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Effect of the Multi Vibration Absorbers on the Nonlinear FG Beam Under Periodic Load with Various Boundary Conditions

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ABSTRACT

A semi-analytical method is used to study the effects of the multi vibration absorbers on the nonlinear functionally graded (FG) Euler-Bernoulli beam subjected to periodic load. The material properties of the beam are assumed to be continuously graded in the thickness direction. The governing equations of functionally graded beam are obtained based on the Hamilton's principle and these equations are solved by using the Rayleigh-Ritz method. To validate the results, comparisons are made with the available solutions for the natural frequencies of isotropic beam. The effects of the multi vibration absorbers and material parameters on the vibration response of functionally graded beam are investigated. For case study the effect of two symmetrical vibration absorbers is considered, these absorbers are applicable in some of the mechanic systems. In those systems, two absorbers are used close to the beginning and end of the structures instead of using them in the middle of these structures. By considering the industrial applications, it is shown that using the two symmetrical vibration absorbers with lower mass is close to the end of functionally graded beam is better than the middle of one. In addition, the effect of different numbers of the vibration absorbers on the nonlinear functionally graded beam with simply supported boundary condition is considered. The results shown that increasing the number of vibration absorbers leads to decreasing the maximum deflection.

Keywords: Functionally graded beam, Vibration absorbers, Vibration response, and Periodic load.

1 INTRODUCTION

ABSORBERS are promising devices for passively reducing the amplitude of vibrations. They have different types and strongly have been used in engineering practice with high efficiency. They are used in different industry such as aerospace industry, bridges, building project, cars, motorcycle etc. In addition, the functionally graded materials are used extensively in a wide range of engineering applications, too. They are used in aerospace structures, fusion energy devices, engine combustion chambers, engine parts and other engineering. According to the application of absorbers and functionally graded material, research on the stability analysis of these structures has been of interest of scientists from many years ago. Various researches have been done in the field of reducing

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vibrations, such as the effect of absorbers on the beams. In without absorbers cases, many studies have been done on vibration analysis of beam. The free vibrations analysis of a stepped Euler–Bernoulli beam consisting of two uniform sections by using Adomian decomposition method was investigated by Mao and Pietrzko [1]. Hsu et al. [2] presented the free vibration response of non-uniform Euler–Bernoulli beam under various supporting conditions. Their study was based on applying the Adomian modified decomposition method. In the above mentioned studies the effects of vibration responses on the functionally graded beam have not been considered. Some studies have been done on vibration analysis of functionally graded beam. Free vibration responses of simply supported FG beam by using different higher order shear deformation theories was investigated by Aydogdu and Taskin [3]. Sina et al. [4] analyzed free vibration of functionally graded beams by using a new beam theory different from the traditional first-order shear deformation theory of beam. Kapuria et al. [5] studied the static and free vibration response for layered functionally graded beams in conjunction with the modified rule of mixtures by using third order zigzag theory. Nonlinear vibration analysis of functionally graded beam containing an open edge crack was studied by Kitipornchai et al. [6]. Their study was based on Timoshenko beam theory and von Ka´rma´n geometric nonlinearity. Alshorbagy et al. [7] presented the free vibration characteristics of functionally graded beam by using finite element method. Wattanasakulpong and Ungbhakorn [8] investigated the free vibration analysis of functionally graded beams supported by arbitrary boundary conditions by using the differential transformation method. A theoretical investigation in free vibration and elastic buckling of functionally graded beams containing open edge cracks was studied by Yang and Chen [9]. They used Bernoulli–Euler beam theory and the rotational spring model at this study. Sahraee and Saidia [10] used a differential quadrature method for free vibration and buckling analysis of deep functionally graded beam-columns composed with elastic foundations. Rahimi et al. [11] presented the post-buckling behavior and free vibration analysis of functionally graded beams by means of an exact solution method. For vibration analysis of beam with vibration absorbers, the forced vibrations analysis of a simply supported beam with non-linear dynamic vibration absorber was investigated by Kojima and Saito [12]. Huang and Chen [13] investigated the reducing of vibration and the interior noise of an aircraft's fuselage by using the vibration absorbers. Yamaguchi [14] studied the effect of the vibration absorber consisting of a spring-viscous damper and a viscoelastic beam. Optimum design of passive vibration absorbers on the long cantilevered beam is studied by Juang [15]. Najafi et al. [16] designed a translational-type absorber and a rotational-type absorber for vibration isolation of a clamped-clamped beam subjected to harmonic point excitation. They used the finite element method to solve that problem. The application of both the undamped and the damped linear dynamic vibration absorbers (DVAs) to a piecewise linear beam system to suppress its first harmonic resonance was presented by Bonsel et al. [17]. Esmailzadeh and Jalili [18] presented a method in designing optimal vibration absorbers for a structurally damped beam system under the harmonic force excitation. Wong et al. [19] studied the effect of a new dynamic vibration absorber combining a translational-type absorber and a rotational-type absorber for isolation of beam vibration under harmonic excitation by using the finite element method. The effect of active and passive vibration control characteristics by used the numerical solution and verified experimentally for carbon/epoxy laminated composite beams with a collocated piezoceramic sensor and actuator is investigated by Kang et al. [20].

A review of studies shows that some studies have been done on the effect of vibration absorbers on the beam and some researchers have been investigated vibration analysis of functionally graded beam but the effect of vibration absorbers on the nonlinear functionally graded beam under periodic load has not been studied. In this study, the effect of the multi vibration absorbers on the functionally graded Euler-Bernoulli beam subjected to periodic load is investigated using semi-analytic approach. The material properties of beam are assumed to be continuously graded in the thickness direction according to a simple power law distribution in terms of volume fraction of constituents. Based on the Hamilton's principle the governing equations are derived and by Rayleigh-Ritz method the vibration problem is solved. In order to valid the formulations, comparisons are made with the previous researches. Results are presented to evaluate the effects of the multi vibration absorbers and material parameters on the vibration response of functionally graded beam.

2 THERORETICAL FORMULATION

2.1 FGM material properties

A functionally graded beam with J (the number of vibration absorbers) vibration absorbers is shown in Fig. 1. The boundary condition in that figure is simply supported but in following of this study, other boundary conditions are also investigated. The beam has thickness h and axial length L. The beam is assumed to be made from a mixture of ceramics and metals that upper surface of beam ($z = h/2$) is ceramic and the lower surface ($y = -h/2$) is metal and in order to keep material continuity.

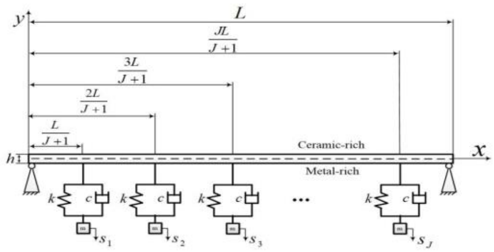


Fig.1
Configuration of functionally graded beam with the multi vibration absorbers.

The volume-fractions by following a simple power law are given by [8, 21]

$$
\begin{aligned}
&V_c = V_c(z) = \left(\frac{2z+h}{2h}\right)^N \\
&V_m = V_m(z) = 1 - V_c(z)
\end{aligned}
\tag{1}
$$

where $-h/2 \le z \le h/2$ and N is the material power law index of the FG beam. Effective properties P_{eff} of FG beam can be determined by linear rule of mixture in the following form

$$
P_{eff} = P_m(z)V_m(z) + P_c(z)V_c \tag{2}
$$

According to the mentioned law, the Young's modulus (E) and mass density (ρ) of the FG beam, can be expressed in the following form

$$
\begin{aligned}
&E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^N, -\frac{h}{2} \le z \le \frac{h}{2} \\
&\rho(z) = \rho_m + (\rho_c - \rho_m)\left(\frac{2z+h}{2h}\right)^N, -\frac{h}{2} \le z \le \frac{h}{2}
\end{aligned}
\tag{3}
$$

where E and ρ are the Young's modulus and mass density of the FG beam, respectively.

2.2 Governing equations

The kinetic and potential energy of beam-absorbers system is presented in Eqs. (4-5). Also, the work of external and damping forces on the beam-absorbers system can be expressed in Eq. (6)

$$
T = \frac{1}{2}\int_0^L \left[\rho(z)A\left(\frac{\partial y}{\partial t}\right)^2 + m\sum_{i=1}^{J}\left(\frac{\partial s_i}{\partial t}\right)^2\right]dx \tag{4}
$$

$$
U = \frac{1}{2}\int_0^L \left[k\sum_{i=1}^{j}(s_i - y)^2\,\delta_k\left(x - \frac{iL}{J+1}\right) + E(z)I\left(\frac{\partial^2 y}{\partial x^2}\right)^2 + E(z)A\left(\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right)^2\right]dx \tag{5}
$$

$$
W = \frac{1}{2}\int_0^L \left[q(x,t) - c\sum_{i=1}^{J}\left(\frac{\partial s_i}{\partial t} - \frac{\partial y}{\partial t}\right)^2\delta_k\left(x - \frac{iL}{J+1}\right)(s - y)\right]dx \tag{6}
$$

where T, U and W are kinetic energy, potential energy and the work of applied forces on the beam-absorbers system, respectively. $y(x,t)$ and $s_i(i=1,2,..,J)$ are displacement of beam and vibration absorbers, respectively. m is the mass of each of absorbers, k is spring stiffness, c is damping coefficient, I is the second moment of cross-section area of beam and δ_k is Dirac delta.

The equations of motion can be derived by applying the Hamilton's principle as follows:

$$\delta\int_0^t \left(T-U+W\right)dt=0 \tag{7}$$

Substituting the Eqs. (4-6) into Eq. (7) and integrating this equation by parts, the equations of motion can be written as:

$$\rho(z)A\frac{\partial^2 y}{\partial t^2}+E(z)I\frac{\partial^4 y}{\partial x^4}-\sum_{i=1}^{J}\left[k(s_i-y)+c\left(\frac{\partial s_i}{\partial t}-\frac{\partial y}{\partial t}\right)\right]\delta_k\left(x-\frac{iL}{J+1}\right)+\frac{E(z)A}{2L}\frac{\partial^2 y}{\partial x^2}\int_0^L\left(\frac{\partial y}{\partial x}\right)^2 dx-q(x,t)=0 \tag{8}$$

$$\begin{cases} i=1: \quad m\dfrac{\partial^2 s_1}{\partial t^2}+\left[k\left(s_1-y\right)+c\left(\dfrac{\partial s_1}{\partial t}-\dfrac{\partial y}{\partial t}\right)\right]\delta_k\left(x-\dfrac{L}{J+1}\right)=0 \\ i=2: \quad m\dfrac{\partial^2 s_2}{\partial t^2}+\left[k\left(s_2-y\right)+c\left(\dfrac{\partial s_2}{\partial t}-\dfrac{\partial y}{\partial t}\right)\right]\delta_k\left(x-\dfrac{2L}{J+1}\right)=0 \\ \qquad\qquad\vdots \\ i=J: \quad m\dfrac{\partial^2 s_J}{\partial t^2}+\left[k\left(s_J-y\right)+c\left(\dfrac{\partial s_J}{\partial t}-\dfrac{\partial y}{\partial t}\right)\right]\delta_k\left(x-\dfrac{JL}{J+1}\right)=0 \end{cases} \tag{9}$$

This system has $J+1$ degrees of freedom ($y,s_1,s_2,...,s_J$) that y and $s_i(i=1,2,..,J)$ are the degrees of freedom for functionally graded beam and J vibration absorbers, respectively.

2.3 Boundary condition

In this section, according to the different boundary condition, the deflection of FG beam is considered as follows:

I. Simply supported end ($x=0$ and $x=L$)

$$y\left(x,t\right)=0,\frac{\partial^2 y\left(x,t\right)}{\partial x^2}=0 \tag{10}$$

According to the Eq. (10), the deflection of beam is considered as [22]

$$y(x,t)=\sum w\left(t\right)\sin\left(\frac{n\pi x}{L}\right) \tag{11}$$

II. Fixed clamped end ($x=0$ and $x=L$)

$$y\left(x,t\right)=0,\frac{\partial y\left(x,t\right)}{\partial x}=0 \tag{12}$$

According to the Eq. (12), the deflection of beam is considered as:

$$y(x,t)=\sum w\left(t\right)\sin^2\left(\frac{n\pi x}{L}\right) \tag{13}$$

III. End of beam is simply supported and other end is fixed clamped

$$y(x,t)=0,\frac{\partial y(x,t)}{\partial x}=0 \qquad at\ x=0$$
$$y(x,t)=0,\frac{\partial^2 y(x,t)}{\partial x^2}=0 \qquad at\ x=L \tag{14}$$

According to the Eq. (14), the deflection of beam is considered as:

$$y(x,t)=\sum w(t)\sin^3\left(\frac{n\pi x}{L}\right) \tag{15}$$

IV. End of beam is fixed clamped and other end is free

$$y(x,t)=0,\frac{\partial y(x,t)}{\partial x}=0 \qquad at\quad x=0$$
$$\frac{\partial^2 y(x,t)}{\partial x^2}=0,\frac{\partial^3 y(x,t)}{\partial x^3}=0 \qquad at\quad x=L \tag{16}$$

According to the Eq. (16), the deflection of beam is considered as:

$$y(x,t)=\sum w(t)\sin^4\left(\frac{n\pi x}{L}\right) \tag{17}$$

where $w(t)$ the time dependent amplitude and n the number of mode shape.

By substituting Eqs. (10-17) in Eqs. (8) and (9) and by applying the Rayleigh-Ritz method in the ranges $0\le x\le L$, for the governing equations with different boundary conditions yield

$$\ddot{w}+a_1w+a_2w^3+a_3\left(kw+c\dot{w}\right)+a_4\left(k\sum_{i=1}^{J}s_i+c\sum_{i=1}^{J}\dot{s}_i\right)-a_4q\left(x,t\right)=0 \tag{18}$$

$$\begin{cases}\ddot{s}_1+b_1\left(ks_1+c\dot{s}_1\right)+c_1\left(kw+c\dot{w}\right)=0\\ \ddot{s}_2+b_2\left(ks_2+c\dot{s}_2\right)+c_2\left(kw+c\dot{w}\right)=0\\ \vdots\\ \ddot{s}_J+b_J\left(ks_J+c\dot{s}_J\right)+c_J\left(kw+c\dot{w}\right)=0\end{cases} \tag{19}$$

where the coefficients $a_i(i=1,2,...,4)$ and $b_j,c_j(j=1,2,...,J)$ for each boundary condition as follows:

I. Simply supported end

$$a_1=\frac{E(z)I}{\rho(z)A}\left(\frac{n\pi}{L}\right)^4;\ a_2=\frac{E(z)}{4\rho(z)}\left(\frac{n\pi}{L}\right)^4;$$
$$a_3=\frac{2}{\rho(z)A}\sum_{i=1}^{J}\sin\left(\frac{n\pi i}{J+1}\right)\left(\frac{1-\cos(n\pi)}{n\pi}\right);\ a_4=\frac{2}{\rho(z)A}\left(\frac{1-\cos(n\pi)}{n\pi}\right) \tag{20}$$
$$b_i=\frac{1}{m};\quad c_i=\frac{1}{m}\sin\left(\frac{n\pi i}{J+1}\right);\quad i=1,2,...,J$$

II. Fixed clamped end

$$a_1=\frac{16E(z)I}{3\rho(z)A}\left(\frac{n\pi}{L}\right)^4;\ a_2=\frac{E(z)}{3\rho(z)}\left(\frac{n\pi}{L}\right)^4;$$
$$a_3=\frac{4}{3\rho(z)A}\sum_{i=1}^{J}\sin^2\left(\frac{n\pi i}{J+1}\right);\ a_4=\frac{4}{3\rho(z)A} \tag{21}$$
$$b_i=\frac{1}{m};\quad c_i=\frac{1}{m}\sin^2\left(\frac{n\pi i}{J+1}\right)\quad;\quad i=1,2,...,J$$

III. End of beam is simply supported and other end is fixed clamped

$$a_1=\frac{9E(z)I}{\rho(z)A}\left(\frac{n\pi}{L}\right)^4;\ a_2=\frac{81E(z)}{160\rho(z)}\left(\frac{n\pi}{L}\right)^4$$
$$a_3=\frac{16}{15\rho(z)A}\sum_{i=1}^{J}\sin^3\left(\frac{n\pi i}{J+1}\right)\left(\frac{\cos^3(n\pi)-3\cos(n\pi)+2}{n\pi}\right)$$
$$a_4=\frac{16}{15\rho(z)A}\left(\frac{\cos^3(n\pi)-3\cos(n\pi)+2}{n\pi}\right) \tag{22}$$
$$b_i=\frac{1}{m};\quad c_i=\frac{1}{m}\sin^3\left(\frac{n\pi i}{J+1}\right)\quad;\quad i=1,2,...,J$$

IV. End of beam is fixed clamped and other end is free

$$a_1=\frac{256E(z)I}{25\rho(z)A}\left(\frac{n\pi}{L}\right)^4;\ a_2=\frac{5E(z)}{7\rho(z)}\left(\frac{n\pi}{L}\right)^4$$
$$a_3=\frac{24}{25\rho(z)A}\sum_{i=1}^{J}\sin^4\left(\frac{n\pi i}{J+1}\right);\ a_4=\frac{24}{25\rho(z)A} \tag{23}$$
$$b_i=\frac{1}{m};\quad c_i=\frac{1}{m}\sin^4\left(\frac{n\pi i}{J+1}\right)\quad;\quad i=1,2,...,J$$

2.4 Free vibration analysis

For the free vibration analysis of functionally graded beam, vibration absorbers and periodic load in Eq. (18) are ignored; it means all terms in Eq. (18) with *k* and *c* coefficients, is vanished, thus, Eq. (18) reduces to

$$\ddot{w}+a_1w=0 \tag{24}$$

According to Eq. (24), the fundamental frequency of natural vibration of functionally graded beam can be determined by

$$\omega_n=\sqrt{a_1} \tag{25}$$

The natural frequency in Eq. (25) is used to validate the present formulations.

3 NUMERICAL RESULTS

3.1 Validation of the present approach

To validate the present formulation, in Table 1, the obtained natural frequencies of simply supported beam present in this study are compared with the results obtained by Majkut [23]. He is studied the natural frequencies of simply supported beam with analytical and finite element method. Comparisons show that the good agreements are obtained.

Table 1
Comparison on the natural frequencies of simply supported beam.

	Natural mode	Present Study	Majkut [23]			
			Analytical method	Erorrs (%)	FEM	Erorrs (%)
Stocky beam						
	1	1178.1	1178.1	0.000	1178.1	0.000
	2	4712.6	4712.6	0.000	4712.6	0.000
	3	10603.3	10603.3	0.000	10603.0	0.003
Slender beam						
	1	441.8	441.8	0.000	441.8	0.000
	2	1767.2	1767.2	0.000	1767.2	0.000
	3	3976.2	3976.2	0.000	3978.2	0.050

3.2 Vibration responses of functionally graded beam

In this section, the vibration analysis of functionally graded Euler-Bernoulli beam with the multi vibration absorbers is investigated. The effects of different geometrical and material parameters such as length of the FG beam, volume fraction of FG material, different place of vibration absorbers, periodic load and different boundary conditions on the vibration responses of functionally graded beam are investigated. The upper surface of the FG beam is ceramic rich and the lower surface of it is metal rich. The FG beam is assumed to be made of aluminum (*Al*) with $E_m = 70GPa, \rho_m = 2702\dfrac{kg}{m^3}$ and alumina (Al_2O_3) with $E_c = 380GPa, \rho_c = 3800\dfrac{kg}{m^3}$, the geometrical parameters of the FG beam with $L = 4m, h = 0.03m, N = 1$ and the parameter of the vibration absorber with $k = 1000\dfrac{N}{m}, c = 10\dfrac{Ns}{m}$, $m = 0.7kg$ are considered. In all solved examples, the number of mode shape (*n*) is assumed equal to 1.

The effect of two vibration absorbers ($J = 2$) on the free vibration response of functionally graded beam with simply supported boundary condition is illustrated in Fig. 2. According to Fig. 2, vibration absorbers strongly decrease the maximum deflection of functionally graded beam.

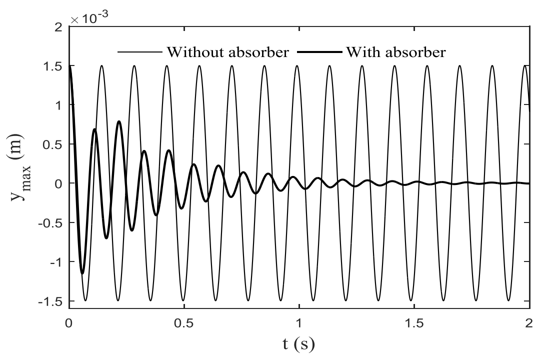
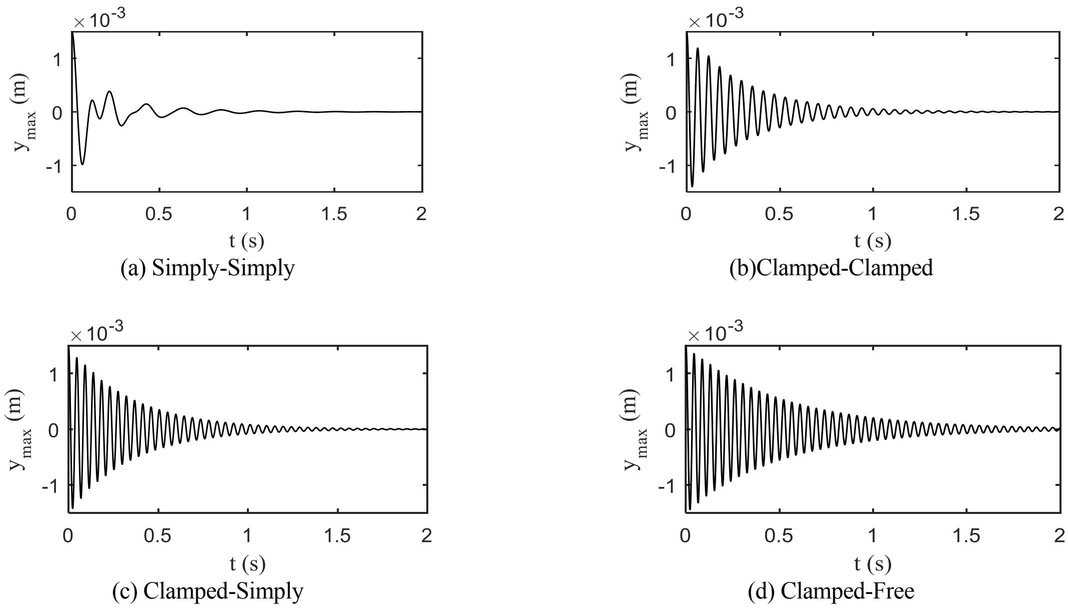


Fig.2
Free vibration responses of functionally graded beam with and without absorber.

In Fig. 3, the effect of two vibration absorbers on the free vibration response of functionally graded beam with different boundary conditions is considered. It can be seen, the effect of two vibration absorbers on reduction of the amplitude vibrations of functionally graded beam with simply supported boundary condition is more than other boundary conditions.

In Fig. 4 the effect of the number of vibration absorbers (*J*) on the nonlinear functionally graded beam with simply supported boundary condition is considered. As can be seen, increasing the number of vibration absorbers leads to decreasing of the maximum deflection.

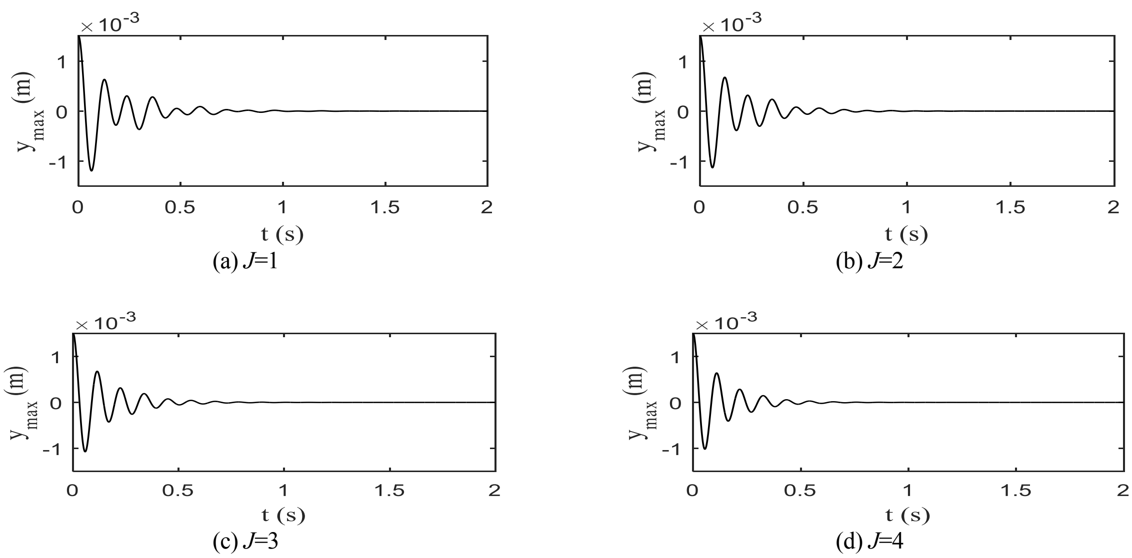
In Fig. 5 the free vibration response with the different damping coefficients is shown. According to this figure, the linear damping coefficients increase the maximum deflection of functionally graded beam when the linear damping coefficients are far from $c = 10Ns/m$. So, the linear damping coefficient about $c = 10Ns/m$, is the best case and it can be decreased the maximum deflection of functionally graded beam.



(a) Simply-Simply (b)Clamped-Clamped

(c) Clamped-Simply (d) Clamped-Free

Fig.3
Free vibration response of functionally graded beam with various boundary conditions.



(a) *J*=1 (b) *J*=2

(c) *J*=3 (d) *J*=4

Fig.4
The effect of number of vibration absorbers on free vibration response of functionally graded beam.

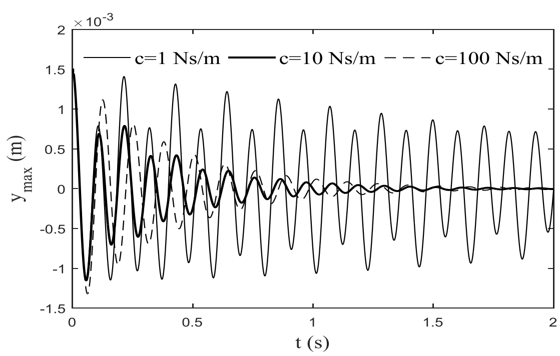


Fig.5
The effect of different damping coefficient on the free vibration responses of functionally graded beam.

The effect of the different spring stiffness on the free vibration response of functionally graded beam in Fig. 6 is investigated. In this section, the spring stiffness about $k = 1000N/m$, as the best case, can be decreased the maximum deflection of functionally graded beam similar to above discussion in previous paragraph.

The effect of material composition of the functionally graded beam with two vibration absorbers on the free vibration response is shown in Fig. 7. According to this figure, ceramic beam and metallic beam have the lowest and highest maximum deflection against the free vibration response, respectively. In addition, the maximum deflection of functionally graded beam is between the maximum deflection of ceramic and metallic beam.

The effect of material power law index on the natural frequencies is considered in Fig. 8. According to this figure, by increasing material power law index, the natural frequencies decrease. Therefore, by increasing the metal properties, the natural frequencies decrease. Therefore, the maximum and minimum natural frequency is related to ceramic and metallic beam, respectively.

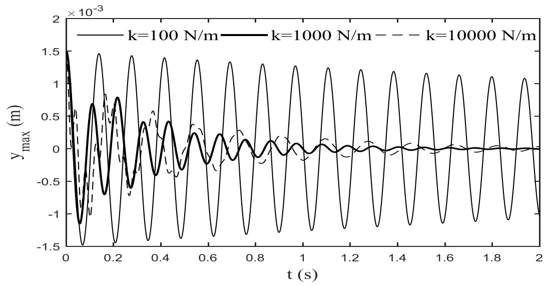


Fig.6
The effect of different spring stiffness on the free vibration responses of functionally graded beam.

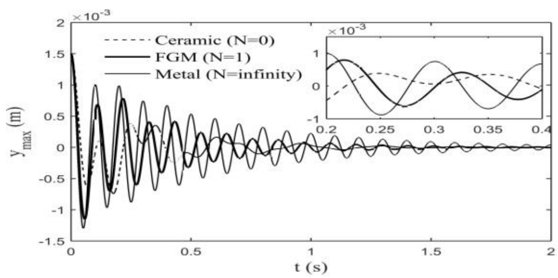


Fig.7
The effect of material composition of functionally graded beam on the free vibration responses.

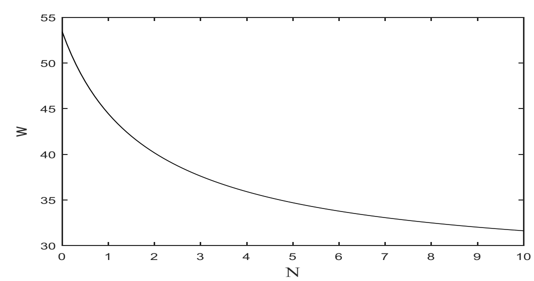


Fig.8
Natural frequencies of functionally graded beam.

In Fig. 9, the effect of two vibration absorbers on the forced vibration response of functionally graded beam is illustrated. The excitation force is considered to equal $q = 5\times10^2 \sin(300t)$. As can be seen, the vibration absorbers decrease the maximum deflection of functionally graded beam.

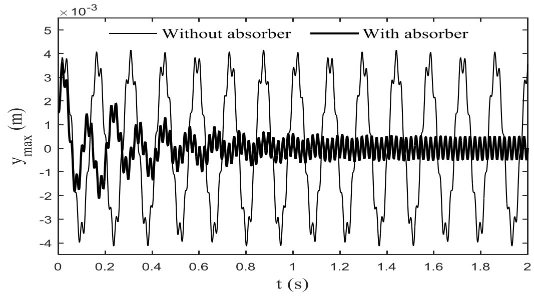
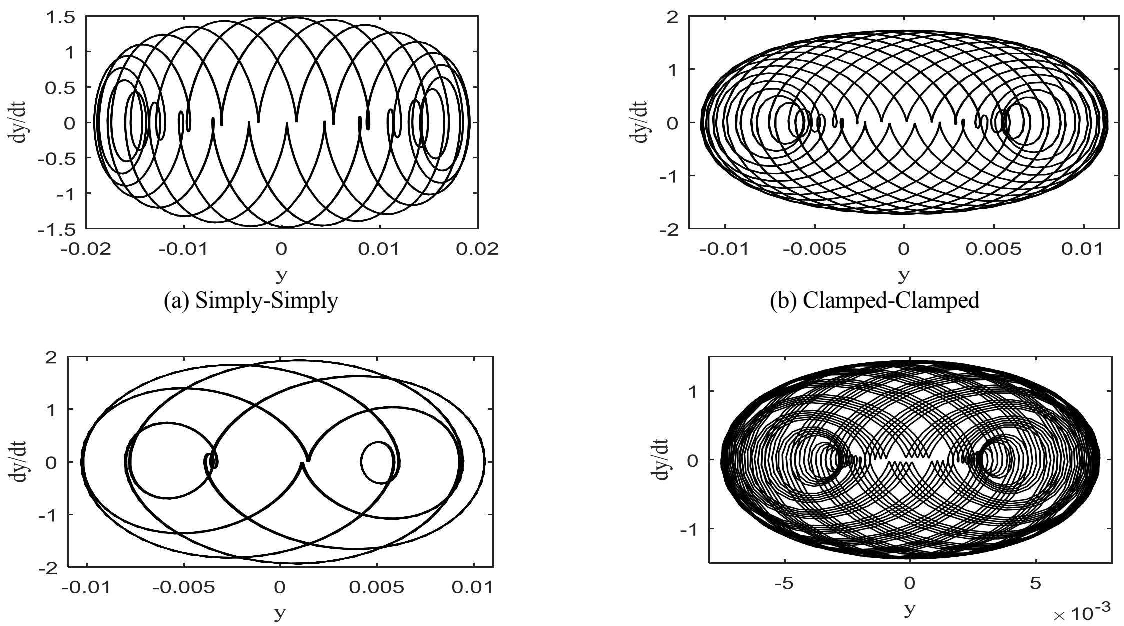


Fig.9
Forced vibration responses of functionally graded beam with and without absorber.

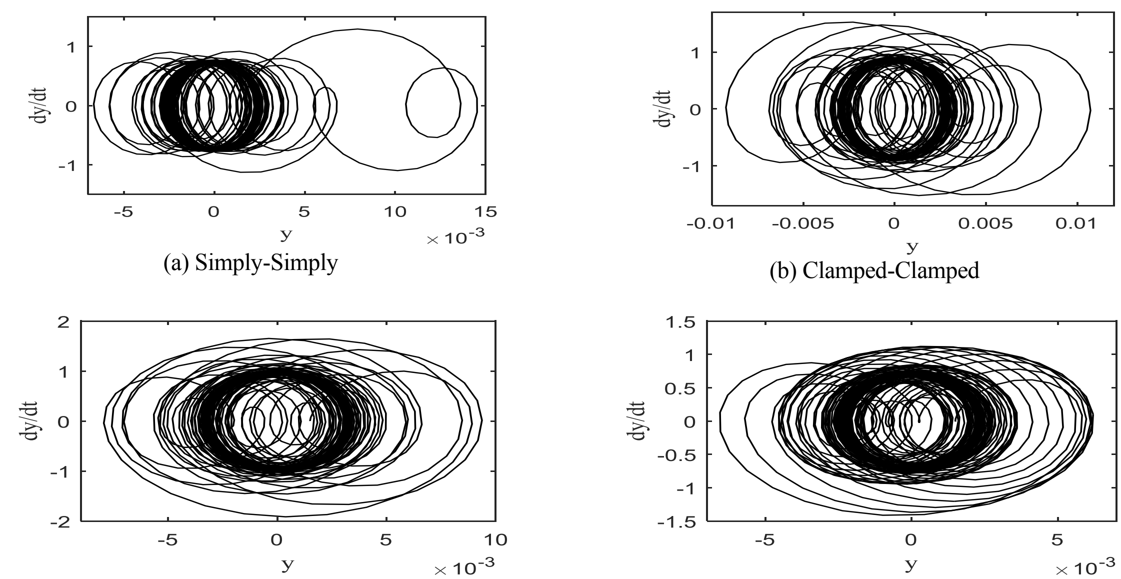
The phase plane of forced vibration without and with two vibration absorbers for different boundary conditions is shown in Figs. 10 and 11, respectively. As can be seen, when the functionally graded beam is without the vibration absorber, the relation of maximum deflection versus velocity has a closed curve while by using two vibration absorbers on the functionally graded beam, at first, the curve of maximum deflection versus velocity has disorder but with the passage of time has reached a limited cycle. Comparison of Fig. 10 to Fig. 11 is illustrated that maximum deflection of functionally graded beam with two vibration absorbers has significantly decreased. For example, due to the Figs. 10(a) and 11(a), maximum deflection of functionally graded simply supported beam in steady state is decreased from 0.02 *m* to 0.003 *m*. In addition, it can be seen, maximum deflection of functionally graded beam with simply supported and clamped-free boundary condition is more and less than other modes, respectively.



(a) Simply-Simply (b) Clamped-Clamped

(c) Clamped-Simply (d) Clamped-Free

Fig.10
Maximum deflection-velocity relation of functionally graded beam without vibration absorber for various boundary conditions.



(a) Simply-Simply (b) Clamped-Clamped

(c) Clamped-Simply (d) Clamped-Clamped

Fig.11
Maximum deflection-velocity relation of functionally graded beam with vibration absorber for various boundary conditions.

4 CONCLUSIONS

A semi-analytical method was used to study the vibration analysis of functionally graded Euler-Bernoulli beam with different boundary conditions subjected to periodic load. The material properties of the beam were assumed to be continuously graded in the thickness direction. The problem formulation was based on the Hamilton's principle. The Rayleigh-Ritz method was used to solve the vibration problem. The effects of different parameters such as material properties, geometrical dimensions and multi vibration absorber on the vibration analysis response of functionally graded beam were examined and the following conclusions were obtained

- The effect of vibration absorbers on the reduction of the amplitude vibrations of functionally graded beam with simply supported boundary condition is more than other boundary condition.
- Increasing the number of vibration absorbers leads to decreasing of the maximum deflection.
- The linear damping coefficient about c=10 *Ns/m* and the spring stiffness about k=1000 *N/m*, as the best case, can be decreased the maximum deflection of functionally graded beam.
- Maximum deflection of functionally graded beam with simply supported and clamped-free boundary condition is more and less than other modes, respectively.
- Ceramic and metallic beam have the lowest and highest maximum deflection against the free vibration response, respectively.
- Maximum deflection of functionally graded beam with simply supported and clamped-free boundary condition is more and less than other modes, respectively.
- By increasing the metal properties, the natural frequencies decrease.

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