Fuzzy Multi-Objective Linear Programming for Project Management Decision under Uncertain Environment with AHP Based Weighted Average Method

Md. Sanowar Hossain[a,*], Shahed Mahmud[a], Md. Mosharraf Hossain[b]

[a] *Assistant Professor, Department of Industrial and Production Engineering, Rajshahi University of Engineering and Technology, Bangladesh*

[b] *Professor, Department of Industrial and Production Engineering, Rajshahi University of Engineering and Technology, Bangladesh*

Received 29 November, 2015; Revised 20 February, 2016; Accepted 26 May, 2016

Abstract

Smooth implementation and controlling conflicting goals of a project through using all related resources by an organization is inherently a complex task to management. Meanwhile, deterministic models are never efficient in practical project management (PM) problems because the related parameters are frequently fuzzy in nature. The project execution time is a major concern of the involved stakeholders (client, contractors and consultants). For optimization of total project cost through time control, here crashing cost is considered as a critical factor. The proposed approach aims to formulate a multi objective linear programming model to simultaneously minimize total project cost, completion time and crashing cost with reference to direct cost, indirect cost in the framework of the satisfaction level of decision maker with fuzzy goals and fuzzy cost coefficients. To make such problems realistic, triangular fuzzy numbers and the concept of minimum accepted level method are employed to formulate the problem. The proposed model leads decision makers to choose the desired compromise solution under different risk levels and the project optimization problems have been solved under multiple uncertainty conditions. The Analytical Hierarchy Process is used to rank multiple objectives to make the problem realistic for the respective case. Here minimum operator and AHP based weighted average operator method is used to solve the model and the solutions are obtained by using LINGO software.

Key Words: Project management, Multi-objective linear programming, Minimum operator, Analytical Hierarchy Process.

1. Introduction

Project management is an activity to ensure smooth implementation of any activity as per its specification. A project is a combination of interrelated activities which must be executed in a certain order which is known as precedence relationship before the entire task is completed. It is the process of planning; scheduling and controlling projects. Therefore, it is truly important for project managers to confirm the project completion that includes quality, effectiveness, the specified completion time and the allocated total budgeted cost. The most commonly used project management techniques are Gantt chart, Work Breakdown Structure, Milestone, Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT). Considering the importance of time-cost optimization, various analytical and heuristic methods have been proposed by many researchers in recent decades including mathematical programming, algorithms and heuristics etc to solve PM decision problems. When any of these traditional techniques are used, however, related parameters are normally assumed to be deterministic / crisp (Lin & Gen 2007; Yin &Wang 2008, Al-Fanzine & Haouari, 2005) which is rather ineffective for changing or uncertain environment. Because in real projects, time and cost of activities may face significant changes due to existing uncertainties such as inflation, economical and social stresses, labor performance, execution errors, design errors, natural events like climate changes and etc.

On the conventional techniques of PM, some modifications have been done by incorporating the concept of fuzzy logic. Han Chung and Liang (2006) used fuzzy critical path method to improve fuzzy airport's ground operation decision analysis assuming fuzzy activity times as trapezoidal fuzzy number. Ling and Han (2004) developed a PM model in fuzzy CPM. Some researchers put emphasis on stochastic policy for project

*Corresponding author E-mail: sanowar@ruet.ac.bd

management decision. Rabbani et al. (2007) developed a resource-constrained PM technique for stochastic networks resource allocation decisions having imprecise duration of each activity with a known distribution function in which the values of activities finish times were determined at decision points. But the critical drawbacks of applying stochastic programming to PM decisions are lack of computational efficiency and inflexible probabilistic consideration.

At the same time the conventional techniques of PM only concern about the cost and time trade-off but ignores the project crashing policy. Generally, the real PM decisions focus on the minimization of project completion time, and/or the minimization of total project costs through crashing or shortening duration of particular activities. The purpose of evaluating time-cost trade-offs is to develop a plan which the decision-maker (DM) can minimize the increase of project total cost and total crashing cost when shortening their total completion time. Liu (2003) formulated the critical path and the project crashing problems by linear programming with fuzzy activity times and then defuzzify the fuzzy activity times following Yager's (1981) ranking method. Again Jiuping Xu and Cuiying Feng (2014) presented multimode resource-constrained project scheduling problem under fuzzy random environment by expectation method, where uncertainty is taken as a normal distribution.

But any solution may be good if the decision maker is satisfied with the result. Various optimization techniques may give optimum or near optimum results of a given PM problem. But this may fail to be achieved in various situations. To deal with this, compromising solution Goal programming is so much effective. The goal programming technique is an analytical framework that a decision maker can use to provide optimal solutions to multiple and conflicting objectives. Mubiru (2010) and Fabiane*et al* (2003) proposed a goal programming model for allocating time and cost in project management and manage the three projects with preemptive goals. Tien-Fu Liang (2010) and Ming-Feng Yang and Yi Lin, (2013) focus on developing a two-phase fuzzy mathematical programming approach for solving the multi-objective project management decision problems in a fuzzy environment. The model was designed to minimize objectives simultaneously by project managers in the framework of fuzzy aspiration levels enabling a decision maker to interactively modify the imprecise data and related parameters until a satisfactory solution is obtained. But here the weight of individual objectives is considered without any analysis.Suo et al (2012) used weighted average operator method for multi-criteria decision making under uncertainty. Again The Analytic Hierarchy Process (AHP) is a mathematically based, multi-objective decision making tool. It uses the pair wise comparison method to rank order alternatives of a problem that are formulated and solved in hierarchical structure (Coyle, 2004; Saaty, 2008). Kamal M. et al (2001) and Hung-Ju Chien (2013) used AHP as a potential decision making method for use in project management.

To solve time-cost trade-off problem, many researchers (e.g. Ghazanfari, 2008 and Liang, 2009) developed an approach by possibility goal programming with fuzzy decision variables. The model was designed to minimize simultaneously total projects costs, total completion time and crashing costs with reference to direct costs, indirect costs, contractual penalty costs, duration of activities, and the constraint of available budget. But possibility linear programming approach for an optimization problem with fuzzy parameters is possibilistic, which leads to the increase of the number of objectives function and constraints of the model. In the above goal programming and possibility goal programming method, the different membership functions are formulated from decision-maker preferences and experiences, but the decision-makers may face difficulties in making tradeoffs between the alternatives because of their inexperience and incomplete information. So there is a need for some analytical ways to define different membership functions. Various types of heuristics have also been developed on the basis of the requirements of PM decision making incorporating with conventional techniques. Leu, Chen and Yang (2001) incorporated fuzzy set theory with genetic algorithms to model uncertainty in time-cost trade-off problem. Abbasnia et al (2008) investigated fuzzy logic based approach called Non-dominated Sorting Genetic Algorithm (NSGA) for time-cost trade-off problem in uncertain environment. This model cannot fully meet uncertainty of practical problems.

Here, fuzzy set theory and Zimmermann's (1976) fuzzy programming technique have been developed into several fuzzy optimization methods to solve imprecise PM decision problems and avoiding unrealistic modeling in an uncertain environment. The minimum operator presented by Bellman and Zadeh (1970) is used to aggregate fuzzy sets, and the original MOLP problem is then converted into an equivalent ordinary LP form.Finally, in this paper, the imprecise nature of the input data is considered by implementing the interactive minimum operators and AHP based weighted average method. The result obtained from minimum operators is used to determine the suitable membership function and seek an efficient solution by AHP based weighted average method.

The organization of this paper is as follows. In Section 2, the problem is introduced, and the notation and assumptions are defined. Section 3 presents computational experiments. Section 4 describes the analysis of the results with various weight of the objectives as well as sensitivity analyses of different parameters to introduce the significant aspects of the model. Finally, in Section 5 the concluding remarks are given and future research directions are provided.

2. The Methodology and Model

2.1 Problem description, assumptions and notations

Assume that a project has 'n' interrelated activities that must be executed in a certain order before the entire task can be completed under uncertain environment. Accordingly, the incremental crashing costs for all activities, variable indirect cost per unit time and total budget are fuzzy. The fuzzy MOLP model designed in this study aims to find out the minimum value of total project costs, total completion time and total crashing costs. The following notation is used after reviewing the literature and considering practical situations [Tien-Fu Liang, 2010, Ming-Feng Yang and Yi Lin, 2013]. The proposed fuzzy mathematical programming model is based on the following assumptions:

(1) All of the objective functions are fuzzy with imprecise aspiration levels.
(2) All of the objective functions and constraints are linear equations.
(3) Direct costs increase linearly as the duration of activity is reduced from its normal time to its crash value.
(4) The normal time and shortest possible time for each activity and the cost of completing the activity in the normal time and crash time are certain.
(5) The available total budget is known over the planning horizons.
(6) The linear membership functions are adopted to specify fuzzy goals, and the minimum operator and AHP based average operator are sequentially used to aggregate fuzzy sets.
(7) The total indirect costs can be divided into fixed costs and variable costs, and the variable costs per unit time are the same regardless of project completion time.

Notations

(I,j)	activity between events i and j,
g	index for objective function, for all g = 1,2,. . . ,K,
Z_1	total project costs
Z_2	total completion time
Z_3	total crashing costs
D_{ij}	normal time for activity (i,j)
d_{ij}	minimum crashed time for activity (i,j)
C_{Dij}	normal (direct) cost for activity (i,j)
C_{dij}	minimum crashed (direct) cost for activity (i,j)
k_{ij}	incremental crashing costs for activity (i,j)
t_{ij}	crashed duration time for activity (i,j)
Y_{ij}	crash time for activity (i,j)
Ei	earliest time for event i
E_1	project start time
E_n	project completion time
T_{nc}	project completion time under normal conditions,
T	specified project completion time,
C_l	fixed indirect costs under normal conditions,
m	variable indirect costs per unit time,
B	Available total budget.

2.2 Fuzzy multi-objective linear programming model

In reality the project management activity is multi directional and multi objective type. Most of the project managers must consider minimizing total project costs, completion duration, crashing costs and contractual penalties, and/or maximizing profits and the maximum utilization of equipments. Among these here three fuzzy objective functions are simultaneously considered during the formulation of the multi-objective PM decision model, as follows.

❖ Minimize total project costs:

$$MinZ_1 = \sum_i \sum_j C_{D_{ij}} + \sum_i \sum_j \tilde{K}_{ij} Y_{ij} + [C_l + \tilde{m}(E_n - T_{nc})] \quad (1)$$

Here the terms, $\sum_i \sum_j C_{D_{ij}} + \sum_i \sum_j \tilde{K}_{ij} Y_{ij}$ denote total direct costs including total normal cost and total crashing cost, obtained using additional direct resources such as overtime, personnel and equipment and the terms $[C_l + \tilde{m}(E_n - T_{nc})]$ denote indirect cost including those of administration, depreciation, financial and other variable overhead cost that can be avoided by reducing total project time.. The symbol '~' represents fuzziness. Here K_{ij} is used to analysis the cost-time slopes for the various activities.

❖ Minimize total completion time:

$$Min\, Z_2 = [E_n - E_1] \quad (2)$$

❖ Minimize total crashing costs

$$MinZ_3 = \sum_i \sum_j \tilde{K}_{ij} Y_{ij} \quad (3)$$

Constraints:

❖ Constraints on the time between events i and j

$$E_i + T_{ij} - E_j \le 0 \quad (4)$$

$$T_{ij} = D_{ij} - Y_{ij} \quad (5)$$

❖ Constraints on the crashing time for activity (i, j)

$$Y_{ij} \le D_{ij} - d_{ij} \quad (6)$$

❖ Constraint on the total budget

$$Z_1 \le \tilde{B} \quad (7)$$

❖ Non-negativity constraints on decision variables

$$t_{ij}, Y_{ij} and E_i \ge 0$$

2.3. Treatment of the fuzzy variable

This work assumes that the decision maker (DM) has already adopted the pattern of triangular possibility distribution to represent the crashing cost, variable indirect costs per unit time and available total budget in the fuzzy linear programming problem. In the process of defuzzification, this work applies Liou and Wang's (1992) fuzzy ranking approach to convert the fuzzy number into a crisp number. That shown in fig: 1. If the minimum acceptable membership level α, then corresponding crisp number of triangular the fuzzy number $\widetilde{K_{ij}} = [\widetilde{K}_{ij}^{p}, \widetilde{K}_{ij}^{m}, \widetilde{K}_{ij}^{o}]$ is:

$$\widetilde{K}_{ij}^{\alpha} = \frac{1}{2}\left[\alpha \widetilde{K}_{ij}^{p} + \widetilde{K}_{ij}^{m} + (1-\alpha)\widetilde{K}_{ij}^{o}\right] \quad (8)$$

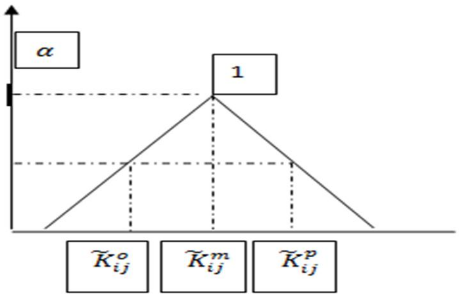

Fig. 1. Membership function of $\widetilde{K_{ij}}$

The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations. For instance, Figure 1 shows the distribution of the triangular fuzzy number k_{ij}.

In practical situations, the triangular distribution of k_{ij} may: (1) the most pessimistic value (K_{ij}^{p}) that has a very low likelihood; (2) the most likely value (K_{ij}^{m}) that definitely belongs to the set of available values; and (3) the most optimistic value (K_{ij}^{p}) that has a very low likelihood of belonging to the set of available values.

2.4. Problem Formulation Using Fuzzy Min Operator

Fuzzy set theory appears to be an ideal approach to deal with decision problems that are formulated as linear programming models with imprecision parameters. In this paper the net relative deviation is considered as fuzzy variable and converted into deterministic form using Zadeh's max-min operator as per Zimmermann (1985). We define a linear membership function by considering suitable upper and lower bounds to the objective function as given below.

First, the positive ideal solution (PIS) and negative ideal solution (NIS) for each of the fuzzy objective functions can be specified as follows:

$$Z_g^{PIS} = \mathrm{Min} Z_g = \text{lower limit and} Z_g^{NIS} = \mathrm{Max} Z_g = \text{upper limit}$$

And then linear membership functions can be specified by the DM to select the interval of goal value [Z_g^{PIS} and Z_g^{NIS}]. Accordingly, the corresponding, non-increasing continuous linear membership functions for the fuzzy objective functions can be expressed by the equation (9) and the fig 2.

$$f_g(Z_g) = \begin{cases} 1, & Z_g \le Z_g^{PIS} \\ \frac{Z_g^{NIS} - Z_g}{Z_g^{NIS} - Z_g^{PIS}}, & Z_g^{PIS} \le Z_g \le Z_g^{PIS}, \quad g = 1,2..k) \\ 0, & Z_g \ge Z_g^{NIS} \end{cases} \quad (9)$$

By introducing a minimum operator β an auxiliary variable, the equivalent fuzzy single goal linear programming problem is as follows:

Maximize β (0≤ β ≤ 1)

Subject to,

$$\beta \le \frac{Z_g^{NIS} - Z_g}{Z_g^{NIS} - Z_g^{PIS}} \quad (10)$$

And equation (4) – (7)

Fig. 2. Linear membership function of fg(Zg)

2.5. The weighted average operator method

The solutions obtained in minimum operator method can be improved by weighted average operator method because in case of multi objective problem, each objective is not equally important to the decision maker. This weighting is done here by analytical hierarchy process with respect to the various characteristics of the case. AHP helps decision-makers choose the best solution from several options and selection criteria. By introducing the auxiliary variable β, the fuzzy MOLP problem can be converted into an equivalent ordinary LP model by the weighted average operator method as follows:

$$Max\beta = \sum_{g=1}^{k} W_g \beta_g \quad (11)$$

Subject to,

$$\beta \le \frac{Z_g^{NIS} - Z_g}{Z_g^{NIS} - Z_g^{PIS}} for \forall g \quad (12)$$

$$\sum_{g=1}^{k} W_g = 1 \ for \forall g \quad (13)$$

$$0 \le W_g \le 1 \ for \forall g \quad (14)$$

Equation (4) – (7)

Where W_g (g = 1, 2, . . . ,K) is the corresponding weight of the g^{th} fuzzy objective function chosen by DM.

2.6. Solutions Procedure

Step1. Formulate the original fuzzy MOLP model for the project management problems according to Eqs. (1) – (7).

Step2. Provide the minimum acceptable membership level, α and then convert the fuzzy variable into crisp ones using the fuzzy ranking number method according to Eqs. (8).

Step3. Specify the degree of membership $f_g(Z_g)$ for several values of each objective function Z_g, g = 1, 2,3 by PIS and NIS.

Step4. Introduce the auxiliary variable β, thus enabling aggregation of the original fuzzy MOLP problem into an equivalent ordinary single-objective LP form using the minimum operator method.
Step5. Solve the ordinary LP problem. If the DM is dissatisfied with the initial solutions, the model should be adjusted until a preferred satisfactory solution is obtained.
Step7. Weighting each of the objective functions by AHP
Step6. Formulate the problem according to the weighted average operator method and solve the fuzzy MOLP problem by an equivalent ordinary single-objective LP model.

3. Computational Experiments

3.1. Data description

Daya Technologies Corporation was used as an industrial case study to demonstrate the practicality of the developed methodology (Wang and Liang 2004; Liang 2009). Daya is the world's first ball screw manufacturer certified to ISO 9001, ISO 14001, and OHSAS18001, and is also the major manufacturer producing the super precision ball screw, linear stage, guide ways, linear bearing and aerospace parts in Taiwan. Its products are mainly distributed throughout Asia, North America and Europe. Table 1 lists the basic data of the Daya case.

Table 1
Summarized data in the Daya case (in US dollar)

(i,j)	D_{ij}day	d_{ij}day	$C_{D_{ij}}$\$	$C_{d_{ij}}$\$	K_{ij}\$/day
1-2	14	10	1000	1600	(132,150, ,162)
1-5	18	15	4000	4540	(164, 180, 198)
2-3	19	19	1200	1200	-
2-4	15	13	200	440	(112, 120, 128)
4-7	8	8	600	600	-
4-10	19	16	2100	2490	(112, 130, 140)
5-6	22	20	4000	4600	(280, 300, 324)
5-8	24	24	1200	1200	-
6-7	27	24	5000	5450	(136, 150, 166)
7-9	20	16	2000	2200	(34, 50, 58)
8-9	22	18	1400	1900	(111, 125, 139)
9-10	18	18	700	1150	(120, 150, 160)
10-11	20	18	1000	1200	(80, 100, 108)

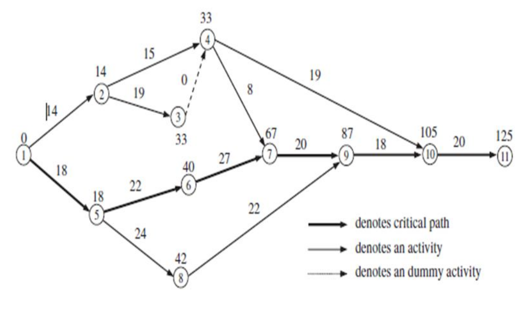

Fig. 3. The project network of the case

Other relevant data are as follows: fixed indirect costs \$12000, saved daily variable indirect costs (\$144, \$150, \$154), available budget (\$40,000, \$45,000, \$51,000), and project duration under normal conditions 125 days. The project start time is set to zero. The minimal acceptable possibility for all imprecise numbers is specified as 0.5. The critical path is 1–5–6–7–9–10–11. And fig 3 describes the project network of the case.

4. Result Analysis

First, formulate the fuzzy MOLP model for solving the multi objective project management problem according to equation (1) – (7) and solve the multi objective project management problem using the ordinary single-objective LP problem by LINGO computer software to obtain the initial solutions for each of the objective functions to determine $Z_1^{PIS} and Z_1^{NIS}$ for objective Z_1, $Z_2^{PIS} and Z_2^{NIS}$ for objective Z_2 and $Z_3^{PIS} and Z_3^{NIS}$ for objective Z_3. After running the program by LINGO computer software the result is shown in table 2.

Table 2
PIS and NIS

Objective function	PIS	NIS
Z_1	35890	45250
Z_2	111	125
Z_3	0	3701

The result is obtained for minimum acceptable membership level α = 0.5.
After finding this NIS and PIS, the linear membership function of each objective functions is defined according to the equation (9). Then the problem presented in equation (10) and equations (4) – (7) is formulated by using fuzzy minimum operator approach as follows,

Maximize β (0≤ β ≤ 1)
Subject to the

$$\beta \leq \frac{452520-Z_1}{45250-35890} \quad (15)$$

$$\beta \leq \frac{125-Z_2}{125-111} \quad (16)$$

$$\beta \leq \frac{3701-Z_3}{3701-0} \quad (17)$$

And equation (4) – (7)

Then LINGO computer software is used to run this ordinary LP model. The optimal solutions are Z_1=38510.8, Z_2= 114.92 and Z_3=\$1036 and the overall level of satisfaction of DM with the given objective values is 0.72 that is shown in table: 8.

4.1. Weighting the objectives

Each objective is not equally important to the decision maker or to the case for various situations. To find the weight of each objective various measure is taken into account and the weighting is done by Analytical Hierarchy Process.

AHP builds a hierarchy (ranking) of decision items using comparisons between each pair of items expressed as a matrix. Paired comparisons produce weighting scores that measure how much importance the items and the criteria have with each other. It is important to note that, since some of the criteria could be contrasting, it is not true in general that the best option is the one which optimizes each single criterion, rather the one which achieves the most suitable trade-off among the different criteria. Various analyses is done to determine the weight like as,

- Pair-wise comparison of satisfaction levels: These solutions indicate that a fair difference and interaction exists in the trade-offs and conflicts among dependent objective functions. Different combinations of arbitrary objective function may influence the objective and β values. These solutions indicate that a fair difference and interaction exists in the trade-offs and conflicts among dependent objective functions. That shown in table 3.

Table 3
Pair-wise comparisons of satisfaction levels

Item	Scenario1	Scenario2	Scenario 3
β	1	1	0.72
Z_1	35890	35890	-
Z_2	111	-	114.8
Z_3	-	0	1036

- Pair-wise comparison for the change in membership value to satisfaction levels: The specific membership value for each of the objective functions strongly affects the overall level of satisfaction of decision maker. If the membership value of first objective functions remain to its original position and changing the lower bound of the membership value of second objective function then the result is β= 0.99. That shown in table 4.

Table 4
Pair-wise comparisons of satisfaction levels for membership value

Increase in lower level of membership value	Z_1	Z_2	Z_3
Z_1	-	0.99	1
Z_2	1	-	0.81
Z_3	1	0.83	-

- Pair-wise comparison for increase α = 0.6: If the minimum acceptable level is increased from 0.5 to higher value, it influence the objective and β values. That shown in table 5.

Table 5
Pair-wise comparisons of satisfaction levels for increase α

Item	Scenario 1	Scenario 2	Scenario 3
β Value	1	1	0.72
Z_1	34890	34890	-
Z_2	111	-	114.78
Z_3	-	0	1047.2

- Pair-wise comparison for decrease in crashing cost: Decrease in crashing cost strongly affects the overall level of satisfaction of decision maker.That shown in table 6.

Table 6
Pair-wise comparisons for Crashing cost decrease

Item	Scenario 1	Scenario 2	Scenario 3
β Value	1	1	0.73
Z_1	36504	36504	-
Z_2	111	-	114.7
Z_3	-	0	887.7

The result can be modified by expectation of decision maker that is how the weight is given by decision maker to the each objective. The value of the relative weights among of multiple goals can be adjusted subjectively based on the DM's experience and knowledge. Then the total average weight is determined from the above table. The criteria and the corresponding values are obtained from the relevant table. Here weighting is done by considering the relationship between each of the objective functions with respect to the satisfaction level. But for the criteria 'with respect to the case analysis' the weighting is done by decision maker by analyzing the case. Analytical Hierarchy Process (AHP) is used for ranking each of the objective value. This process is done by weighting the pair-wise objective with respect to the satisfaction value by (1-5) where 1 means equal relation and 5 means extreme relation. Bellow the table 7 shows the result.

Table 7
Total average weight

Criteria	Z_1	Z_2	Z_3
With respect to the case analysis	0.42	0.25	0.33
With respect to the relation of satisfaction levels with each objectives	0.35	0.3	0.35
With respect to the membership value to satisfaction levels with each objectives	0.38	0.25	0.3
With respect to the increase in α	0.37	0.3	0.33
With respect to the decrease in project crashing cost	0.38	0.3	0.32
Average weight	0.38	0.28	0.34

Then by using the above weight from the table: 7 and the positive ideal solution (PIS) and negative ideal solution (NIS) for each of the fuzzy objective functions are taken as lower limit and upper limit respectively the above problem is formulated. Then the problem presented in equations (11) – (14) and (4) – (7) is formulated by Using Fuzzy Weighted Average Operator method as follow,

$$\text{Max}\,\beta = 0.38 * \beta_1 + 0.28 * \beta_2 + 0.34 * \beta_3 \quad (18)$$

Subject to the

$$\beta \leq \frac{452520 - Z_1}{45250 - 35890} \quad (19)$$

$$\beta \leq \frac{125 - Z_2}{125 - 111} \quad (20)$$

$$\beta \leq \frac{3701 - Z_3}{3701 - 0} \quad (21)$$

$$W_1 + W_2 + W_3 = 1 \quad (22)$$

$$0 \leq W_g \leq 1 \quad (23)$$

And equations (4) – (7)
Where W_g (g = 1,2, . . . ,K) is the corresponding weight of the g[th] fuzzy objective function chosen by DM.
After that LINGO computer software is used to run this ordinary LP model and the result obtained is shown in table 8. The proposed model provides the overall levels of DM satisfaction (β value) that gives the multiple fuzzy goal values (Z_1, Z_2, and Z_3). If the solution is β =1, then each goal is fully satisfied; if 0 < β < 1, then all of the goals are satisfied at the level of β, and if β =0, then none of the goals are satisfied. For example, the overall degree of DM satisfaction (β) with the goal values (Z_1=$38510.8, Z_2= 114.92 days and Z_3=$1036) was initially generated as 0.72. At the same time if the decision maker weight the three objective functions by 0.38, 0.28 and 0.34 respectively then for the minimum acceptable membership level α=0.5, the overall degree of DM satisfaction (β) is 0.83 with the goal values (Z_1=$37,481, Z_2= 113.4 days and Z_3= $630).

Table 8
Result of two methods

Item	Initial optimum solution with min operator	Improved solution with respect to weighted average method
Goal values	β = 0.72, Z_1=$38510.8, Z_2= 114.92 days and Z_3=$1036	β = 0.83 Z_1=$37,481, Z_2= 113.4days and Z_3= $630
Y_{ij}days)	Y_{12}=0, Y_{15}=1.08, Y_{23}=0, Y_{24}=0, Y_{47}=0, Y_{410}=0, Y_{56}=0, Y_{58}=0, Y_{67}=3, Y_{79}=4, Y_{89}=0, Y_{910}=0, Y_{1011}=2	Y_{12}=0, Y_{15}=3, Y_{23}=0, ,Y_{24}=0 Y_{47}=0 Y_{410}=0, Y_{56}=0, Y_{58}=0 Y_{67}=3, Y_{79}=4, Y_{89}=0, Y_{910}=0, Y_{1011}=2
t_{ij}(days)	t_{12}=14, t_{15}=16.9, t_{23}=19, t_{24}=15, t_{47}=8, t_{410}=19, t_{56}=22, t_{58}=24, t_{67}=24, t_{79}= 16, t_{89}=22, t_{910}=18, t_{1011}=18	t_{12}=14, t_{15}=15, t_{23}=19 , t_{24}=15, t_{47}=8 t_{410}=19, t_{56}=22 , t_{58}=24, t_{67}=24, t_{79}= 16 , t_{89}=22, t_{910}=18, t_{1011}=18
E_{ij}(days)	E_1=0 ,E_2=14, E_3=33, E_4=29, E_5=16.9, E_6=38.9, E_7=63, E_8=40.9,E_9=78.9, E_{10}=96.6,E_{11}=114.9	E_1=0 , E_2=14, E_3=33, E_4=29, E_5=15, E_6=37, E_7=61, E_8=39, E_9=77, E_{10}=95, E_{11}=113

So, the proposed method yields an efficient compromise solution. Generally, the β value may be adjusted to identify better results if the DM does not accept the initial overall degree of this satisfaction value. Additionally, the optimal solution yielded by the minimum operator method may not be an efficient solution, and the computational efficiency of the solution is not been assured. The minimum operator is preferable when a DM wants to make values of the optimal membership functions approximately equal or when a DM believes that the minimum operator is an approximate representation. To overcome the disadvantage of using the minimum operator the compensatory weighted average operator is employed for to obtain overall DM satisfaction degree.

5. Conclusion

This research focused on realistic and flexible project management decision by developing AHP based weighted average method with respect to the case. The above mentioned fuzzy multi-objective linear programming (FMOLP) method for project management considers multiple conflicting objectives uncertain environment. Here minimum operator method is used to aggregate all fuzzy set at first and then an AHP based weighted average method is develop with respect to the results obtained from minimum operator method. The flexibility of decision maker is one of the important parameters of this work. The flexibility of this proposed model lies in its ability to identify the optimal value by interactive and analytical hierarchy processes. The proposed method helps decision makers to choose the desired compromise solution for time-cost trade off within a time limit under different risk levels that varies with respect to the minimum acceptable level.Here the problem considers completion time in a suitable range for multi-objective project management (PM) decisions and this time is balanced with respect to the project crashing cost. The proposed model simultaneously minimizes the total project costs, crashing cost and the total time with reference to fuzzy data in the framework of satisfaction level of decision maker interactively.
The developed model could also be further extended by adopting systematic approaches, such as Analytical Network Process (ANP) and other types of fuzzy membership functions like flexible membership function and dynamic membership function.

References

Abbasnia, R., Afshar, A., & Eshtehardian, E. (2008). Time - cost trade-off problem in construction project management, based on fuzzy logic. Journal of Applied Sciences, 8(22), 4159-4165.

Al-Fanzine, M. A., & Haouari, M. (2005). A bi-objective model for robust resourceconstrained project scheduling. International Journal of Production Economics, 96,175–187.

Arikan, F., &Gungor, Z. (2001).An application of fuzzy goal programming to a multi-objective project network problem. Fuzzy Sets and Systems, 119, 49–58.

Bellman, R.E. and Zadeh, L.A. (1970), Decision-making in a fuzzy environment. Management Science, Vol. 17, P. 141–164.

Chanas, S., & Zielinski, P. (2001). Critical path analysis in the network with fuzzy activity times. Fuzzy Sets and Systems, 122, 195-204.

Coyle, G. (2004), 'The Analytic Hierarchy Process', Pearson Education Limited, USA

Fabiane.D.O, Neida.M.P, Carlos.R.S.(2003). Goal programming in a planning problem.Applied Mathematics and Computation 140: 165-178.

Ghazanfari, M., Yousefli, A., Ameli, M. S. J., &Amiri, A. B. (2008).A new approach to solve time-cost trade-off problem with fuzzy decision variables. Int. J. Adv. Manuf. Technol,

Hung-JuChien., Stephen Barthorpe (2013). Using Analytic Hierarchy Process to Analyse the Critical Success Factors for Performance Management of the Taiwanese Veterans Home.International Journal of Applied Science and Technology Vol. 3 No. 7; October 2013

Jiuping Xu and Cuiying Feng (2014).Multimode Resource-Constrained Multiple Project Scheduling Problem under Fuzzy Random Environment and Its Application to a Large Scale Hydropower Construction Project.The Scientific World Journal Volume 2014, Article ID 463692, 20 pages

Kamal M. Al-Subhi Al-Harbi (2001). Application of the AHP in project managementInternational Journal of Project ManagementVolume 19, Issue 1, January 2001, Pages 19–27

Liu, S. T. (2003). Fuzzy activity times in critical path and project crashing problems. Cybernetics and Systems, 34, 161-172.

Liang, G. S., & Han, T. C. (2004). Critical path analysis based on fuzzy concept. International Journal of Information and Management Sciences, 15(4), 29-40.

Liang, T. F. (2009). Application of fuzzy sets to multi-objective project management decisions in uncertain environments. International Journal of General System, 38, 311–330.

Liou, T. S. and Wang, M.T. Ranking fuzzy numbers with integral value, Fuzzy Sets and Systems, 50, 1992, pp. 247–255

Ming-Feng Yang and Yi Lin (2013) Applying fuzzy multi-objective linear programming to project management decisions with the interactive two-phase method.Computers & Industrial Engineering 66 (2013) 1061–1069.

Rabbani, M., Ghomi, F., Jolai, F., Lahiji, N.S.: A new heuristic for resource-constrained project scheduling networks using critical chain concept. Eur. J. Operat. Res. 176, 794–808 (2007).

Saaty, T. L. (2008), 'Decision making with the Analytic Hierarchy Process', International Journal of Service Sciences, Vol. 1 No. 1, Inderscience Enterprises Ltd.

Suo, M. Q., Li, Y. P., & Huang, G. H. (2012). Mult-icriteria decision making under uncertainty: An advanced ordered weighted averaging operator for planning electric power systems. Engineering Applications of Artificial Intelligence, 25(1), 72–81.

Tien-Fu Liang (2010) Applying fuzzy goal programming to project management decisions with multiple goals in uncertain environments. Expert Systems with Applications 37.8499–8507

Tamiz M, Jones DF, Romero C (1998). Goal programming for decision making: an overview of the current state-of-the-art. European Journal of Operational Research; 111:569–81.

Yin, P. Y., & Wang, J. U. (2008).Optimal multiple-objective resources allocation using hybrid particle swarm optimization and adaptive resources bounds technique. Journal of Computational and Applied Mathematics, 15, 73–86.

Zadeh LA (1978). Fuzzy sets as a basis for a theory of possibility. Fuzzy Set. Syst. 1:328.

Zimmermann HJ (1978). Fuzzy programming and linear programming with several objective functions. Fuzzy Set. Syst. 1:45-55.