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Vibration Analysis of Rotating Disk Carrying Annular Concentrated Masses in Turbo-pump System

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Abstract: Vibration analysis of rotating disks is one of the most important problems in turbomachines. In this study, a new method has been presented which analyzed the radial vibration of a turbo-pump rotating disk carrying two annular concentrated masses located on the disk and at its end. Natural frequencies have been calculated in different rotating speeds; then results have been compared with each other. The effects of concentrated masses position and intensity on natural frequencies have been investigated. The results show that concentrated masses always have been decreased the value of first natural frequency, but in the case of second and third natural frequencies, depending on the mass concentration magnitude and its position, the magnitude of natural frequency has been increased or decreased. The vibration of the rotating disk without considering the concentrated mass, was examined. Then the resulting solution was generalized for two connected disks in internal concentrated mass location. The effect of concentrated masses, one on the disk body and the other on the outside of the disk, is considered as boundary conditions in the two disk Equations. The results show that increasing in angular velocity of rotating disk reduces the natural frequency. Concentrated masses always reduce the first natural frequency. At the second and third natural frequencies, concentrated masses may increase or decrease the natural frequency, which depends on the value and position of concentrated mass. Concentrated mass has the most impact when it is in a position that has the most radial displacement.

Keywords: Concentrated Masses, Free Vibrations, Rotating Disk, Turbo-pump

Biographical notes: Behrooz Shahriari was born in Isfahan, Iran in 1975. He received the B.S. in Mechanical Engineering and MSc. and PhD degrees in Aerospace Engineering from Malek Ashtar University of Technology in 2002 and 2012 and 2016, respectively. **Mostafa Nazemizadeh** received his PhD in Mechanical Engineering from Amirkabir University of Technology, in 2016. His current research interests are robotic nonlinear dynamics and optimal control, micro-to-nano dynamics and nonlinear vibration. **M. A. Shirvani** received the B.Sc. in Aerospace Engineering from Malek Ashtar University of Technology in 2020. He is MSc. Student in Malek Ashtar University of Technology.

Research paper

1 INTRODUCTION

Rotating disks are one of the most important components of turbines and their structures design are very important. From decades ago, due to the importance of rotating disk vibration, this issue has been considered by engineers and scientists. Disks are used in very wide areas of technology such as, turbomachines and hydraulic and power generating machines, turbines, turbo pumps, turbo-super chargers, centrifugal pumps, centrifugal compressors, fans, molecular pumps, centrifuges, turbo-generators, turbo alternator, rotary dynamos, synchronous and asynchronous electric cables, different types of planes, propulsion units on turboprop, turbofan and turbojet aircrafts, rotary ships, turbine of tanker ships, anchor wheels, wind turbine spindles, flywheel extractors and expanders. Disks have different natural frequencies at each angular velocity. And angular velocities that are equal to the natural frequency are known as critical speed [1]. If critical speed is equal to the working speed, the rotary system will suffer from extreme vibrations or in other words, resonance will happen and may result in damages or even fracture in machine part or in entire system.

Lamb and Southwell [2] investigated the axial vibrations of the spinning disk with a constant thickness and rotating speed. They mentioned that the vibration of a nonrotating disks was previously investigated by Kirchhoff. In order to disregard the bending forces, they assumed that the disk was sufficiently thin and had high rotating speed. The Southwell [3] investigated the free transverse vibrations of a uniform circular disk clamped at its center and the effects of rotation. His article is mainly divided into two parts:1) Free vibrations of a nonrotating disk which is clamped in the center of disk; he used the Kirchhoff and Riley Equation to ignore the inertial effects of shear deformation. 2) The effect of the rotation was considered and problem was solved again with neglecting the flexural hardness and the applying of bending and centrifugal stresses obtained from the results of a rotating disk article mentioned in [2]. Bhuta and Jones [4] examined the Symmetric planar vibrations of a rotating disk. These vibrations include radial and tangential vibration of a thin disk. They used an uncertain coordinate system. They also found out that there are two types of instability for the discs: Static resonances and classical instability. Dodson and Eversman [5] investigated free vibration of a centrally clamped spinning circular disk. They provided an exact solution for the transverse vibration of the rotating disk with considerable flexural hardness. Barasch and Chen [6], also investigated the vibration of a rotating disk. But they used the modified Adams numerical method to solve the governing differential Equations. Radcliffe and Mote [7] tried to identify and control the rotating disk vibration. They used the FFT method to identify the dominant mode of the disk in different conditions. Tomioka et al. [8] studied the Analysis of free vibration of rotating disk–blade coupled systems by using artificial springs and orthogonal polynomials. They used the Ritz method to solve the problem and assumed that there is a hypothetical spring between the blades and the disk. Finally, some of the results were compared with results obtained from the finite element method. Parker and Sathe [9] calculated an exact solution for free and forced vibration of a rotating disk-spindle system. They provided a closed form solution to obtain Eigen values and assumed that the disk and shaft were completely in elastic field. Luo and Mote [10] studied the nonlinear vibration of rotating thin disks. They used Galerkin's approach to obtain an analytical solution.

In this paper, the vibrations of rotating disk carrying two peripheral concentrating masses are investigated. The purpose of this study is to investigate the effect of position and intensity of concentrated mass on the vibration of a rotating disk. These concentrated masses can be considered as balance ones on the disk body and blades on the outer end of the rotating disk in gas turbine, steam turbine and axial compressors.

2 HOMOGENEOUS ROTATING DISKS WITHOUT CONCENTRATED MASS

At the first, we investigate the mathematical Equations governing thin Homogeneous rotating disk without concentrated mass which is constrained at center of disk. Consider the element of a rotating disk shown in "Fig. 1".

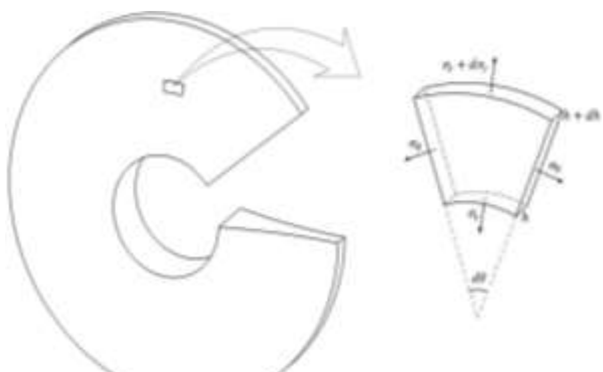

Fig. 1 Volume element of a rotating disk.

By using Newton's second law, we can write the following Equation for a given element in the radial direction:

$$(\sigma_r + d\sigma_r)(r+dr)(hd\theta) - \sigma_r rh\,d\theta - 2\sigma_\theta h\,dr\,\frac{d\theta}{2} + r^2 drd\theta h\rho\omega^2 = rdrd\theta h\rho\,\frac{d^2u}{dt^2} \quad (1)$$

Where, u is radial displacement. Also, in accordance with the geometry of the element, the following Stresses are neglected.
"Eq. (1)" can be rewritten in a summarized form below:

$$\sigma_z = 0 \;\;,\;\; \tau_{r\theta} = 0 \quad (2)$$

$$\frac{\partial\sigma_r}{\partial r} + \frac{(\sigma_r - \sigma_\theta)}{r} + \rho r\omega^2 = \rho\,\frac{\partial^2 u}{\partial t^2} \quad (3)$$

According to hook's law, the relationship between radial and tangential stresses with radial displacement is:

$$\sigma_r = \frac{E}{1-\nu^2}\left(\frac{\partial u}{\partial r} + \nu\frac{u}{r}\right) \qquad \sigma_\theta = \frac{E}{1-\nu^2}\left(r\frac{\partial u}{\partial r} + \frac{u}{r}\right) \quad (4)$$

Substituting "Eq. (4)" into "Eq. (3)", the Equation of motion as the following expression can be reached:

$$\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r\partial r} - \frac{u}{r^2} + \frac{(1-\nu^2)}{E}\rho r\omega^2 = \frac{(1-\nu^2)}{E}\rho\ddot{u} \quad (5)$$

Which the number of dots on u, indicates the number of derivatives relative to time. The parameters and new defined variables are expressed to simplify the Equation of motion:

$$1/c^2 = [(1-\nu^2)\rho]/E \qquad \boldsymbol{z = r/R} \;\;,\;\; \boldsymbol{\tau = \omega t} \quad (6)$$

Where, R is outer radius. Consequently, the Equation of motion will change into new form below:

$$\frac{\partial^2 u}{\partial z^2} + \frac{\partial u}{z\partial z} - \frac{u}{z^2} = \left[\frac{(R^2\omega^2)}{c^2}\right](u'' - Rz) \quad (7)$$

Where the prime sign denotes the derivative of τ .
The boundary conditions for this disk were considered as follows:
-Disk is constrained clamped in inner radius and cannot move in radial direction.
-Disk is free in outer radius and there is no radial force. On the other words:

$$u_{z=0} = 0 \qquad \boldsymbol{\sigma_r|_{z=1} = 0 \Rightarrow \left[\frac{\partial u}{\partial z} + \nu\frac{u}{z}\right]_{z=1} = 0} \quad (8)$$

"Eq. (7)" is a non-homogeneous second order differential Equation, which can be considered as a solution consisting of two functions as follows:

$$\boldsymbol{u(z,\tau) = u_1(z) + u_2(z,\tau)} \quad (9)$$

Therefore, the differential Equation is divided into two separate differential Equations:

$$\boldsymbol{\frac{d^2u_1}{dz^2} + \frac{du_1}{zdz} - \frac{u_1}{z^2} = -\alpha^2 Rz} \quad (10)$$

$$\boldsymbol{\frac{\partial^2 u_2}{\partial z^2} + \frac{\partial u_2}{z\partial z} - \frac{u_2}{z^2} = \alpha^2 u''_2} \quad (11)$$

Where:

$$\boldsymbol{\alpha^2 = (R^2\omega^2)/c^2} \quad (12)$$

The physical interpretation of the "Eq. (9)" is that the overall radial displacement of a disk element consists of two parts, first the static elastic displacement due to centrifugal force and the other is oscillatory displacement due to vibrations.
"Eq. (10)" is an ordinary differential Equation which general solution is considered as:

$$\boldsymbol{u_{1c} = z^L} \quad (13)$$

Substituting "Eq. (13)" into "Eq. (10)", L can be obtained as follows:

$$z^2L(L-1)z^{L-2} + zLz^{L-1} - z^L = 0 \Rightarrow L^2 = 1 \quad (14)$$

Therefore, the general solution of the "Eq. (10)" will be obtained as follows:

$$\boldsymbol{u_{1c} = C_1 z + C_2\frac{1}{z}} \quad (15)$$

Where, C_1 and C_2 coefficients will be determined according to the boundary conditions. Also, the particular solution of the "Eq. (10)" will be considered as follows:

$$\boldsymbol{u_{1p} = Az^3} \quad (16)$$

By substituting "Eq. (16)" into "Eq. (10)", A will be obtained:

$$z^2 A6z + zA3z^2 - Az^3 = -\alpha^2 R z^3 \Rightarrow A = \frac{-\alpha^2 R}{8} \quad (17)$$

And the complete solution of "Eq. (10)" will be as follows:

$$u_1(z) = u_{1c} + u_{1p} = C_1 z + C_2 \frac{1}{z} + \frac{-\alpha^2 R}{8} z^3 \quad (18)$$

By using the method of isolating variables to separate the position variable from time, the solution of "Eq. (11)" can be considered as follows:

$$\boldsymbol{u_2(z,\tau) = U(z) g(\tau)} \quad (19)$$

By substituting "Eq. (19)" into "Eq. (11)", we will have:

$$\frac{d^2U}{dz^2} g(\tau) + \frac{dU}{zdz} g(\tau) - \frac{U}{z^2} g(\tau) = \alpha^2 U \frac{d^2 g(\tau)}{d\tau^2} \quad (20)$$

And as a result:

$$\frac{\frac{d^2U}{dz^2} + \frac{dU}{zdz} - \frac{U}{z^2}}{U\alpha^2} = \frac{\frac{d^2 g(\tau)}{d\tau^2}}{g(\tau)} = cte \quad (21)$$

Where, the "Eq. (21)" is a constant value. In order to make the system response to be oscillatory and intermittent, this constant value should be a negative value, so we will have it:

$$\boldsymbol{\frac{\frac{d^2 g(\tau)}{dt^2}}{g(\tau)} = -p^2 \Rightarrow g(\tau) = exp(ip\tau)} \quad (22)$$

And:

$$\boldsymbol{\frac{d^2U}{dz^2} + \frac{dU}{zdz} - \frac{U}{z^2} = -\alpha^2 p^2 U}$$
$$z^2 \frac{d^2U}{dz^2} + z\frac{dU}{dz} + (z^2\alpha^2p^2 - 1)U = 0 \quad (23)$$

Therefore, the part related to the time of u_2 is specified and also by solving "Eq. (23)", the part related to position will be calculated. The "Eq. (23)" is a Bessel differential Equation and also, the solution will be Bessel function. With a variable change, this Equation can be converted into Bessel Equation.

$$\boldsymbol{kz = \eta} \quad (24)$$

By substituting "Eq. (24)" into "Eq. (23)", the suitable value of k for converting an "Eq. (23)" to a first type of Bessel Equation will be obtained:

$$\eta^2 U'' + \eta U' + \left(\frac{\eta^2}{k^2}\alpha^2 p^2 - 1\right) U = 0 \Rightarrow k^2 = \alpha^2 p^2 \Rightarrow k_1 = \alpha p, k_2 = -\alpha p$$
$$\Rightarrow \eta^2 U'' + \eta U' + (\eta^2 - 1)U = 0 \quad (25)$$

Given that the negative value of k can be considered in the positive value of k, therefore we only consider the positive values. The general solution of this homogeneous Equation will be:

$$\boldsymbol{U(\eta) = C_3 J_1(k_1 z) + C_4 Y_1(k_1 z)} \quad (26)$$

Where, J_1 and Y_1 respectively, are the first and second types of Bessel function. In order to calculate the natural frequencies, we apply boundary conditions in "Eq. (26)" which will be considered as follows:

$$u_2(z_{in},\tau) = 0$$
$$\Rightarrow C_3 J_1(k_1 z) + C_4 Y_1(k_1 z)|_{z=z_{in}} = 0$$
$$\sigma_r|_{z=1} = 0 \Rightarrow \left[\frac{\partial u_2}{\partial z} + \nu \frac{u_2}{z}\right]_{z=1} = 0$$
$$\Rightarrow C_3 \left[J'_1(k_1 z) + \frac{\nu J_1(k_1 z)}{z}\right]_{z=1} + \boldsymbol{C_4 \left[Y'_1(k_1 z) + \frac{\nu Y_1(k_1 z)}{z}\right]_{z=1} = 0} \quad (27)$$

Where Z_{in} is dimensionless internal radius of disk. This Equation is simplified in following matrix form:

$$\boldsymbol{\begin{bmatrix} J_1(k_1 z_{in}) & Y_1(k_1 z_{in}) \\ J'_1(k_1) + \frac{\nu J_1(k_1)}{z} & Y'_1(k_1) + \frac{\nu Y_1(k_1)}{z} \end{bmatrix} \begin{bmatrix} C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}} \quad (28)$$

In order to obtain non-binary solutions, the square matrix determinants stated in "Eq. (28)" must be zero. So, given that p or in other words natural frequencies is the only unknown parameters of determinant inside the k_1, so the natural frequencies are calculated by using this fact. In other words, roots of the determinant Equation are natural frequencies of desire rotating disk, and is:

$$J_1(k_1 z_{in})\left[Y'_1(k_1)+\frac{\nu Y_1(k_1)}{1}\right]- Y_1(k_1 z_{in})\left[J'_1(k_1)+\frac{\nu J_1(k_1)}{1}\right]=0 \tag{29}$$

The graph of the frequency Equation is shown in “Fig. 2”. As previously stated, the root of this graph are the natural frequencies.

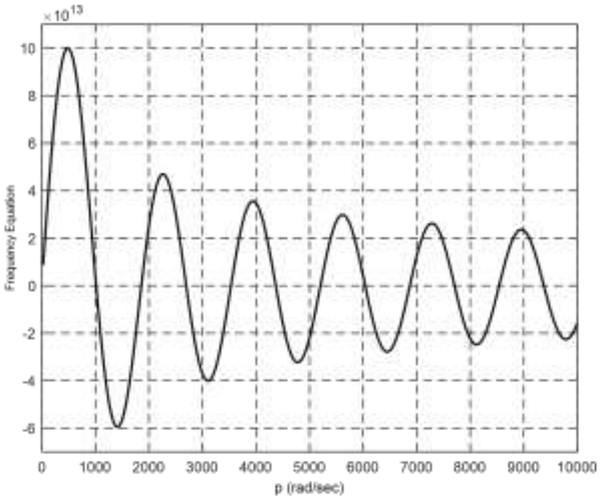

Fig. 2 Distribution of natural frequencies.

After determining the natural frequencies, we have to calculate the mode shapes. To obtain that, we use the “Eq. (26) and Eq. (28)”. Since the natural frequencies are determined, so the value of k_1 is also known. By matrix multiplication in “Eq. (28)” and using the first Equation, the ratio of unknown coefficients is obtained:

$$C_3^i J_1(K_1^i z_{in}) + C_4^i Y_1(K_1^i z_{in}) = 0 \Rightarrow \frac{C_3^i}{C_4^i} = -\frac{Y_1(K_1^i z_{in})}{J_1(K_1^i z_{in})} \tag{30}$$

The i upshot represents i^{th} coefficient of k_1 corresponding to the i^{th} natural frequencies. Because only the available ratio of these coefficients was derived from the boundary conditions, the value of C_4 is considered to be 1. By applying these coefficients in “Eq. (26)”, the mode shape Equation will be obtained for each natural frequency.

$$U^i(z) = -\frac{Y_1(K_1^i z_{in})}{J_1(K_1^i z_{in})} J_1(K_1^i z_{in}) + Y_1(K_1^i z_{in}) \tag{31}$$

3 ROTATING DISKS CARRYING TWO CONCENTRIC ANNULAR MASSES

Rotating disk carrying two concentric annular masses is shown in “Fig. 3”. As shown in Fig.3, one of the masses located at the end of disk and the other is on the disk body.

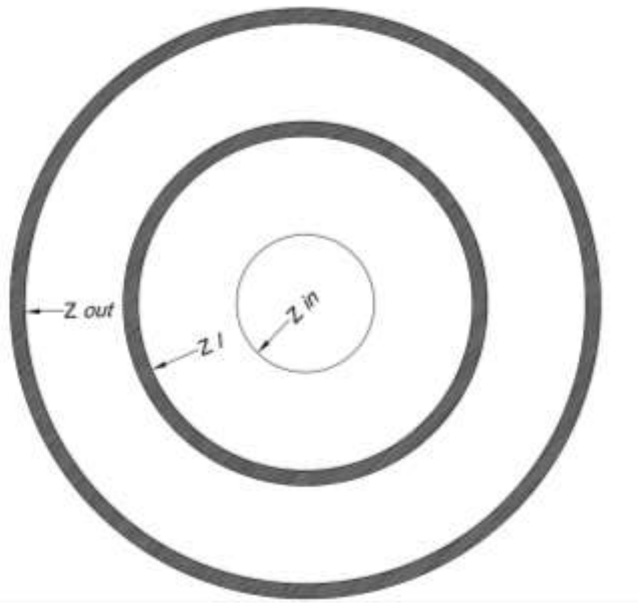

Fig. 3 Rotating disk carrying two concentric peripheral masses.

Using the Equations obtained for annular rotating disk in section 2.1, we assume that disk shown in “Fig. 3” consists of two separate disks connected to each other in z_1 or internal concentrated mass position.
The value of radial stress produced by the concentrated masses can be divided in two parts. First, which is due to centrifugal force that apply from disk on concentrated mass that have effect on u_1 and the second part which is due to the mass inertia and is concentrated on the radial direction of the disk which have effect on u_2. To study radial vibrations, its only effect on u_2 was studied.

$$\sigma_r = \frac{F}{A} = \frac{dm\,\ddot{u}}{rd\theta h} = \frac{\rho'}{r}\ddot{u} \tag{32}$$

r is radial position of concentrated mass, ρ' is the density of concentrated mass defined as $m = \rho' 2\pi h$ and its unit is kg/m, also, $\ddot{u}$ is acceleration in radius direction. The ratio of concentrated mass to disk mass can also be obtained by using “Eq. (33)”:

$$n = \frac{m}{M} = \frac{2\rho'}{(r_o^2 - r_i^2)\rho} \tag{33}$$

As mentioned earlier, a rotating disk with two concentric masses was considered as two disconnected disks connected to each other in a dimensionless radius of z_1 or the same internal concentrated mass position. Therefore, the following boundary conditions can be expressed for a rotating disk.

- The radial displacement in inner radius is zero (because of being constrained)
- External radial displacement of the internal disk is equal to the internal radius of the external disk.
- Radial stress in the internal radius of the external disk is equal to the radial stress in the external radius of the internal disk plus the stress caused by the concentrated mass.
- The radial stress generated in the external radius of the external disk is equal to the stress created by the concentrated mass.

Which the mathematical form of these four conditions were considered as follows:

$$u_{2i}(z_{in},\tau)=0 \Rightarrow \boldsymbol{C_3 J_1(k_1 z)+C_4 Y_1(k_1 z)/_{z=z_{in}}=0} \tag{34}$$

$$u_{2i}(z_l,\tau)=u_{2o}(z_l,\tau) \Rightarrow C_3 J_1(k_1 z)+C_4 Y_1(k_1 z)|_{z=z_l} = \boldsymbol{C_5 J_1(k_1 z)+C_6 Y_1(k_1 z)|_{z=z_l}} \tag{35}$$

$$\sigma_{ro}|_{z=z_l}=\sigma_{ri}|_{z=z_l}+\sigma_{rcm1} \Rightarrow C_5\left[J'_1(k_1 z)+\frac{\nu J_1(k_1 z)}{z}\right]_{z=l} + C_6\left[Y'_1(k_1 z)+\frac{\nu Y_1(k_1 z)}{z}\right]_{z=l} = C_3\left[J'_1(k_1 z)+\frac{\nu J_1(k_1 z)}{z}\right]_{z=l} + C_4\left[Y'_1(k_1 z)+\frac{\nu Y_1(k_1 z)}{z}\right]_{z=l} + \frac{\rho'_i}{z}(-p^2)\frac{(1-\nu^2)}{E}[C_3 J_1(k_1 z) + C_4 Y_1(k_1 z)]\Big|_{z=z_l} \tag{36}$$

$$\sigma_{ro}|_{z=z_{out}}=\sigma_{rcm2} \Rightarrow C_5\left[J'_1(k_1 z)+\frac{\nu J_1(k_1 z)}{z}\right]_{z=1} + C_6\left[Y'_1(k_1 z)+\frac{\nu Y_1(k_1 z)}{z}\right]_{z=1} = = \frac{\rho'_o}{z}(-p^2)\frac{(1-\nu^2)}{E}[C_5 J_1(k_1 z) + C_6 Y_1(k_1 z)]\Big|_{z=1} \tag{37}$$

Where, the $\boldsymbol{i}$ and o indexes and C_1, C_2, C_3 *and* C_4 coefficients are respectively related to the internal and external disks. Also $cm1$ and $cm2$ indexes are respectively related to internal and external concentrated masses. Like “Eq. (27) and Eq. (28)”, we rewrite the above Equations in the following matrix form:

$$s\begin{bmatrix}C_3\\C_4\\C_5\\C_6\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$kk=(-p^2)\frac{(1-\nu^2)}{E}$$

$$ss=\begin{bmatrix}ss_{11}&ss_{12}&ss_{13}&ss_{14}\\ss_{21}&ss_{22}&ss_{23}&ss_{24}\\ss_{31}&ss_{32}&ss_{33}&ss_{34}\\ss_{41}&ss_{42}&ss_{43}&ss_{44}\end{bmatrix}$$

$$ss_{11}=J_1(k_1 z_{in}),\ ss_{12}=Y_1(k_1 z_{in})$$
$$ss_{21}=J_1(k_1 z_l),\ ss_{22}=Y_1(k_1 z_l)$$
$$ss_{23}=-J_1(k_1 z_l),\ ss_{24}=-Y_1(k_1 z_l)$$

$$ss_{31}=\left[\begin{matrix}J'_1(k_1 z)+\frac{\nu J_1(k_1 z)}{z}+\\ \frac{\rho'_i}{z}kk\,J_1(k_1 z)\end{matrix}\right]_{z=l}$$

$$ss_{32}=\left[\begin{matrix}Y'_1(k_1 z)+\frac{\nu Y_1(k_1 z)}{z}+\\ \frac{\rho'_i}{z}kk\,Y_1(k_1 z)\end{matrix}\right]_{z=l} \tag{38}$$

$$ss_{33}=-\left[J'_1(k_1 z)+\frac{\nu J_1(k_1 z)}{z}\right]_{z=l}$$

$$ss_{34}=\left[Y'_1(k_1 z)+\frac{\nu Y_1(k_1 z)}{z}\right]_{z=l}$$

$$ss_{43}=\left[\begin{matrix}J'_1(k_1 z)+\frac{\nu J_1(k_1 z)}{z}-\\ \frac{\rho'_o}{z}kk\,J_1(k_1 z)\end{matrix}\right]_{z=1}$$

$$ss_{44}=\left[\begin{matrix}Y'_1(k_1 z)+\frac{\nu Y_1(k_1 z)}{z}-\\ \frac{\rho'_o}{z}kk\,Y_1(k_1 z)\end{matrix}\right]_{z=1}$$

$$ss_{13}=ss_{14}=ss_{41}=ss_{42}=0$$

The deterministic matrix of coefficients in “Eq. (38)” is desire frequency Equation and the graph results will be like “Fig. 2”.

4 NUMERICAL RESULTS

Consider an annular rotating disk with the following specifications:

$$E=2\times10^{11}(pa)\ ,\rho=7800\left(\frac{kg}{m^3}\right) \tag{39}$$

$\nu = 0.3$, $R_{out} = 0.2(m)$, $R_{in} = 0.05$ (m)

At the first, we assume a disk without concentrated mass. Also, the angular velocity is $\omega = 100\, rad/sec$. The three first mode shapes and corresponding natural frequencies are respectively shown in “Fig. 4” and “Table 1”.

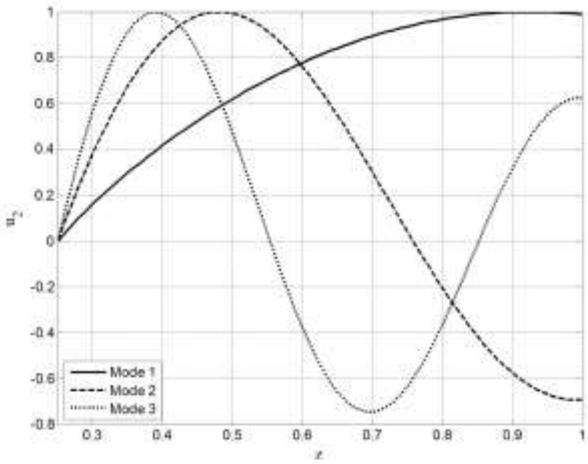

Fig. 4 Three first mode shape of rotating disk.

Table 1 Three first natural frequency $\omega = 100\, rad\ /sec$

Frequency number	1	2	3
p (rad/sec)	608	1709	2808

By repeating above solution for a same disk at a speed of $\omega = 300\, rad/sec$, the results were calculated again and shown in “Table 2”.

Table 2 Three first natural frequency $\omega = 300\, rad/sec$

Frequency number	1	2	3
p (rad/sec)	203	570	936

Comparison between “Tables 1 and 2” shows that, increasing the angular velocity leads to decrease in natural frequencies.
Figure 5 shows the first natural frequency of the rotating disk at different angular velocities. Also, the ratio of concentrated mass to disk mass is considered in different quantities. As shown in this Figure, the first natural frequency decreases by increasing the angular velocity. With the increase in the amount of concentrated mass, the first natural frequency is also decreased.

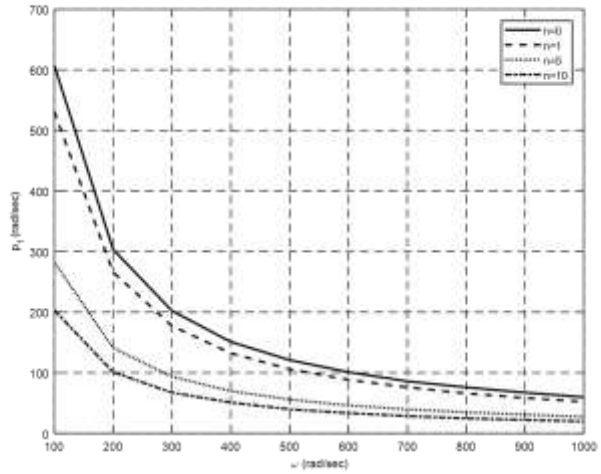

Fig. 5 The first natural frequency of rotating disk carrying concentrated masses in term of angular velocity and for different values of concentrated masses ($\boldsymbol{z_l = 0.55}$).

As shown in “Fig. 5”, “Fig. 6” shows the second frequency of the rotating disk. With an increase in angular velocity, the value of the second natural frequency has been decreased. However, by increasing the value of concentrated mass, the second natural frequency does not show a steady behavior. The second natural frequency increases first and then remains approximately constant.

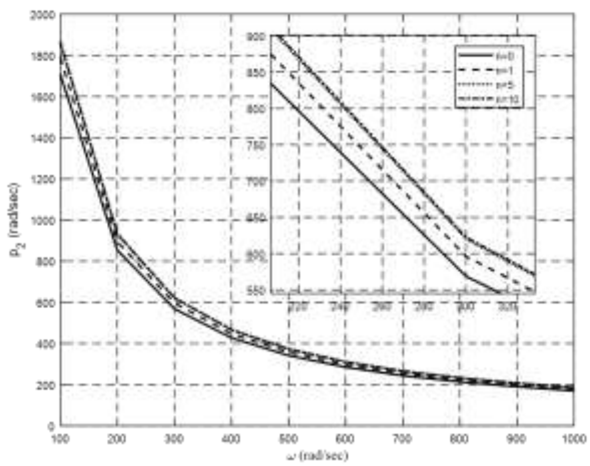

Fig. 6 The second natural frequency of rotating disk carrying concentrated masses in term of angular velocity and for different values of concentrated masses ($\boldsymbol{z_l = 0.55}$).

Fig. 7 shows the third natural frequency of the desired rotating disk. Like the first and second frequencies, the third natural frequency also decreases with increasing the angular velocity. As with the second natural frequency, by increasing the concentrated mass, the third natural frequency does not have a stable behavior. The third natural frequency increases first and then decrease.

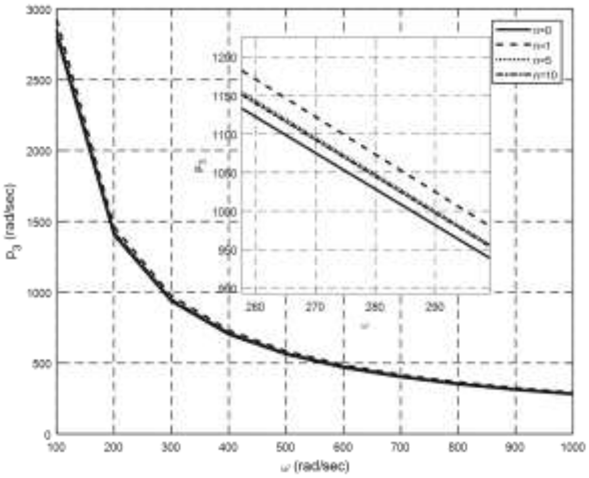

Fig. 7 The third natural frequency of rotating disk carrying concentrated masses in term of angular velocity and for different values of concentrated masses ($z_l = 0.55$).

Fig. 8 shows the first natural frequency of the rotating disk, which it's internal concentrated mass located in different positions.

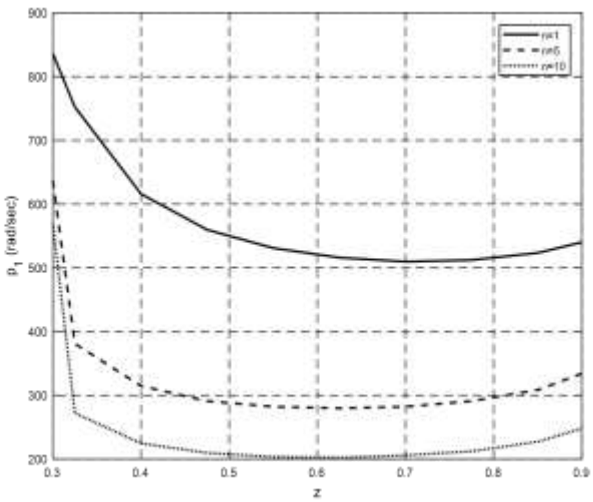

Fig. 8 The first natural frequency of rotating disk carrying concentrated masses in term of internal concentrated mass location and for different values of concentrated masses ($\omega = 100\ rad\ /sec$).

In fact, this graph shows the effect of concentrated mass location on the first natural frequency. As shown in this figure and by comparison with "Fig. 4", when the mass is in the position of first mode shape, which has a higher displacement, the frequency reduction is also higher. It also reduces the natural frequency by increasing the concentrated mass.

Fig. 9 and Fig. 10, like "Fig. 8", show the second and third natural frequencies of the disk, respectively. It is clear that the location of concentrated mass in some places increases the natural frequency and, in some places, decreases the natural frequency. It is also increasing the concentrated mass impact by placing the mass in locations where more radial displacements occur.

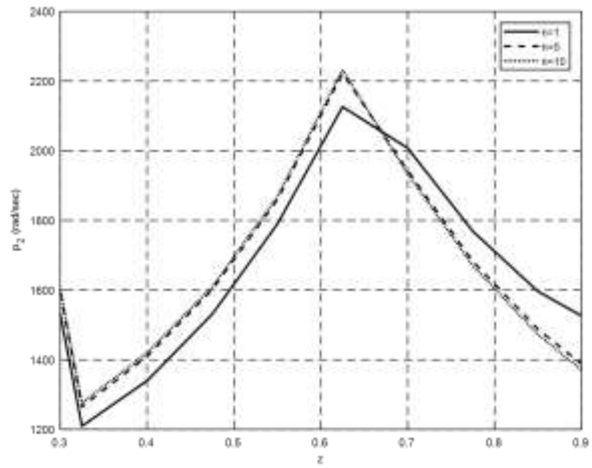

Fig. 9 The second natural frequency of rotating disk carrying concentrated masses in term of internal concentrated mass location and for different values of concentrated masses ($\omega = 100\ rad\ /sec$).

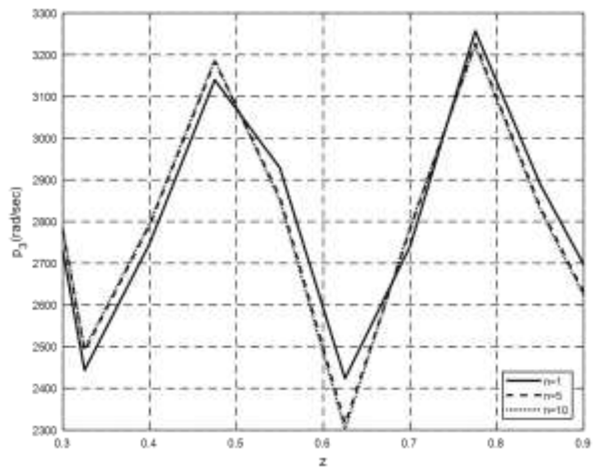

Fig. 10 The third natural frequency of rotating disk carrying concentrated masses in term of internal concentrated mass location and for different values of concentrated masses ($\omega = 100\ rad\ /sec$).

5 CONCLUSIONS

In this research, a free radial vibration analysis for an annular thin homogeneous rotating disk with two concentrating masses in turbo-pump system is presented. At the first, the vibration of the rotating disk was examined without considering the concentrated mass. Then, the resulting solution was generalized for two connected disks in internal concentrated mass location. Also, the effect of concentrated masses, one on the disk body and the other on the outside of the disk, is considered as boundary conditions in the two disk Equations. The results show that increasing in angular velocity of rotating disk reduces the natural frequency. Also concentrated masses always reduce the first natural frequency. In the case of second and third natural frequencies, concentrated masses may increase or decrease the natural frequency, which depends on the value and position of concentrated mass. At the end, concentrated mass has the most impact when it is in a position that has the most radial displacement. The results of this research can be used in the preliminary design of rotor structures in turbomachines.

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