Numerical Study on Normalized Split-step Fourier Method and Fourth-order Runge-Kutta Method in Laser Pulses Generation by Use of Photonic Crystal Fibers

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ABSTRACT:
In this work, we study the generation of ultrashort femtosecond laser pulses in photonic crystal fibers through the widely used symmetric split-step Fourier method (S-SSFM). The results are compared with the method of fourth-order Runge-Kutta method (RK4) as the most recognized and powerful method in the analysis of the evolution of ultrashort femtosecond laser pulses. The results show that although S-SSFM is a widely used method, its accuracy decreases for the selected relatively large step sizes of Δz in comparison with RK4. In contrast, the accuracy of the mentioned methods becomes closer to each other by selecting relatively small step sizes of Δz.

KEYWORDS: Ultrashort Laser Pulse, Optical Pulse, Runge-Kutta, Split-step, Photonic Crystal Fiber.

1. INTRODUCTION

Theoretical and experimental investigations on the propagation process of femtosecond ultrashort laser pulses through photonic crystal fibers (PCFs) have been the target of intensive research groups in recent years [1-5]. As previously stated in Ref [6] and [7], because of the special properties of the above-mentioned fibers, they are special candidates for the generation of highly qualified ultrashort laser pulses which can be extensively used in several aspects of industries such as military, telecommunications, medical health systems and several applications such as optical sensors, optical coherent tomography (OCT), WDM, lasers generation and so on [5],[6-9].

Because of the importance of these kinds of optical pulses, the accurate analysis of the evolution of the propagated pulses along the relevant optical fibers seems to be necessary. The evolution of optical pulses along the waveguides can be presented by simulating the nonlinear Schrödinger equation (NLSE) which is a reduction of nonlinear coupled-mode equations (NLCME) and is derived by solving Maxwell's equations [7-10]. For solving such equations, numerical techniques must be added to the accounts [6-8]. Several methods such as the finite element method (FEM), inverse scattering method (ISM), second and fourth-order Runge-Kutta method (RK2,4), split-step Fourier method (SSFM), and so on [4-8] have been investigated in many kinds of researches for simulating the Schrödinger equations so far. For instance, Chi *et al.* investigated the accuracies of both normalized and general SSFM in a conventional optical fiber in 2002 [10]. Raja *et al.* reported in 2008 the investigation results on soliton propagation in conventional PCFs by using SSFM [11]. Moreover, a comprehensive study was done by Long *et al.* in 2008 on numerical solutions for solving NLSE including RK4 and SSFM [12]. Also, recently, Safaei *et al.* presented a novel fast optimized SSFM for solving NLSE by implementing some changes in the relevant algorithm [13]. Furthermore, in 2021, Bourdine *et al.* published a novel algorithm for solving a system of coupled nonlinear Schrödinger equations [14].

As a complementary example, Jiang *et al.* proposed a novel method for solving NLSE which was faster than SSFM [15].

Among all of the above-mentioned examples, we can clearly see that SSFM is the basis. It means that most of the mentioned examples have been done based on the SSFM initial algorithm or in comparison with it.

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Totally, SSFM is known as a widely used and common technique to integrate several types of nonlinear partial differential equations [3], [4], [12]. Although SSFM is now a widely used technique, however, its accuracy must be analyzed and estimated in comparison with the other recognized and proven methods such as RK4 as the most popular and accurate technique, especially when the ultrashort femtosecond laser pulse generation is under consideration. So to speak, in order to achieve reliable results by using SSFM, it is necessary to determine the convergence range of SSFM to a very accurate method such as RK4. This is exactly the missing point between the published results so far. In the following study, to define the range of the deviations relevant to the different mathematical calculations of the wave equations, we resolve the NLSE by using the two methods of SSFM and RK4, define the convergence region exactly, and also compare the results in the term of the propagation of femtosecond optical pulses along an optimized designed highly nonlinear photonic crystal fiber (HNL-PCF).

2. GENERALIZED NONLINEAR SCHRODINGER EQUATION (GNLSE)

As stated before, the general equations forms that can define all the electromagnetic phenomena are Maxwell's equations whose integral and differential formats can be used from [3-6]. After passing long complicated mathematical procedures [3, 4-8], the generalized nonlinear form of the wave equation which is named GNLSE can be extracted as:

$$\frac{\partial \mathrm{E}(z,t)}{\partial z} = -\frac{\alpha}{2}\,\mathrm{E} - \left[\sum_{m=2}^{N} \frac{\mathrm{i}^{m-1}}{m!}\,\beta_m \frac{\partial^m}{\partial T^m}\right]\mathrm{E} + \mathrm{i}\gamma\left[\,|\mathrm{E}^2|\mathrm{E} + \mathrm{i}\frac{1}{\omega_0}\frac{\partial}{\partial T}\left(|\mathrm{E}^2|\mathrm{E}\right) - \mathrm{T}_R\,\mathrm{E}\frac{\partial|\mathrm{E}^2|}{\partial T}\right] \quad (1)$$

Where, E is the envelope of the pulse, β is the propagation constant, z presents the length of the fiber, α is the loss of the fiber, and T_R is the Raman coefficient. It is noteworthy that, the last parameter, T_R, can be calculated as:

$$\mathrm{T}_R = f_\mathrm{R} \int_0^\infty t h_\mathrm{R}\,(t)dt \quad (2)$$

Where, f_R is the Raman response Function and can be calculated by using the related information in Ref. [6] for different materials.

3. NUMERICAL SOLUTIONS FOR SOLVING GNLSE

3.1. Generalized Split-Step Fourier Method (S-SSFM)

In simulating nonlinear Schrödinger systems, the SSF method is strongly used rather than the other existing methods because is often a more user-friendly and time-saving method [2, 12-16]. Considering one of the easiest NLS-type systems, the equation contains the terms of higher order dispersions (HODs), loss, and nonlinearity (See Eq. (1)). To solve Eq. (1) by using SSFM, we re-wrote the above-mentioned equation in the following applicable format [3], [7], [12],

$$\frac{\partial \mathrm{A}(z,t)}{\partial z} = [\mathrm{L}+\mathrm{N}]\mathrm{A}(z,t) \quad (3)$$

Where, N and L are the nonlinear and linear parts of Eq. (1), respectively, and can be described as follows:

$$\mathrm{L} = -\frac{\alpha}{2}\,\mathrm{E} - \left[\sum_{m=2}^{N} \frac{\mathrm{i}^{m-1}}{m!}\,\beta_m \frac{\partial^m}{\partial T^m}\right]$$

$$\mathrm{N} = \mathrm{i}\gamma\left[\,|\mathrm{E}^2|\mathrm{E} + \mathrm{i}\frac{1}{\omega_0}\frac{\partial}{\partial T}\left(|\mathrm{E}^2|\mathrm{E}\right) - \mathrm{T}_R\,\mathrm{E}\frac{\partial|\mathrm{E}^2|}{\partial T}\right] \quad (4)$$

The following form of the solution has been proposed extensively as the final solution of SSFM [3, 12]. So, by applying Eq. (4) in the following solution format and by using a small space interval of Δz, the evolution of the propagated pulses along the z-direction of the fibers can be presented in the both time and frequency domain [12].

$$\mathrm{E}\,(z+\Delta z, T) = \exp\left[\frac{\Delta z}{2}\,\mathrm{L}\right] \exp\left[\frac{\Delta z}{2}\,\left(N(z) + N(z+\Delta z)\right)\right] \exp\left(\frac{\Delta z}{2}\,\mathrm{L}\right)\mathrm{E}\,(z,T) \quad (5)$$

It is noteworthy that the aforementioned solution is called a symmetrized SSFM and has an accuracy of second-order in comparison with the first-order common solution of SSFM [3-7].

3.2. Forth Order Runge-Kutta Method (RK4)

Eq. (1) can be computed by using the Runge-Kutta algorithm. From the theory [17-21], in this method, the time discretization and computations of time partial derivatives are similar to the SSFM method [12, 21-24], but the spatial derivatives must be calculated by the Runge-Kutta algorithm. In this stage, we can use both second and fourth-order Runge-Kutta methods, however, for achieving the most accurate result, we selected to continue with RK4. Denoting,

$$\mathrm{V} = \exp\left[\left(\frac{i\omega^2}{2} - i\omega^3\,\frac{\beta_3}{6|\beta_2|T_0}\right)z\right]\mathrm{F}[\mathrm{E}] \quad (6)$$

We can re-write Eq. (1) in the following form:

$$\frac{\partial \mathrm{V}}{\partial z} = \mathrm{f}(z,E) \quad (7)$$

Where, $f(z,E)$ can be calculated from [8,9]. Using the fourth-order Runge-Kutta algorithm for Eq. (7), the function of V can be calculated in the distance of z+Δz as:

$$\mathrm{V}(z+\Delta z)=V(z)+0.16666\,[\mathrm{K}_1+2(\mathrm{K}_2+\mathrm{K}_3)+ +\mathrm{K}_4] \quad (8)$$

Where, the variables of K_{i} (i=1-4) can be computed theoretically as follows:

-------Second-order RK (RK2) -------

$$K_1=\Delta z.\,\mathrm{f}\big(z,E(z,T)\big) \quad (9a)$$
$$K_2=\Delta z.\,\mathrm{F}\left(z+\frac{\Delta z}{2},E(z,T)+0.5K_1\right)$$

-------fourth-order RK (RK4) -------

$$K_3=\Delta z.\,\mathrm{F}\left(z+\frac{\Delta z}{2},U(z,T)+0.5K_2\right) \quad (9b)$$

$$K_4=\Delta z.\,\mathrm{F}(z+\Delta z,E(z,T)+K_3)$$

By referring to and applying the vital mentioned variables and the relevant coefficients and also following the stated process, the final solution can be presented in the below form along the z-direction of the fiber:

$$\mathrm{E}\,(z+\Delta z)= \mathrm{F}^{-1}\left[\mathrm{V}(z+\Delta z)\exp\left[\left(\frac{-i\omega^2}{2}+i\omega^3\frac{\beta_3}{6|\beta_2|T_0}\right)(z+\Delta z)\right)\right] \quad (10)$$

It is noteworthy that, Eq. (1) and consequently Eq. (10) can be also changed to their normalized forms, by using the relevant variables as follows [2, 3, 12-18]:

$$\zeta=\frac{z}{L_{\mathrm{D}}},\; L_{NL}=\frac{1}{\gamma P_0},\quad N^2=\frac{L_{\mathrm{D}}}{L_{\mathrm{N}}},\quad S=\frac{1}{\omega_0 T_0}, \quad (11)$$
$$T_{\mathrm{R}}=\frac{\tau_{\mathrm{R}}}{T_0},\; \mathrm{U}\,(z,\tau)=\frac{1}{\sqrt{P_0}}A(z,\tau),\; \tau=\frac{T}{T_0}, L_D=\frac{T_0^2}{\beta_2}$$

Where, $\zeta=\frac{z}{L_{\mathrm{D}}}$ is the normalized distance, z displays the length of the fiber, dispersion length is shown by $L_{\mathrm{D}}.L_{NL}$ depicts the nonlinear length of the fiber, N represents the soliton order which corresponds to the input power. S shows the parameter of the self-steepening, β_2 is the second-order dispersion (GVD), $\left||\mathrm{U}\,(z,\tau)|\right|^2$ is the normalized intensity, T_0 is the initial pulse width, and τ_{R} is the normalized Raman parameter to T_0. The value of τ_{R} is estimated to be 3fs for silica material [2], [3], [8-18]. As stated before, the normalized forms of Eq. (1) and also the normalized solutions forms of Eq.(5) and Eq.(10) can be derived as bellow, respectively [12, 18-24],

---- Normalized GNLSE model----

$$\frac{\partial U}{\partial \zeta}=-\frac{i}{2}sgn(\beta_2)\frac{\partial^2 U}{\partial \tau^2}+\frac{\beta_3}{6|\beta_2|T_0}\frac{\partial^3 U}{\partial \tau^3}+\cdots+iN^2(|U^2|U+. ..+iS\frac{\partial}{\partial \tau}(|U^2|U)-\frac{\tau_{\mathrm{R}}}{T_0}U\frac{\partial |U^2|}{\partial \tau}) \quad (12a)$$

-----Normalized Solution Format for SSFM----

$$\mathrm{U}\;(\zeta+\Delta\zeta,\tau)=\exp[\frac{\Delta\zeta}{2}\;\mathrm{L}]\;\exp\;[\frac{\Delta\zeta}{2}\,(N(\zeta)+N(\zeta+\Delta\zeta))]\exp\,(\frac{\Delta\zeta}{2}\,\mathrm{L})\;\mathrm{U}\,(\zeta,\tau) \quad (12b)$$

----Normalized Solution Format for RK4M----

$$\mathrm{U}\,(\zeta+\Delta\zeta)= \mathrm{F}^{-1}[\mathrm{V}(\zeta+\Delta\zeta)\exp\left[\left(\frac{-i\omega^2}{2}+i\omega^3\frac{\beta_3}{6|\beta_2|T_0}\right)(\zeta+\Delta\zeta)\right)] \quad (12c)$$

4. RESULTS AND DISCUSSIONS

To deeply investigate the evolution of ultrashort femtosecond laser optical pulses in both SSFM and RK4M, first, we considered the parameters of a highly nonlinear PCF as listed in Table 1.

Table 1. Parameters of the PCF [16].

Parameters	Values
$\beta_2(^{ps^2}/_{km})$	-1500
$\beta_3(^{ps^3}/_{km})$	0
$P_0(w)$	396
N(soliton-order)	6.5
T_0(fs)	100
$\lambda(nm)$	2500

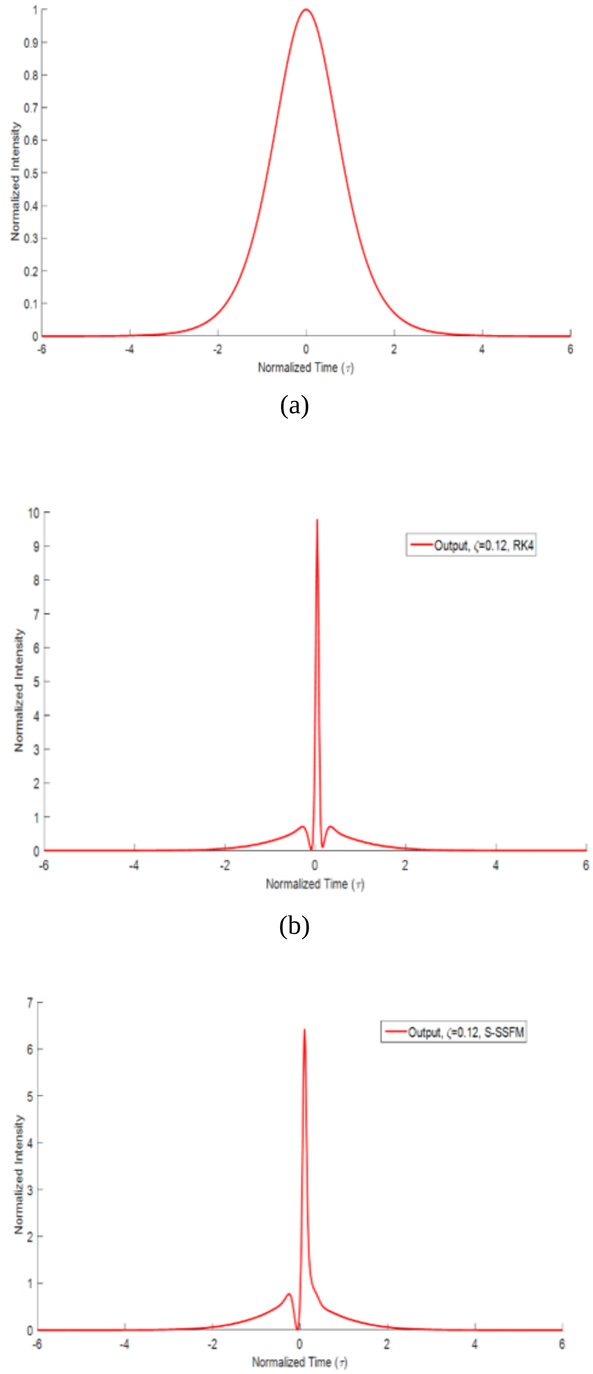
Then, we divided the simulation process based on the following steps,

A. Performing the relevant simulations based on a fixed step size of 100 (Δz =100) and the input optical pulse of 100fs using both methods.
B. Performing the relevant simulations based on a fixed step size of 10000 (Δz =10000) for S-SSFM and (Δz =10000) for RK4M. Both for the input pumped pulses width of 100fs.

First of all, in Fig. 1(a), we can see the schematic of the input pulse of T_0 =100 fs as a function of normalized time ($\tau=\frac{T}{T_0}$). In the following, we can see that by applying a fixed step size of 100 (Δz =100) in the simulation process and by using the relevant parameters of the fiber through Table 1, the superficial deviations of the generated laser pulses in both methods become more critical (Fig. 1b and 1c). As we can observe, by using RK4M, and using the soliton order of 6.5 (corresponds to the peak power of 396 W), a laser pulse of 8 fs can be generated at the normalized distance of $\zeta = \frac{z}{L_D} = 0.12$. While, by using the same parameters in the S- SSFM, a 13fs laser pulse can be generated at the same normalized distance of 0.12.

The mentioned differences arise from the higher effects of nonlinear terms along the simulation process and the applied approximations during the derivation of NLSE from Maxwell equations. We repeated the simulation for two fixed step sizes of 10000 for S-SSFM and 10000 for RK4M, respectively. In both cases, the simulation parameters were considered to be N=6.5 (corresponds to the peak power of 396 W) and an input pumped pulse width of 100 fs. The results are shown in Fig. 2. As we can see, the accuracies of the propagated laser pulses become closer to each other. Actually, two laser pulses of 8 fs and 8.5 fs are generated at the normalized distance of 0.12 using RK4M and SSFM, respectively.

The convergence of the two methods is because of the fact that by the selection of more steps with small sizes during the process of simulations, high oversampling rates of signal occur. Particularly, Symmetric SSFM needs high rates of oversampling and small step sizes (Δz) to be converged to the real results of the GNLSE. On the other hand, by comparing Fig. 2a with Fig. 1c, it is clear that SSFM, itself depicts two different results when two step sizes of 100 and 10000 are considered in the account. These discrepancies show the unreliability of SSFM when using step sizes lower than 10000. Therefore, although the implementation of more steps with small sizes is more time-consuming, however, for using SSFM, it seems to be a necessary action to achieve exact and reliable results. In contrast, we can also recognize that the RK4M is more accurate than S-SSFM and more effective even in the case of the selection of lower steps with bigger sizes. Because, based on the simulation results, its accuracy does not show any sensible changes to the selection of lower steps with bigger sizes.

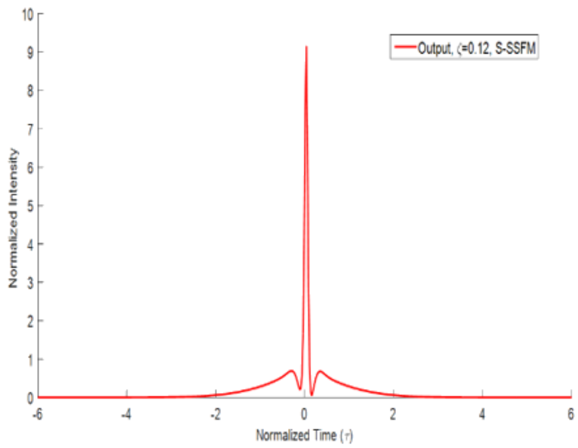


(a)

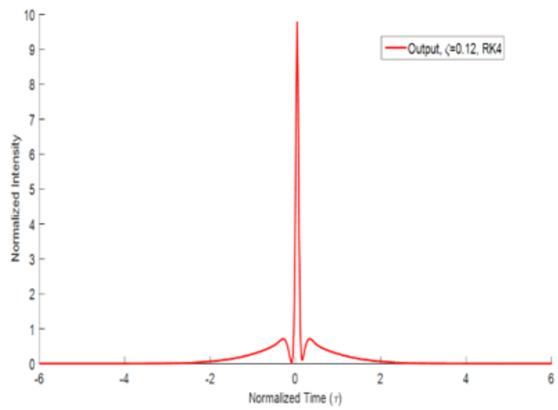
(b)

(c)

Fig. 1. Schematics of (a) 100 fs optical input pulse with the peak power of 396 W, (b) generated laser pulse of 8 fs at the normalized distance of 0.12 using RK4 method and step sizes of 100, and (c) generated laser pulse of 13 fs at the normalized distance of 0.12 using S-SSF method and step sizes of 100.



(a)



(b)

Fig. 2. Schematics of (a) a generated 8.5 fs optical laser pulse using S-SSFM and step-sizes of 10000 and (b) a generated 8 fs optical laser pulse using RK4 and step-sizes of 10000; both at the normalized distance of 0.12.

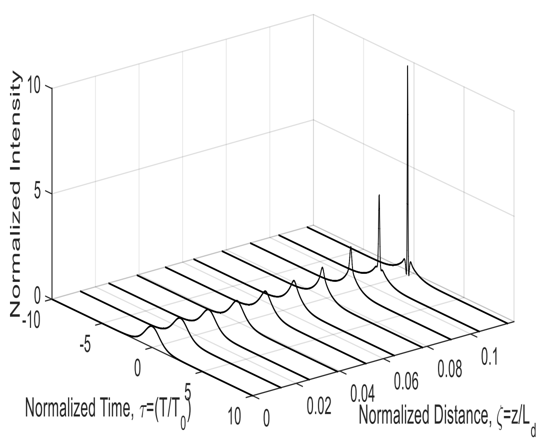


Fig. 3. Evolution of the propagated optical pulses along the fiber based on RK4M.

The form of the response time characteristic obtained due to modeling, according to the proposed algorithm of RK4 has been shown in Fig. 3. This figure shows the evolution of the propagated pulses along the fiber in a 3-D schematic. The relevant information of the mentioned figure has been summarized in the form of Table 2.

Table 2. Optical characteristics of Fig. 3.

Parameters	Values
Input power (W)	396
Input Normalized Intensity	1
Soliton Order	6.5
Input Pulse Width (fs)	100
Output Compressed Pulse (fs)	8
Output Peak Power (kW)	≈168
Output normalized Intensity	10
Output normalized Distance	0.12
Output Distance (mm)	0.8
Number of Plots	10
Normalized Time Duration (τ)	-6< ($\tau = \frac{T}{T_0}$) <6

The above-obtained results can be compared with the results obtained from SSFM. For this reason, NLSE with selected step sizes of 10000 was resolved by using SSFM and new software with the commercial name of NLSE.S which solves the GNLSE (Fig. 4). As expected, the results were in very good agreement with the mentioned results in Table 2.

From another point of view, we listed a comparison between the wasted times in applying both methods in the simulations in the form of Tab.3. We can see that by the selection of step size of 1000 for SSFM and 100 for RK4M, the needed time for completing the simulations in RK4M is less than the S-SSFM. The computational intricacy of the S-SSFM mostly arises from a large number of times of conversions between time and frequency domains using the FFT (fast Fourier transform) and its inverse (F^{-1}), and also the exponential calculations in the nonlinear operator.

Table 3. Needed time for completing the simulations.

Methods	Selected Step Sizes (Δz)	Needed Time (s)
S-SSFM	10000	37.5
S-SSFM	100	7.2
RK4M	100	5.5
RK4M	10000	32.3
SSFM-NLSE software	10000	38.6

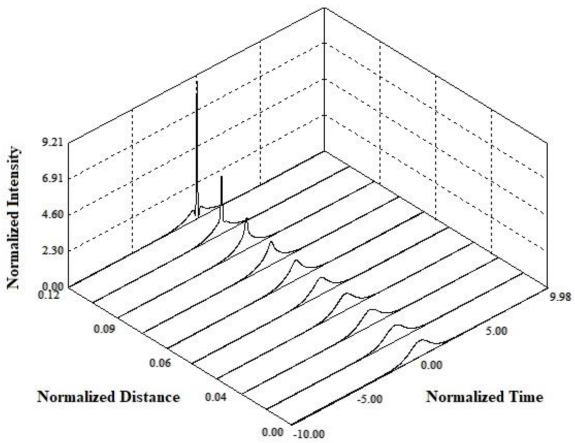


Fig. 4. 8.5 fs laser pulse generation process using NLSE simulator software along the 0.12 length of the fiber and using 10000 step sizes in the simulation algorithm.

5. CONCLUSIONS

In this study, a comprehensive investigation of the methods of normalized symmetric split-step Fourier method and fourth-order Runge-Kutta method (RK4M) in the generation of ultrashort femtosecond optical pulses has been done. We showed that applying smaller step sizes was needed in the case of using the normalized split-step Fourier method to achieve accurate results. Applying smaller step sizes in the process of the computations increases the time-consuming factor. So, the results of using S-SSFM with selected larger step sizes cannot be reliable especially when the femtosecond optical pulse propagation was under consideration. On the other side, RK4M shows reliable results in both cases of selected larger and smaller step sizes in comparison with S-SSFM. The results showed that the needed time for completing RK4M in spite of involving smaller step sizes is less than using S-SSFM in a similar condition. Totally, since the SSFM is now a widely used method for solving the NLSE, the relevant steps must be chosen more than $(\Delta z \gg 10000)$ to achieve more accurate and referable results.

REFERENCES

[1] P. V. Mamyshev, S. V. Chernikov, "**Ultrashort-pulse propagation in optical fibers**", *Journal of optics letter*, Vol.15, No.19,1990.

[2] S. Garzo, E. Mendoza, "**Simulation for fiber optics pulse propagation through a numerical solution of the nonlinear schrödinger equation**", *Conference on physics, ETSETB*, Universität Polytechnical de Catalunya, 2020.

[3] G. P. Agrawal, **"Nonlinear fiber optics"**, *4th Edition, Academic Press*, 2007.

[4] P. C. T. Munaweera, K.A.I.L.W. Gamalath, "**Simulation of pulse propagation in optical fibers**", *International letters of chemistry, physics and astronomy*, ISSN: 2299-3843, Vol. 64, pp 159-170, 2016.

[5] F. Sun, "**Propagation of solitons in optical fibers with generalized Kudryashov's refractive index**", *Journal of results in physics*, Vol. 28, No. 2021, pp.1-8, 2021.

[6] A.F. Garcia, "**Nonlinear pulse propagation in optical fibers**", *Master of science thesis, University of NTNU*, pp.1-142, 2012.

[7] MH. Mozaffari, A. Farmani, "**On-Chip single-mode optofluidic microresonator dye laser sensor**", *IEEE sensors Journal*, Vol. 20, No. 8, pp. 3556-3563, 2020.

[8] M. Heydari, A.R. Zali, R.E. Gildeh, A. Farmani, "**Fully integrated, 80 GHz bandwidth, 1.3μ m InAs/InGaAs CW-PW quantum dot passively colliding-pulse mode-locked (CPM) lasers for IR sensing application**",*IEEE sensors journal*, Vol. 22, No. 7, pp.6528-6535, 2022.

[9] A. Farmani, M. Farhang, M.H. Sheikhi, "**High performance polarization-independent quantum dot semiconductor optical amplifier with 22 dB fiber to fiber gain using mode propagation Tuning without additional polarization controller**", *Journal of optics and laser technology*, Vol. 93, No. 2017, pp.127-132,2017.

[10] C.F. Chen, S. Chi, B. Luo, "**Femtosecond soliton propagation in an optical fiber**", *Journal of Optik*, Vol. 116, No. 6, pp. 267-271, 2002.

[11] R.V.J. Raja, K. Porseian, "**Numerical Investigation of Soliton Pulse Propagation in Photonic Crystal Fiber**", *Asia Optical Fiber Communication and Optoelectronic Exposition and Conference, ISBN*: 978-1-55752-863-6, 2008.

[12] V.C. Long, H.N. Viet, M. Tripenbach, K.D. Xuan, "**Propagation technique for ultrashort pulses III: pulse splitting of ultrashort pulses in a Kerr medium**", *Journal of computational methods in science and technology*, Vol. 14, No.1, 2008.

[13] A. Safaei, M.A. Bolorizadeh, "**Solving nonlinear Schrödinger equation by a new and fast method**", *Journal of IOP- Physica Scripta*, Vol. 96, No.12, 2021.

[14] A.V. Bourdine, V.A. Bourdine, O.G. Morozov, "**Algorithm for solving a system of coupled nonlinear Schrödinger equations by the split-step method to describe the evolution of a high-power femtosecond optical pulse in an optical polarization maintaining Fiber**", *Journal of Fibers-MDPI*, Vol.10, No.3, 2021.

[15] X. Jiang, D. Wang, Q. Fan, M. Zhang, C. Lu, A.P.T. Lau, "**Solving the nonlinear Schrödinger equation in optical fibers using physics-informed neural network**", *Optical fiber communication conference*, ISBN: 978-1-943580-86-6, 2021.

[16] A. Ghanbari, A. Sadr, A. Kashaninia, "**Supercontinuum generation with femtosecond optical pulse compression in silicon photonic crystal fibers at 2500 nm**", *Journal of optical and quantum electronics*, Vol. 40, No. 411, pp. 1-12, 2018.

[17] J. Hault, "**A Fourth-Order Runge–Kutta in the Interaction Picture method for simulating**

supercontinuum generation in optical fibers", *Journal of lightwave technology*, Vol.25, No.12, pp.3770-3775, 2007.

[18] A. Ghanbari, "**Modelling highly nonlinear photonic crystal fibers for efficient propagation of ultrashort optical pulses**", *PhD Thesis, Islamic Azad University*, Tehran central branch, 2018.

[19] S. Wen, Y. Wang, W. Su, Y. Xiang, X. Fu and D. Fan, "**Modulation instability in nonlinear negative-index material**", *Journal of Physics Review*, Vol. 73, No. 3. pp. 036617-03621, 2006.

[20] M. Shackeerali, A.K. Shafeeque, A. Uthayakumar, "**Impact of higher-order effects on modulation instability in negative index materials**", *Optik*, Vol. 256, Article Number 168660, 2022.

[21] H. Saghei, "**Chalcogenide highly nonlinear photonic crystal fibers for OCT applications**", *PhD Thesis, Islamic Azad University, Science and Research Branch*, pp.1-135, 2015.

[22] V. A. Trofimov, S. Stepanenko, A. Razgulin, "**Conservation laws of femtosecond pulse propagation described by generalized nonlinear Schrödinger equation with cubic nonlinearity**", *Journal of mathematics and computers in simulation*, Vol. 182, No. 2021, pp. 366-396, 2021.

[23] M. Conforti, C.M. Arabi, A. Mussot, A. Kudlinski, "**Fast and accurate modeling of nonlinear pulse propagation in graded-index multimode fibers**", *Journal of optical society of America*, Vol. 42, No. 19, pp. 4004-4007, 2017.

[24] H. Pakarzadeh, Z. Delirian, "**Propagation of short pulses in dispersion-engineered silicon nanowires: impact of chirp parameter**", *Journal of springer*, Vol. 14, No. 54, pp. 1221-1225, 2021.