this paper, for dynamic and static analysis and also time domain simulation of DIGSILENT software is used [30]. We also evaluate effect of industrial electrical loads on different aspects of dynamic voltage stability by the proposed method

3.1. Constant Power Load Scenario

We examined constant power Load scenarios including increasing constant power load of bus 14 in IEEE14 test system.

Sample results for the IEEE 14 bus test system are shown in Fig. 7 and 8. In Fig. 9, eigenvalue locus for increasing load of bus 14 is shown. Those eigenvalues of Jacobian matrix which finally cross the imaginary axis and cause HB point are named critical conjugate eigenvalues. A single real one or a pair of complex conjugate eigenvalues can be critical eigenvalue.

Table 1. The characteristics of test systems.

Test System	Bus	Branch	Generator	Load
IEEE 14-Bus	14	20	2	11
New England	39	46	10	19

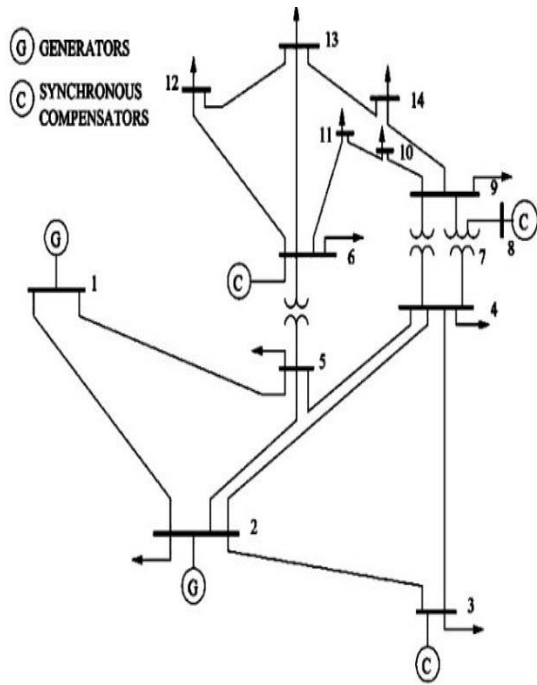**Fig. 5**. Single line diagram of the IEEE 14 bus test system.

Fig. 6. Single line diagram of New England test system.

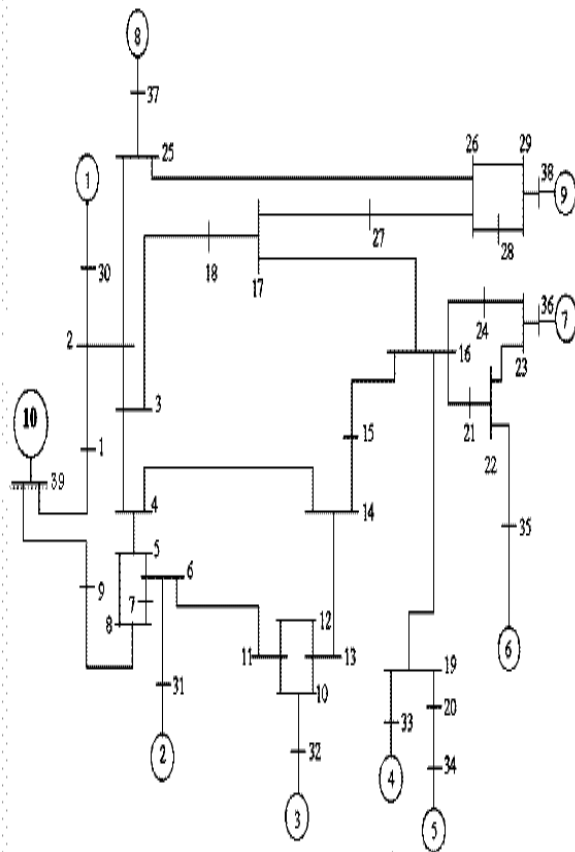**Fig. 7**. Eigenvalue locus for increasing load of bus 14 in the IEEE 14 bus test system.

Bifurcation curve for constant power load scenarios in the IEEE 14 bus test system is shown in Fig. 8.

Also, in Table 2, participation factors for examinations of Fig. 7 are shown. The participation factors represent contribution of each state of the power system in the critical modes [31]. In this Table, eight states with the largest participation factors (normalized with respect to their maximum) at HB point are sorted. For instance, state speed of bus 1 has the largest participation factor in the critical modes.
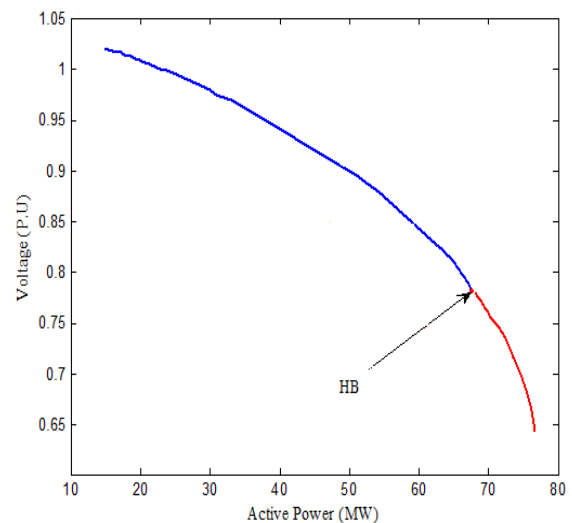

Fig. 8. Bifurcation curve for constant power load scenarios in the IEEE 14 bus test system.

Table 2. Participation factors for the load scenarios of Fig. 7 and 8 (HB point).

| 67.72+j23.016 MVA (HB)		Participation Factor
State	Bus	
Speed	1	1
Speed	3	0.483
Speed	2	0.313
Speed	6	0.312
Speed	8	0.264
Excitation-Flux	1	0.260
Rotor-Angle	1	0.038
Excitation-Flux	2	0.032

3.2. Hybrid Load Scenario

We examined effect of hybrid load (static and dynamic) in the voltage stability studies. In this section, we increase constant power load of bus 14 of IEEE 14 bus system, while buses 5,6,9,11,12,13,14 are contained hybrid loads, 40% dynamic load (induction motor) and 60 % static load (constant power load).

In Fig. 9 locus of eigenvalues of the reduced Jacobian *A* close to imaginary axis is shown. The nonlinear behavior of the dominant eigenvalues has been presented in the previous works with respect to the load variation [1]. In [1], we clarified where the dominant eigenvalues in the current operating point can differ from the critical ones. Here, a similar condition is seen for the reduced Jacobian matrix *A* where the closest eigenvalues to the imaginary axis are changed by the load variation. For instance, in the first parts of Fig. 9, the critical eigenvalues (19-20) appear after three other ones. By increasing load (4), the critical eigenvalues appear in the second parts of Fig. 9 are 22-23 and finally, by further increasing load (4), the critical eigenvalues 18-19 cross the imaginary axis and cause HB.

Also, we examined effect of hybrid load in the voltage stability studies of the New England test system. In this section, we increase constant power load of bus 23, while buses 3, 12, 15, 18, 20, 23, 29 and 31 are contained hybrid loads, 40% dynamic load (induction motor) and 60 % static load (constant power load). In Fig. 10 locus of eigenvalues of the reduced Jacobian matrix *A* close to imaginary axis is shown.

In addition, we change load of bus 23 of constant power into constant impedance load. Result of this test is shown in Table 3. It is observed that DLM is improved by constant impedance load in bus 23. However, dynamic load margin (DLM) in power system is dependent to the load type. In the other words, we can conclude that the critical eigenvalues in different types of loads are fixed, but the speed by which these eigenvalues come close to the imaginary axis is sensitive on the load type.
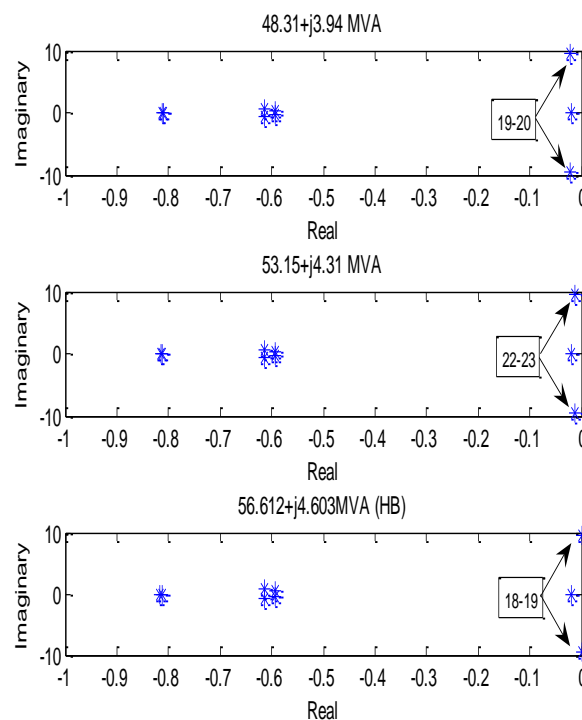
Fig. 9. Eigenvalue locus for increasing load of bus 4 in the IEEE 14 bus test system in hybrid load scenarios.
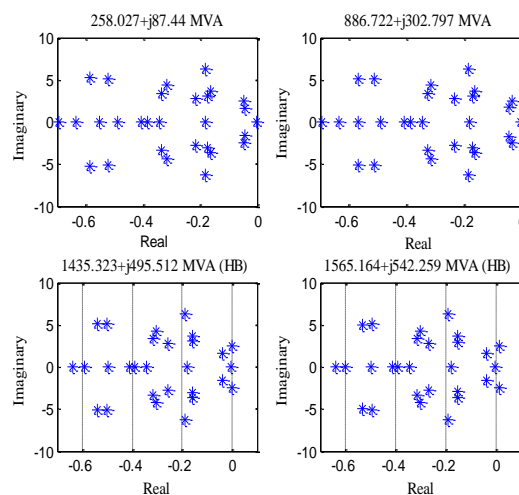
Fig. 10. Eigenvalue locus for increasing load of bus 23 in the New England test system in hybrid load scenarios.

Table 3. Result of change in load of bus 23.

Bus	Type of Load	Load in HB Point
23	Constant Power	1435.323+j495.512
23	Constant Impedance	1828.150+j624.895

4. CONCLUSION

In order to study voltage stability in a power system, different boundaries of dynamic stability are considered

in this paper. Hopf Bifurcation point (HB) (as one of the boundaries of dynamic stability) is determined by analyses of eigenvalues locus of dynamic algebraic Jacobian matrix of power system. Also, we tested effect of industrial loads (static and dynamic) e.g. induction motors on boundaries of dynamic voltage stability. In addition, Bifurcation curve and participation factors for each test system were obtained and also were investigated. According to results, we can conclude that the critical eigenvalues in different types of loads are fixed, but the speed by which these eigenvalues come close to the imaginary axis is sensitive on the load type. Anyway, the proposed solution approach can help operator of power systems to monitor their systems with reasonable computation times more accurately
.

5. REFERENCES

[1] N. Amjady, M. R. Ansari "**Small Disturbance Voltage Stability Assessment of Power Systems by Modal Analysis and Dynamic Simulation**". *Energy Conversion and Management*, Vol. 49(10), pp. 2629-2641, 2008.

[2] S. Repo, "**On-line Voltage Stability Assessment of Power Systems-An Approach of Black Box Modeling**", *Tampere University of Technology Publications 344*, 2001.

[3] N. Amjady, M. R. Ansari, "**Small disturbance voltage stability evaluation of power systems**", *Transmission and Distribution Conference and Exposition*, 2008

[4] P. Kundur, J. Paserba, V. Ajjarapu, G. Anderson, A. Bose, C. Canizares, N. Hatziargyriou, D. Hill, A. Stankovic, C. Taylor, T.V. Cutsem, and V. Vittal. "**Definition and Classification of Power System Stability**", *IEEE Trans. on Power Systems*, Vol. 19(3), pp. 1387-1401, Aug. 2004.

[5] T. V. Cutsem, C. Vournas, "**Voltage Stability of Electric Power Systems**", *Kluwer Academic Publishers*, Boston, USA, 1998.

[6] F. Zhihong., "**The static voltage stability analysis methods for many generators power system– singularity decoupled method**", *Proceedings of CSEE.*, Vol. 12(3): pp. 10-18, 1992.

[7] H. Chen., Y. Wang, R. Zhou "**Transient and voltage stability enhancement via coordinated excitation and UPFC control**", *IEE Proc. Gener Transm Distrib.* Vol. 148(3), pp. 201-208, 2001.

[8] N. Amjady, M. H. Velayati, "**Evaluation of Hopf Bifurcation Considering the Effect of Load Models and Excitation System Parameters**", *International Review of Electrical Engineering*, Vol. 6(5), pp. 2419-2427, 2011.

[9] N. Amjady, M. H. Velayati, "**Evaluation of the Maximum Loadability Point of Power Systems Considering the Effect of Static Load Models**", *Energy Conversion and Management*, Vol. 50(12), pp. 3202-3210, 2009.

[10] GM. Huang, L. Zhao, X. Song, "**new bifurcation analysis for power system dynamic voltage stability studies**", *In: Proc. IEEE power eng. Soc. winter meeting*. 2002, pp. 882-887, 2002.

[11] J. Hongjie., Y. Xiaodan, Y. Yixin, "**An improved**

[12] A. P. Lerm, "**Control of Hopf Bifurcation in Multi Area Power Systems via a Secondary Voltage. Regulation Scheme**". *IEEE Trans. Power Syst.*, Vol. 23(3), pp. 245-254, 2002.

[13] C. F. Yang, G. G. Lai, C. H. Lee, C. T. Su, G. W. Chang, "**Optimal Setting of Reactive Compensation Devices with an Improved Voltage Stability Index for Voltage Stability Enhancement**". *Electrical Power and Energy Systems*, Vol. 37(1). pp. 50-57, 2012.

[14] H. Raoufi, M. Kalantar, "**Reactive Power Rescheduling with Generator Ranking for Voltage Stability Improvement**". *Energy Conversion and Management*, Vol. 50(4), pp. 1129-1135, 2009.

[15] A. Kargarian, M. Raoofat, M. Mohammadi, "**Reactive Power Market Management Considering Voltage Control Area Reserve and System Security**", *Applied Energy*, Vol. 88(11), pp. 3832-3840, 2011.

[16] M. Moohseni-Bonab, A. Rabiee, B. Mohammadi-Ivatloo "**Voltage stability constrained multi-objective optimal reactive power dispatch under load and wind power uncertainties: A stochastic approach**", Renewable Energy, Vol. 85, pp. 598-609, Jan. 2016.

[17] M. Nojavan, H. Seyedi, B. Mohammadi-Ivatloo "**Voltage Stability Margin Improvement Using Hybrid NLP and MBPSO Algorithm Considering Optimal Transmission Line Switching**", IET Generation, Transmission & Distribution, Vol. 12(4), pp. 815 – 823, Feb. 2018

[18] N. Amjady, M. Esmaili, "**Improving voltage security assessment and ranking vulnerable bus with consideration of power system limits**," *Elect. Power Energy Syst.*, Vol. 25(9), pp. 705–715, Nov. 2003.

[19] H. Chen, Y. Wang, R. Zhou, "**Transient and voltage stability enhancement via coordinated excitation and UPFC control**," *IEE Proc. Gener. Transm. Distrib.*, Vol. 148(3), pp. 201-208, May 2001.

[20] F. Karbalaei, S. Abasi, "**Quadratic Approximation of PV Curve Path Based On Local Measurements On Presence of Voltage Dependent Loads**" Majlesi Journal of Electrical Engineering, Vol. 7 (3), pp. 8-13, Sep. 2013

[21] R. J. Avalos, C. Canizares, F. Milano, A. j. Conejo, "**Equivalency of Continuation and Optimization Methods to Determine Saddle-node and Limit-induced Bifurcations in Power Systems**". *IEEE Trans. Circuits and Systems–I: Regular Paper.*, Vol. 56(1), pp. 210-223, 2009.

[22] F. Jabari, B. Mohammadi Ivatloo, "**Static Voltage Stability Assessment Using Probabilistic Power Flow to Determine the Critical PQ Buses**" Majlesi Journal of Electrical Engineering, Vol. 8 (4), pp. 17-23, Dec. 2014.

[23] P. W. Sauer and M. A. Pai, "**Power System Dynamics and Stability**", Upper Saddle River, NJ: Prentice-Hall, 1998.

[24] W. Marszalek, Z. W. Trzaska, "**Singularity-Induced Bifurcations in Electrical Power Systems**," *IEEE*

Trans. on Power Systems, Vol. 20(1), pp. 312-320, Feb. 2005.

[25]   N. Amjady, "**Dynamic voltage security assessment by a neural network based method**," *Elect. Power Syst. Res.*, vVol. 66(3), pp. 215–226, Sep. 2003.

[26]   K. Hongesombut, Y. Mitani, and K. Tsuji, "**Power System Stabilizer Tuning in Multimachine Power System Based on a Minimum Phase Control Loop Method and Genetic Algorithm**", *14th PSCC, Sevilla*, pp. 1-7, June 2002.

[27]   S. Nojavan,, V. Fathi,, M. Nojavan, K. Zare, "**A New Combined Index Applied for Anticipatory Load Shedding with Voltage Stability Consideration"**

Majlesi Journal of Electrical Engineering, Vol. 9 (3), pp. 53-60, Sep. 2015.

[28]   M. A. Pai, Energy Function Analysis for Power System Stability, Kluwer, Boston, 1989.

[29]   http://www.ee.washington.edu/research/pstca/.

[30]   DIgSILENT User Manual Toolbox, Available: http://www.digsilent.de/.

[31]   H. K. Nam, Y. K. Kim, K. S. Shim, and K. Y. Lee, "**A new Eigen-sensitivity theory of augmented matrix and its applications to power system stability analysis**," *IEEE Trans. Power Syst.*, Vol. 15(1), pp. 363–369, Feb. 2000.
.