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An Efficient Method for Solving the Fuzzy AH1N1/09 Influenza Model Using the Fuzzy Atangana-Baleanu-Caputo Fractional Derivative

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A B S T R A C T

One of the most devastating viruses that has significantly impacted human life is the AH1N1/09 influenza virus. Its examination is crucial since the virus is unstable and new varieties with distinct properties are produced every year. To describe these disorders, numerous mathematical models have been presented. In order to investigate this virus, mathematical modeling using fractional differential equations with the Atangana-Baleanu-Caputo derivative and initial values is suggested in this research. The fuzzy model of the virus is examined due to the confusing and imprecise nature of the virus and the way it affects the human body. The proposed model is solved numerically using tools such as r-cut, generalized Hakuhara difference, ABC fractional derivative, and ABC-PI numerical method. Finally, the applicability of the method is shown via a numerical example.

1. Introduction

Influenza disease is an acute respiratory infection caused by influenza viruses that have different types, the most dangerous of which is the influenza A virus. This virus causes a local epidemic of influenza every winter, despite previous infections with the common influenza virus in the region. Wider genetic changes may occur in the virus, and as a result, a new virus may emerge that can cause more severe pathogenicity and even a global pandemic [24]. The H1N1/A influenza virus was first reported in 2009 in Mexico and several states of the United States, with a different pathogenicity and form than the seasonal types. It then spread rapidly to most parts of the world, and according to the World Health Organization (WHO), two months after the first report, it became an epidemic in the whole world. This virus emerged as a result of the simultaneous infection of pigs with common subtypes of influenza type A and the simultaneous multiplication and displacement of their genomes, which led to more severe pathogenicity than other seasonal subtypes [6, 21]. According to the WHO

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report, in a short time after the outbreak of the disease, many people lost their lives around the world [29]. H1N1 influenza is a type of swine flu that is considered a dangerous disease due to the transformation of this virus and its attack on the human body, and it causes a more severe disease than other types. This virus is so unstable that new types with different characteristics emerge every year. Consequently, it is very important to study and investigate this model to provide immunity against new types. So far, extensive research has been conducted in this field [1,3,4,7,8, 12,17, 20, 25, 29].

In [5], a new interesting mathematical model was proposed according to age and spatial structure in order to estimate the potential impact of the H1N1 virus in Vietnam.

Thereafter, the issue of modeling epidemic waves throughout the world gained attention. In this regard, the classic Susceptible-Exposed-Infectious-Removed (SEIR) model was proposed for the dynamic transmission of the AH1N1/09 virus. Moreover, mathematical models based on systems of ordinary differential equations with two independent and dependent communities were proposed to reproduce two-wave profiles [11].

Lohm et al. [16] stated that the influenza virus basically involves uncertainty. Therefore, they investigated how the general public can understand the 'ambiguous' concept of the influenza virus and adapt themselves to infection control actions.

Recently, the SEIR epidemic model for propagation of the AH1N1 influenza under the Caputo-Fabrizio fractional-order derivative has been discussed in [23], and by applying the fixed-point theorem, the existence and uniqueness of the solution were investigated, and then the numerical solution of the mathematical model was achieved. In [22], the synthesis and inhibition of the H1N1 influenza virus by a propargylaminoalkyl derivative of lithocholic acid were studied. Due to the fact that the two sides of the disease, i.e., human and virus, are ambiguous in nature, fuzzy modeling of the diseases has recently received attention [13,14,18,28].

Verma et al. analyzed a model of influenza spread with an asymptotic transmission rate, wherein the disease transmission and death rates were considered fuzzy sets [28]. In [19], a fuzzy SEIR epidemic model was proposed for human amoebiasis infection. Allahvirenloo and Ghanbari proposed the ABC fractional derivative on fuzzy set-valued functions in a parametric interval [2].

In this paper, we use the ABC fractional derivative to investigate the AH1N1/09 influenza model in fuzzy form. The main contributions of this research are: 1) all the parameters are considered fuzzy; and 2) the ABC fractional derivative is utilized to discuss and examine the influenza virus model.

The structure of the paper is as follows: In Section 2, some necessary basic theories are presented. In Section 3, the AH1N1/09 influenza model with fuzzy variables is studied based on the fractional ABC-fuzzy derivative. A numerical example is provided in Section 4, and the conclusion is given in Section 5.

2. Preliminaries

In this section, the basic definitions required for the next sections are presented.

Definition 1. A fuzzy number u in parametric form is a pair $u[r] = [\underline{u}(r), \overline{u}(r)]$ of functions $\underline{u}(r)$ and $\overline{u}(r)$, $0 \leq r \leq 1$, that satisfy the following requirements:

- $\underline{u}(r)$ is a bounded non-decreasing left continuous function in $(0,1]$, and right continuous at 0,
- $\overline{u}(r)$ is a bounded non-increasing left continuous function in $(0,1]$, and right continuous at 1,
- $\underline{u}(r) \leq \overline{u}(r),\ 0 \leq r \leq 1$.

Definition 2. For arbitrary fuzzy numbers $u[r] = [\underline{u}(r), \overline{u}(r)]$ and $v[r] = [\underline{v}(r), \overline{v}(r)]$, addition and scalar multiplication are defined as follows for $0 \leq r \leq 1$:

- $(u \oplus v)[r] = [\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r)]$,
- $(\lambda u)[r] = \begin{cases} [\lambda \underline{u}(r), \lambda \overline{u}(r)], & \lambda \geq 0 \\ [\lambda \overline{u}(r), \lambda \underline{u}(r)] & \lambda < 0 \end{cases}$
- $(u \odot v)[r] = [\underline{u}(r)\underline{v}(r), \overline{u}(r)\overline{v}(r)] (\tilde{u}, \tilde{v} \geq 0)$.

Suppose a fuzzy-valued function $y(t) \epsilon C^F(I) \cap L^F(I)$, then its parametric interval form can be written as:

$$[y(t)]_r = \left[\underline{y}(t;r), \overline{y}(t;r)\right], 0 \leq r \leq 1.$$

Definition 3. [26] The generalized Hukhara difference of two fuzzy numbers is defined as follows:

$$u \ominus_{gh} v = w \Leftrightarrow \begin{cases} (i)\ u = v \oplus w \\ (ii)\ v = u \oplus (-1)w \end{cases}.$$

The first case is equivalent to the definition of the Hukuhara difference, denoted by $u \ominus v$.

Definition 4. [26] Let $x_0 \epsilon (a,b)$ and h be such that $x_0 + h \,\epsilon\, (a,b)$, then the gH-derivative of a function $f:(a,b) \to I$ can be defined as:

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) \ominus_{gh} f(x_0)}{h}$$

If $f'(x_0)\epsilon I$ fulfills the above equation, we say that f is generalized Hukuhara differentiable (gH-differentiable, for short).

Definition 5. [10] The ABC fractional derivative in the sense of Caputo is defined in two cases as follows:

$$[{}^{ABC}_{0}D_t^{i,\alpha} y(t)]_r = \left[{}^{ABC}_{0}D_t^{\alpha} \underline{y}(t;r), {}^{ABC}_{0}D_t^{\alpha} \overline{y}(t;r)\right] \quad (1)$$

$$[{}^{ABC}_{0}D_t^{ii,\alpha} y(t)]_r = \left[{}^{ABC}_{0}D_t^{\alpha} \overline{y}(t;r), {}^{ABC}_{0}D_t^{\alpha} \underline{y}(t;r)\right] \quad (2)$$

where

$$\left[{}^{ABC}_{0}D_t^{i,\alpha} y(t)\right]_r = \frac{B(\alpha)}{1-\alpha} \int_0^t \left[\underline{y}(t;r), \overline{y}(t;r)\right] E_\alpha\left(-\frac{\alpha}{1-\alpha}(t-\tau)^\alpha\right) d\tau \quad (3)$$

$$\left[{}^{ABC}_{0}D_t^{ii,\alpha} y(t)\right]_r = \frac{B(\alpha)}{1-\alpha} \int_0^t \left[\overline{y}(t;r), \underline{y}(t;r),\right] E_\alpha\left(-\frac{\alpha}{1-\alpha}(t-\tau)^\alpha\right) d\tau \quad (4)$$

On other hand

$$\begin{gathered} {}^{AB}_{0}I_t^{\alpha}\left({}^{AB}_{0}D_t^{\alpha} \underline{y}(t,r)\right) = \underline{y}(t,r) - \underline{y}(0,r) \\ {}^{AB}_{0}I_t^{\alpha}\left({}^{AB}_{0}D_t^{\alpha} \overline{y}(t,r)\right) = \overline{y}(t,r) - \overline{y}(0,r) \end{gathered} \quad (5)$$

in which

$${}^{AB}_{0}I_t^{\alpha}(y(t)) = \frac{1-\alpha}{B(\alpha)} y(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t y(\tau)\,(t-\tau)^{\alpha-1}\, d\tau \quad (6)$$

where $B(\alpha) > 0$ is a normalization function satisfying $B(0) = B(1) = 1$.

3. Mathematical Modeling of AH1N1/09 Influenza Virus Transmission under a Fuzzy Environment Using Fuzzy ABC Fractional Derivative

In this section, we introduce and solve the AH1N1/09 influenza virus transmission modeling problem in a fuzzy environment using the fuzzy ABC fractional derivative.

Definition 6. A fuzzy SEIR model of AH1N1/09 influenza virus transmission using the ABC derivative is defined as below:

$$\begin{cases} {}^{ABC}_{0}D_t^{*_1,\alpha_1} \tilde{S}(t) = \tilde{\kappa} \ominus_{gh} \tilde{\beta} \odot \tilde{S}(t) \odot \tilde{I}(t) \ominus_{gh} \tilde{m} \odot \tilde{S}(t), \\ {}^{ABC}_{0}D_t^{*_2,\alpha_2} \tilde{E}(t) = \tilde{\beta} \odot \tilde{S}(t) \odot \tilde{I}(t) \ominus_{gh} (\tilde{m} \oplus \tilde{\delta}) \odot \tilde{E}(t), \\ {}^{ABC}_{0}D_t^{*_3,\alpha_3} \tilde{I}(t) = \tilde{\delta} \odot \tilde{E}(t) \ominus_{gh} (\tilde{m} \oplus \tilde{\mu}) \odot \tilde{I}(t), \\ {}^{ABC}_{0}D_t^{*_4,\alpha_4} \tilde{R}(t) = \tilde{\mu} \odot \tilde{I}(t) \ominus_{gh} \tilde{m} \odot \tilde{R}(t), \\ \tilde{S}(0) = \tilde{S}_0, \\ \tilde{E}(0) = \tilde{E}_0, \\ \tilde{I}(0) = \tilde{I}_0, \\ \tilde{R}(0) = \tilde{R}_0, \end{cases} \quad (7)$$

where $*_1, *_2, *_3, *_4 \epsilon \{i, ii\}$, $0 < \alpha_1, \alpha_2, \alpha_3, \alpha_4 < 1$, $0 < t < T < \infty$, $T \epsilon R$, and fuzzy variables are as follows:

$\tilde{S}(t)$	Susceptible individuals
$\tilde{E}(t)$	Exposed individuals
$\tilde{I}(t)$	Infected individuals
$\tilde{R}(t)$	Recovered individuals
$\tilde{\kappa}$	Birth rate of people
$\tilde{m}$	Death rate of people
$\tilde{\beta}$	Transmission rate of infection from I to S
$\tilde{\delta}$	Transmission rate of people from E to I
$\tilde{\mu}$	Recovery rate of infected people

Solving method. To calculate the fuzzy solution for (7), we assume:

$$f\left(\tilde{S}(t), \tilde{E}(t), \tilde{I}(t), \tilde{R}(t)\right) = \tilde{\kappa} \ominus_{gh} \tilde{\beta} \odot \tilde{S}(t) \odot \tilde{I}(t) \ominus_{gh} \tilde{m} \odot \tilde{S}(t)$$

$$g\left(\tilde{S}(t), \tilde{E}(t), \tilde{I}(t), \tilde{R}(t)\right) = \tilde{\beta} \odot \tilde{S}(t) \odot \tilde{I}(t) \ominus_{gh} (\tilde{m} \oplus \tilde{\delta}) \odot \tilde{E}(t)$$

$$h\left(\tilde{S}(t), \tilde{E}(t), \tilde{I}(t), \tilde{R}(t)\right) = \tilde{\delta} \odot \tilde{E}(t) \ominus_{gh} (\tilde{m} \oplus \tilde{\mu}) \odot \tilde{I}(t)$$

$$K\left(\tilde{S}(t), \tilde{E}(t), \tilde{I}(t), \tilde{R}(t)\right) = \tilde{\mu} \odot \tilde{I}(t) \ominus_{gh} \tilde{m} \odot \tilde{R}(t).$$

The model (7) is transformed to a fuzzy ABC fractional differential equations system with initial values as follows:

$$\begin{cases} {}^{ABC}_{0}D_t^{*_1,\alpha_1} \tilde{S}(t) = f\left(\tilde{S}(t), \tilde{E}(t), \tilde{I}(t), \tilde{R}(t)\right) \\ {}^{ABC}_{0}D_t^{*_2,\alpha_2} \tilde{E}(t) = g\left(\tilde{S}(t), \tilde{E}(t), \tilde{I}(t), \tilde{R}(t)\right) \\ {}^{ABC}_{0}D_t^{*_3,\alpha_3} \tilde{I}(t) = h\left(\tilde{S}(t), \tilde{E}(t), \tilde{I}(t), \tilde{R}(t)\right) \\ {}^{ABC}_{0}D_t^{*_4,\alpha_4} \tilde{R}(t) = K\left(\tilde{S}(t), \tilde{E}(t), \tilde{I}(t), \tilde{R}(t)\right). \\ \tilde{S}(0) = \tilde{S}_0 \\ \tilde{E}(0) = \tilde{E}_0 \\ \tilde{I}(0) = \tilde{I}_0 \\ \tilde{R}(0) = \tilde{R}_0 \end{cases} \tag{8}$$

Consider the parametric form of the system of nonlinear fuzzy ABC fractional differential equations (8) as below:

$$\begin{cases} \left[{}^{ABC}_{\ 0}D_t^{*_1,\alpha_1}\tilde{S}(t)\right]_r = \left[f\left(\tilde{S}(t),\tilde{E}(t),\tilde{I}(t),\tilde{R}(t)\right)\right]_r \\ \left[{}^{ABC}_{\ 0}D_t^{*_2,\alpha_2}\tilde{E}(t)\right]_r = \left[g\left(\tilde{S}(t),\tilde{E}(t),\tilde{I}(t),\tilde{R}(t)\right)\right]_r \\ \left[{}^{ABC}_{\ 0}D_t^{*_3,\alpha_3}\tilde{I}(t)\right]_r = \left[h\left(\tilde{S}(t),\tilde{E}(t),\tilde{I}(t),\tilde{R}(t)\right)\right]_r \\ \left[{}^{ABC}_{\ 0}D_t^{*_4,\alpha_4}\tilde{R}(t)\right]_r = \left[k\left(\tilde{S}(t),\tilde{E}(t),\tilde{I}(t),\tilde{R}(t)\right)\right]_r . \\ \left[\tilde{S}(0)\right]_r = \left[\tilde{S}_0\right]_r \\ [\tilde{E}(0)]_r = [\tilde{E}_0]_r \\ [\tilde{I}(0)]_r = [\tilde{I}_0]_r \\ [\tilde{R}(0)]_r = [\tilde{R}_0]_r \end{cases} \tag{9}$$

Assume that:

$$\left[f\left(\tilde{S}(t),\tilde{E}(t),\tilde{I}(t),\tilde{R}(t)\right)\right]_r = [\underline{f}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right),$$
$$\overline{f}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)]$$
$$\left[g\left(\tilde{S}(t),\tilde{E}(t),\tilde{I}(t),\tilde{R}(t)\right)\right]_r = [\underline{g}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right),$$
$$\overline{g}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)]$$
$$\left[h\left(\tilde{S}(t),\tilde{E}(t),\tilde{I}(t),\tilde{R}(t)\right)\right]_r = [\underline{h}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right),$$
$$\overline{h}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)]$$
$$\left[k\left(\tilde{S}(t),\tilde{E}(t),\tilde{I}(t),\tilde{R}(t)\right)\right]_r = [\underline{k}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right),$$
$$\overline{k}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) \tag{10}$$

When solving system (8), different cases can occur as follows:

Case	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$*_1$	*i*	*ii*	*i*	*i*	*i*	*ii*	*i*	*i*	*ii*	*i*	*ii*	*i*	*ii*	*ii*	*ii*	*ii*
$*_2$	*i*	*i*	*ii*	*i*	*i*	*ii*	*ii*	*i*	*i*	*ii*	*i*	*ii*	*i*	*ii*	*ii*	*ii*
$*_3$	*i*	*i*	*i*	*ii*	*i*	*i*	*ii*	*ii*	*i*	*i*	*ii*	*ii*	*ii*	*i*	*ii*	*ii*
$*_4$	*i*	*i*	*i*	*i*	*ii*	*i*	*i*	*ii*	*ii*	*ii*	*i*	*ii*	*ii*	*ii*	*i*	*ii*

We explain the method for the first case. As it is similar in other cases, they will not be discussed to avoid repetition.

Case 1) $*_1 = *_2 = *_3 = *_3 =$ i

Using the fuzzy ABC derivative and placing (1), (2), and (10) in (9), we obtain:

$$\left[{}^{ABC}_{\ 0}D_t^{\alpha}\underline{S}(t,r),\ {}^{ABC}_{\ 0}D_t^{\alpha}\overline{S}(t,r)\right] = [\underline{f}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right),$$
$$\overline{f}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)]$$
$$\left[{}^{ABC}_{\ 0}D_t^{\alpha}\underline{E}(t,r),\ {}^{ABC}_{\ 0}D_t^{\alpha}\overline{E}(t,r)\right] = [\underline{g}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right),$$
$$\overline{g}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)]$$

$$\left[{}^{ABC}_{0}D_t^{\alpha}\underline{I}(t,r), {}^{ABC}_{0}D_t^{\alpha}\overline{I}(t,r)\right] = [\underline{h}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right), \overline{h}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)]$$

$$\left[{}^{ABC}_{0}D_t^{\alpha}\underline{R}(t,r), {}^{ABC}_{0}D_t^{\alpha}\overline{R}(t,r)\right] = [\underline{k}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right), \overline{k}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)] \quad \textbf{(11)}$$

Therefore, applying the fractional integral on both sides of Equations (5) and (6) results in:

$$\underline{S}(t,r)-\underline{S}(0,r) = \frac{1-\alpha}{B(\alpha)}\underline{f}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t \underline{f}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)(t-\tau)^{\alpha-1}\,d\tau$$

$$\overline{S}(t,r)-\overline{S}(0,r) = \frac{1-\alpha}{B(\alpha)}\overline{f}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t \overline{f}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)(t-\tau)^{\alpha-1}\,d\tau$$

$$\underline{E}(t,r)-\underline{E}(0,r) = \frac{1-\alpha}{B(\alpha)}\underline{g}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t \underline{g}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)(t-\tau)^{\alpha-1}\,d\tau$$

$$\overline{E}(t,r)-\overline{E}(0,r) = \frac{1-\alpha}{B(\alpha)}\overline{g}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t \overline{g}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)(t-\tau)^{\alpha-1}\,d\tau$$

$$\underline{I}(t,r)-\underline{I}(0,r) = \frac{1-\alpha}{B(\alpha)}\underline{h}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t \underline{h}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)(t-\tau)^{\alpha-1}\,d\tau$$

$$\overline{I}(t,r)-\overline{I}(0,r) = \frac{1-\alpha}{B(\alpha)}\overline{h}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t \overline{h}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)(t-\tau)^{\alpha-1}\,d\tau$$

$$\underline{R}(t,r)-\underline{R}(0,r) = \frac{1-\alpha}{B(\alpha)}\underline{k}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t \underline{k}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)(t-\tau)^{\alpha-1}\,d\tau$$

$$\overline{R}(t,r)-\overline{R}(0,r) = \frac{1-\alpha}{B(\alpha)}\overline{k}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t \overline{k}\left(\underline{S}(t,r),\overline{S}(t,r),\underline{E}(t,r),\overline{E}(t,r),\underline{I}(t,r),\overline{I}(t,r),\underline{R}(t,r),\overline{R}(t,r)\right)(t-\tau)^{\alpha-1}\,d\tau \quad (12)$$

By placing $t = t_n = 0 + nh$ in the above equations and using the ABC-PI formula [25-26], we have:

$$\underline{S}(t_n,r)-\underline{S}(0,r) = \frac{\rho\, h^{\rho}}{K(\rho)}\left(\varepsilon_n \underline{f}\left(\underline{S}(0,r),\overline{S}(0,r),\underline{E}(0,r),\overline{E}(0,r),\underline{I}(0,r),\overline{I}(0,r),\underline{R}(0,r),\overline{R}(0,r)\right) + \sum_{i=1}^{n}\mu_{n-i}\,\underline{f}\left(\underline{S}(t_i,r),\overline{S}(t_i,r),\underline{E}(t_i,r),\overline{E}(t_i,r),\underline{I}(t_i,r),\overline{I}(t_i,r),\underline{R}(t_i,r),\overline{R}(t_i,r)\right)\right)$$

$$\overline{S}(t_n,r)-\overline{S}(0,r) = \frac{\rho\, h^{\rho}}{K(\rho)}\left(\varepsilon_n \overline{f}\left(\underline{S}(0,r),\overline{S}(0,r),\underline{E}(0,r),\overline{E}(0,r),\underline{I}(0,r),\overline{I}(0,r),\underline{R}(0,r),\overline{R}(0,r)\right) + \sum_{i=1}^{n}\mu_{n-i}\,\overline{f}\left(\underline{S}(t_i,r),\overline{S}(t_i,r),\underline{E}(t_i,r),\overline{E}(t_i,r),\underline{I}(t_i,r),\overline{I}(t_i,r),\underline{R}(t_i,r),\overline{R}(t_i,r)\right)\right)$$

$$\underline{E}(t_n,r)-\underline{E}(0,r) = \frac{\sigma\, h^{\rho}}{K(\sigma)}\left(\varepsilon'_n \underline{g}\left(\underline{S}(0,r),\overline{S}(0,r),\underline{E}(0,r),\overline{E}(0,r),\underline{I}(0,r),\overline{I}(0,r),\underline{R}(0,r),\overline{R}(0,r)\right) + \sum_{i=1}^{n}\vartheta_{n-i}\,\underline{g}\left(\underline{S}(t_i,r),\overline{S}(t_i,r),\underline{E}(t_i,r),\overline{E}(t_i,r),\underline{I}(t_i,r),\overline{I}(t_i,r),\underline{R}(t_i,r),\overline{R}(t_i,r)\right)\right)$$

$$\overline{E}(t_n,r)-\overline{E}(0,r) = \frac{\sigma\, h^{\rho}}{K(\sigma)}\left(\varepsilon'_n \overline{g}\left(\underline{S}(0,r),\overline{S}(0,r),\underline{E}(0,r),\overline{E}(0,r),\underline{I}(0,r),\overline{I}(0,r),\underline{R}(0,r),\overline{R}(0,r)\right) + \sum_{i=1}^{n}\vartheta_{n-i}\,\overline{g}\left(\underline{S}(t_i,r),\overline{S}(t_i,r),\underline{E}(t_i,r),\overline{E}(t_i,r),\underline{I}(t_i,r),\overline{I}(t_i,r),\underline{R}(t_i,r),\overline{R}(t_i,r)\right)\right),$$

$$\underline{I}(t_n,r)-\underline{I}(0,r) = \frac{\theta\, h^{\rho}}{K(\theta)}\left(\varepsilon''_n \underline{h}\left(\underline{S}(0,r),\overline{S}(0,r),\underline{E}(0,r),\overline{E}(0,r),\underline{I}(0,r),\overline{I}(0,r),\underline{R}(0,r),\overline{R}(0,r)\right) + \sum_{i=1}^{n}\gamma_{n-i}\,\underline{h}\left(\underline{S}(t_i,r),\overline{S}(t_i,r),\underline{E}(t_i,r),\overline{E}(t_i,r),\underline{I}(t_i,r),\overline{I}(t_i,r),\underline{R}(t_i,r),\overline{R}(t_i,r)\right)\right),$$

$$\overline{\mathrm{I}}(\mathrm{t_n},\mathrm{r})-\overline{\mathrm{I}}(0,\mathrm{r}) = \frac{\theta\, h^{\rho}}{K(\theta)}\Bigg(\varepsilon''_n\overline{h}\left(\underline{S}(0,r),\overline{S}(0,r),\underline{E}(0,r),\overline{E}(0,r),\underline{I}(0,r),\overline{I}(0,r),\underline{R}(0,r),\overline{R}(0,r)\right) + \sum_{i=1}^{n}\gamma_{n-i}\,\overline{h}\left(\underline{S}(t_i,r),\overline{S}(t_i,r),\underline{E}(t_i,r),\overline{E}(t_i,r),\underline{I}(t_i,r),\overline{I}(t_i,r),\underline{R}(t_i,r),\overline{R}(t_i,r)\right)\Bigg)$$

$$\underline{\mathrm{R}}(\mathrm{t_n},\mathrm{r})-\underline{\mathrm{R}}(0,\mathrm{r}) = \frac{\theta' h^{\rho}}{K(\theta')}\Bigg(\varepsilon'''_n\underline{k}\left(\underline{S}(0,r),\overline{S}(0,r),\underline{E}(0,r),\overline{E}(0,r),\underline{I}(0,r),\overline{I}(0,r),\underline{R}(0,r),\overline{R}(0,r)\right) + \sum_{i=1}^{n}\gamma'_{n-i}\,\overline{k}\left(\underline{S}(t_i,r),\overline{S}(t_i,r),\underline{E}(t_i,r),\overline{E}(t_i,r),\underline{I}(t_i,r),\overline{I}(t_i,r),\underline{R}(t_i,r),\overline{R}(t_i,r)\right)\Bigg)$$

$$\overline{\mathrm{R}}(\mathrm{t_n},\mathrm{r})-\overline{\mathrm{R}}(0,\mathrm{r}) = \frac{\theta' h^{\rho}}{K(\theta')}\Bigg(\varepsilon'''_n\overline{k}\left(\underline{S}(0,r),\overline{S}(0,r),\underline{E}(0,r),\overline{E}(0,r),\underline{I}(0,r),\overline{I}(0,r),\underline{R}(0,r),\overline{R}(0,r)\right) + \sum_{i=1}^{n}\gamma'_{n-i}\,\overline{k}\left(\underline{S}(t_i,r),\overline{S}(t_i,r),\underline{E}(t_i,r),\overline{E}(t_i,r),\underline{I}(t_i,r),\overline{I}(t_i,r),\underline{R}(t_i,r),\overline{R}(t_i,r)\right)\Bigg)$$

Finally, using fuzzy arithmetic operations in this case, system (8) has solutions as follows:

$$\tilde{S}(t_n)=\tilde{S}(0)\oplus\frac{\rho\, h^{\rho}}{K(\rho)}\left(\varepsilon_n f\left(\tilde{S}(0),\tilde{E}(0),\tilde{I}(0),\tilde{R}(0)\right)\oplus\sum_{i=1}^{n}\mu_{n-i}\, f\left(\tilde{S}(t_i),\tilde{E}(t_i),\tilde{I}(t_i),\tilde{R}(t_i)\right)\right)$$

$$\tilde{E}(t_n)=\tilde{E}(0)\oplus\frac{\sigma\, h^{\rho}}{K(\sigma)}\left(\varepsilon'_n g\left(\tilde{S}(0),\tilde{E}(0),\tilde{I}(0),\tilde{R}(0)\right)\oplus\sum_{i=1}^{n}\vartheta_{n-i}\, g\big(t_i,x(t_i),y(t_i),z(t_i)\big)\right)$$

$$\tilde{I}(t_n)=\tilde{I}(0)\oplus\frac{\theta\ h^{\rho}}{K(\theta)}\left(\varepsilon''_n h\left(\tilde{S}(0),\tilde{E}(0),\tilde{I}(0),\tilde{R}(0)\right)\oplus\sum_{i=1}^{n}\gamma_{n-i}\, h\big(t_i,x(t_i),y(t_i),z(t_i)\big)\right)$$

$$\tilde{R}(t_n)=\tilde{R}(0)\oplus\frac{\theta'\ h^{\rho}}{K(\theta')}\left(\varepsilon'''_n k\left(\tilde{S}(0),\tilde{E}(0),\tilde{I}(0),\tilde{R}(0)\right)\oplus\sum_{i=1}^{n}\gamma'_{n-i}\, k\big(t_i,x(t_i),y(t_i),z(t_i)\big)\right)\qquad(13)$$

where

$$K(x)=1-x+\frac{x}{\Gamma(x)},\ \ x=\rho\ or\ \sigma or\ \theta or \theta'$$

$$\varepsilon_n=\frac{(n-1)^{\rho+1}-n^{\rho}(n-\rho-1)}{\Gamma(\rho+2)},\qquad \varepsilon'_n=\frac{(n-1)^{\sigma+1}-n^{\sigma}(n-\sigma-1)}{\Gamma(\sigma+2)}$$

$$\varepsilon''_n=\frac{(n-1)^{\theta+1}-n^{\theta}(n-\theta-1)}{\Gamma(\theta+2)},\qquad \varepsilon'''_n=\frac{(n-1)^{\theta'+1}-n^{\theta'}(n-\theta'-1)}{\Gamma(\theta'+2)}$$

$$\mu_j=\begin{cases}\frac{1}{\Gamma(\rho+2)}+\frac{1-\rho}{\rho h^{\rho}} & j=0\\ \frac{(j-1)^{\rho-1}-2j^{\rho+1}+(j+1)^{\rho+1}}{\Gamma(\rho+2)} & j=1,2,\dots,n-1\end{cases}$$

$$\vartheta_j = \begin{cases} \dfrac{1}{\Gamma(\sigma+2)} + \dfrac{1-\sigma}{\sigma h^{\sigma}} & j = 0 \\ \dfrac{(j-1)^{\rho-1} - 2j^{\rho+1} + (j+1)^{\rho+1}}{\Gamma(\rho+2)} & j = 1,2,\dots,n-1 \end{cases}$$

$$\gamma_j = \begin{cases} \frac{1}{\Gamma(\theta+2)} + \frac{1-\theta}{\theta h^{\theta}} & j = 0 \\ \frac{(j-1)^{\theta-1} - 2j^{\theta+1} + (j+1)^{\theta+1}}{\Gamma(\theta+2)} & j = 1,2,\dots,n-1 \end{cases}$$

$$\gamma'_j = \begin{cases} \dfrac{1}{\Gamma(\theta'+2)} + \dfrac{1-\theta'}{\theta' h^{\theta'}} & j = 0 \\ \dfrac{(j-1)^{\theta'-1} - 2j^{\theta'+1} + (j+1)^{\theta'+1}}{\Gamma(\theta'+2)} & j = 1,2,\dots,n-1 \end{cases}$$

Remark 1. In this paper, a method is proposed to solve the system of fractional differential integral equations in the form of (7). The proposed method can also be applied to the general form of the system of fractional differential integral equations as follows:

$$\begin{cases} {}^{ABC}_{0}D_t^{*_1,\alpha_1}\tilde{u}_1(t) = f_{11}(t)\tilde{u}_1(t) + f_{12}(t)\tilde{u}_2(t) + \cdots + f_{1n}(t)\tilde{u}_{\mathrm{n}}(t) \\ {}^{ABC}_{0}D_t^{*_2,\alpha_2}\tilde{u}_2(t) = f_{21}(t)\tilde{u}_1(t) + f_{22}(t)\tilde{u}_2(t) + \cdots + f_{2n}(t)\tilde{u}_{\mathrm{n}}(t) \\ \quad\vdots \\ {}^{ABC}_{0}D_t^{*_n,\alpha_n}\tilde{u}_{\mathrm{n}}(t) = f_{n1}(t)\tilde{u}_1(t) + f_{n2}(t)\tilde{u}_2(t) + \cdots + f_{nn}(t)\tilde{u}_{\mathrm{n}}(t) \end{cases}$$

Where

$*_1, *_2, \dots, *_{\mathrm{n}} \in \{i, ii\}$, $0 < \alpha_1, \alpha_2, \dots, \alpha_n < 1$, $0 < t < T < \infty$, $T \in R$, $\tilde{u}_1(t), \tilde{u}_2(t), \dots, \tilde{u}_{\mathrm{n}}(t) \in C^F(I) \cap L^F(I)$, $I = [0, T] \subseteq R$,
and $f_{kj}(t)$ are real-valued functions for $1 \leq k, j \leq n$.

Remark 2. In solving the model, it is assumed that the coefficients are positive and that a generalized Hakuhara difference exists.

4. Numerical Example

In this section, an example is presented to verify the effectiveness of the proposed method.

Example 1. Consider the following AH1N1/09 influenza transmission fuzzy mathematical model:

$$\begin{cases} {}^{ABC}_{0}D_t^{\mathrm{ii},\alpha_1}\tilde{S}(t) = \left[\frac{0.015}{52}, \frac{0.015}{52}\right] \ominus [3 + 0.58r, 4 - 0.42r] \odot \tilde{S}(t) \odot \tilde{I}(t) \ominus \left[\frac{0.015}{52}, \frac{0.015}{52}\right] \odot \tilde{S}(t) \\ {}^{ABC}_{0}D_t^{\mathrm{i},\alpha_2}\tilde{E}(t) = [3 + 0.58r, 4 - 0.42r] \odot \tilde{S}(t) \odot \tilde{I}(t) \ominus \left(\left[\frac{0.015}{52}, \frac{0.015}{52}\right] \oplus [0.2r, 0.3 - 0.1r]\right) \odot \tilde{E}(\\ {}^{ABC}_{0}D_t^{\mathrm{ii},\alpha_3}\tilde{I}(t) = [0.2r, 0.3 - 0.1r] \odot \tilde{E}(t) \ominus_{gh} \left(\left[\frac{0.015}{52}, \frac{0.015}{52}\right] \oplus [0.1 + 0.043r, 0.3 - 0.157r]\right) \odot \tilde{I} \\ {}^{ABC}_{0}D_t^{\mathrm{i},\alpha_4}\tilde{R}(t) = [0.1 + 0.043r, 0.3 - 0.157r] \odot \tilde{I}(t) \ominus_{gh} \left[\frac{0.015}{52}, \frac{0.015}{52}\right] \odot \tilde{R}(t) \\ \tilde{S}(0) = [0.8 + 0.199\mathrm{r}, 0.999] \\ \tilde{E}(0) = 0 \\ \tilde{I}(0) = [0.001\mathrm{r}, 0.002 - 0.001\mathrm{r}] \\ \tilde{R}(0) = 0 \end{cases} \quad (15)$$

For the above parametric form, the following system is considered:

$$\begin{cases} {}_{0}^{ABC}D_t^{\mathrm{ii},\alpha_1}\overline{\mathrm{S}}(t,r) = \dfrac{0.015}{52} - (3+0.58r)(\underline{\mathrm{S}}(t,r).\underline{\mathrm{I}}(t,r)) - \dfrac{0.015}{52}\underline{\mathrm{S}}(t,r) \\ {}_{0}^{ABC}D_t^{\mathrm{ii},\alpha_1}\underline{\mathrm{S}}(t,r) = \dfrac{0.015}{52} - (4-0.42r)(\overline{\mathrm{S}}(t,r).\overline{\mathrm{I}}(t,r)) - \dfrac{0.015}{52}\overline{\mathrm{S}}(t,r) \\ {}_{0}^{ABC}D_t^{\mathrm{i},\alpha_2}\underline{\mathrm{E}}(t,r) = (3+0.58r)\left(\underline{\mathrm{S}}(t,r).\underline{\mathrm{I}}(t,r)\right) - \left(\dfrac{0.015}{52} + 0.2r\right)\underline{\mathrm{E}}(t,r) \\ {}_{0}^{ABC}D_t^{\mathrm{i},\alpha_2}\overline{\mathrm{E}}(t,r) = (4-0.42r)\left(\overline{\mathrm{S}}(t,r).\overline{\mathrm{I}}(t,r)\right) - \left(\dfrac{0.015}{52} + 0.3 - 0.1r\right)\overline{\mathrm{E}}(t,r) \\ {}_{0}^{ABC}D_t^{\mathrm{ii},\alpha_3}\overline{\mathrm{I}}(t,r) = 0.2r\left(\underline{\mathrm{E}}(t,r)\right) - \left(\dfrac{0.015}{52} + 0.1 + 0.043r\right)\underline{\mathrm{I}}(t,r) \\ {}_{0}^{ABC}D_t^{\mathrm{ii},\alpha_3}\underline{\mathrm{I}}(t,r) = (0.3-0.1r)\left(\overline{\mathrm{E}}(t,r)\right) - \left(\dfrac{0.015}{52} + 0.3 + 0.157r\right)\overline{\mathrm{I}}(t,r) \\ {}_{0}^{ABC}D_t^{\mathrm{i},\alpha_4}\underline{\mathrm{R}}(t,r) = (0.1+0.043r)\underline{\mathrm{I}}(t,r) - \dfrac{0.015}{52}\underline{\mathrm{R}}(t,r) \\ {}_{0}^{ABC}D_t^{\mathrm{i},\alpha_4}\overline{\mathrm{R}}(t,r) = (0.3-0.157r)\overline{\mathrm{I}}(t,r) - \dfrac{0.015}{52}\overline{\mathrm{R}}(t,r) \\ \underline{\mathrm{S}}(0,r) = 0.8 + 0.199\mathrm{r}, \qquad \overline{\mathrm{S}}(0,r) = 0.999 \\ \underline{\mathrm{E}}(0,r) = 0, \qquad \overline{\mathrm{E}}(0,r) = 0 \\ \underline{\mathrm{I}}(0,r) = 0.001\mathrm{r}, \qquad \overline{\mathrm{I}}(0,r) = 0.002 - 0.001\mathrm{r} \\ \underline{\mathrm{R}}(0,r) = 0, \qquad \overline{\mathrm{R}}(0,r) = 0 \end{cases} \tag{16}$$

As in the previous section, by using the ABC-PI method, the solution to the above system and then the fuzzy solution to (15) can be easily obtained.

5. Conclusions

Since the two sides of the influenza disease transmission model, i.e., human and virus, are ambiguous in nature, it is necessary to develop these models in a fuzzy state. Considering the importance of solving these models and the fact that the more accurate the model, the closer the solution is to the real-world problem, in this paper we introduced the mathematical modeling of AH1N1/09 influenza virus transmission under a fuzzy environment using the fuzzy ABC fractional derivative. By defining new symbols, this model was considered a system of fuzzy fractional differential equations and was solved using the r-cut and ABC-PI methods. One of the advantages of the proposed method is its practicality, but it is not easy to solve the systems of the form (12). Therefore, MATLAB software was utilized to solve it based on the ABC-PI method. In the future, we will examine this model using the fuzzy Caputo-Fabrizio derivative, try to simplify the complexities of the equations, and then compare the results with those of the current research.

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