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On weakly e^*-open and weakly e^*-closed functions

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Abstract. The aim of this paper is to introduce and study two new classes of functions called weakly e^*-open functions and weakly e^*-closed functions via the concept of e^*-open set defined by Ekici [9]. The notions of weakly e^*-open and weakly e^*-closed functions are weaker than the notions of weakly β-open and weakly β-closed functions defined by Caldas and Navalagi [6], respectively. Moreover, we investigate not only some of their fundamental properties, but also their relationships with other types of existing topological functions.

Keywords: e^*-open set, e^*-open function, weakly e^*-open function, weakly e^*-closed function, contra e^*-closed.

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1. Introduction and preliminaries

The notions of open functions and closed functions are important objects of topology. Recently, many researchers have been introduced and studied weak and strong forms of these concepts such as α-open functions [16], almost open functions [21], semiopen functions [5], preclosed functions [12], β-open and β-closed functions [1], weakly open functions [19], weakly closed functions [20], preopen functions [14], weakly eR-open functions [18]. After then, the concepts of weakly β-open and weakly β-closed functions which are

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weaker than the concept of weakly open and weakly closed functions are defined and investigated by Caldas and Navalagi [6].

In this paper, we define and study the notions of weakly e^*-open and weakly e^*-closed functions which are weaker than the notions of weakly β-open and weakly β-closed functions. Also, we obtain various characterizations of weakly e^*-open and weakly e^*-closed functions and investigate some of their fundamental properties.

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let X be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively. The family of all closed (open) sets of X is denoted by $C(X,\tau)(O(X))$. Recall that a set A is called regular open [22] (resp. regular closed [22]) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A point $x \in X$ is said to be θ-cluster point of A if $A \cap cl(U) \neq \emptyset$ for each open neighborhood U of x. The set of all θ-cluster points of A is called the θ-closure [23] of A and is denoted by $\theta\text{-}cl(A)$. If $A = \theta\text{-}cl(A)$, then A is called θ-closed, and the complement of a θ-closed set is called θ-open. A subset A of a space X is called e^*-open [9] (resp. β-open [1], preopen [15], α-open [16], a-open [7, 8, 10]) if $A \subseteq cl(int(\delta\text{-}cl(A)))$ (resp. $A \subseteq cl(int(cl(A)))$, $A \subseteq int(cl(A))$, $A \subseteq int(cl(int(A)))$, $A \subseteq int(cl(\delta\text{-}int(A)))$). The complement of an e^*-open (resp. β-open, preopen, α-open, a-open) set is called e^*-closed [9](resp. β-closed [1], preclosed [15], α-closed [16], a-closed [8]). The intersection of all e^*-closed (resp. β-closed, regular closed) sets of X containing A is called the e^*-closure [9] (resp. β-closure [1], δ-closure [23]) of A and is denoted by $e^*\text{-}cl(A)$ (resp. $\beta\text{-}cl(A)$, $\delta\text{-}cl(A)$). The union of all e^*-open (resp. β-open, regular open) sets of X contained in A is called the e^*-interior [9](resp. β-interior [1], δ-interior [23]) of A and is denoted by $e^*\text{-}int(A)$ (resp. $\beta\text{-}int(A)$, $\delta\text{-}int(A)$). The family of all e^*-open (resp. e^*-closed, θ-closed, preopen, α-open, δ-open, regular open) subsets of X is denoted by $e^*O(X)$ (resp. $e^*C(X)$, $\theta C(X)$, $PO(X)$, $\alpha O(X)$, $\delta O(X)$, $RO(X)$). The family of all e^*-open (resp. e^*-closed, θ-closed, preopen, α-open, δ-open, regular open) sets of X containing a point x of X is denoted by $e^*O(X,x)$ (resp. $e^*C(X,x)$, $\theta C(X,x)$, $PO(X,x)$, $\alpha O(X,x)$, $\delta O(X,x)$, $RO(X,x)$).

Definition 1.1 A space X is called e^*-connected [10] if X is not the union of two disjoint nonempty e^*-open sets.

Definition 1.2 A function $f : (X,\tau) \to (Y,\sigma)$ is called:
(1) strongly continuous [13] if for every subset A of X, $f[cl(A)] \subseteq f[A]$.
(2) weakly open [19] if $f[U] \subseteq int(f[cl(U)])$ for each open subset U of X.
(3) weakly closed [20] if $cl(f[int(F)]) \subseteq f[F]$ for each closed subset F of X.
(4) relatively weakly open [4] if $f[U]$ is open in $f[cl(U)]$ for every open subset U of X.
(5) almost open in the sense of Singal and Singal (briefly, *a.o.S.*) [21] if the image of each regular open subset U of X is open set in Y.
(6) preopen [14] (resp. β-open [1], α-open [16], semiopen [5]) if for each open subset U of X, $f[U]$ is preopen (resp. β-open, α-open, semiopen) set in Y.
(7) preclosed [12] (resp. β-closed [1], α-closed [16], semiclosed [5]) if for each closed subset F of X, $f[F]$ is preclosed (resp. β-closed, α-closed, semiclosed) set in Y.
(8) contra-open [3] (contra-closed [3]) if $f[U]$ is closed (open) in Y for each open (closed) subset U of X.
(9) weakly β-open [6] if $f[U] \subseteq \beta\text{-}int(f[cl(U)])$ for each open set U of X.
(10) weakly β-closed [6] if $\beta\text{-}cl(f[int(F)]) \subseteq f[F]$ for each closed set F of X.

Lemma 1.3 [23] Let X be a regular space. Then $O(X) = \delta O(X)$.

Lemma 1.4 [2] Let A and B be two subsets of a topological space X. If $A \in aO(X)$

and $B \in e^*O(X)$, then $A \cap B \in e^*O(X)$.

Lemma 1.5 [9] Let A be a subset of a space X. Then the following properties hold.
a) $e^*\text{-}int(A) = A \cap cl(int(\delta\text{-}cl(A)))$,
b) If $\mathcal{A} \subseteq 2^X$, then $\bigcup_{A\in\mathcal{A}} e^*\text{-}int(A) \subseteq e^*\text{-}int(\bigcup_{A\in\mathcal{A}} A)$.

2. Weakly e^*-open functions

Definition 2.1 A function $f : (X, \tau) \to (Y, \sigma)$ is called weakly e^*-open if for each open set U of X such that $f[U] \subseteq e^*\text{-}int(f[cl(U)])$.

Definition 2.2 A function $f : (X, \tau) \to (Y, \sigma)$ is called e^*-open if for each open set U of X, $f[U]$ is e^*-open set in Y.

Remark 1 *From Definition 2.1 and Definition 2.2, we have the following diagram. The converses of these implications are not true in general as shown by the following examples.*

$$\begin{array}{ccc} \text{weakly } \beta\text{-open} & \to & \text{weakly } e^*\text{-open} \\ \uparrow & & \uparrow \\ \beta\text{-open} & \to & e^*\text{-open} \end{array}$$

***Example* 2.3** Let $X := \{a, b\}$, $\tau_1 := 2^X$ and $Y := \{1, 2\}$, $\tau_2 := \{\emptyset, Y, \{2\}\}$. Define the function $f : (X, \tau_1) \to (Y, \tau_2)$ by $f := \{(a, 1), (b, 2)\}$. Then f is e^*-open and so weakly e^*-open, but it is neither weakly β-open nor β-open.

***Example* 2.4** Let $X := \{a, b, c\}$, $\tau_1 := \{\emptyset, X, \{b\}, \{b, c\}\}$ and $Y := \{1, 2, 3\}$, $\tau_2 := \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$. Define the function $f : (X, \tau_1) \to (Y, \tau_2)$ by $f := \{(a, 1), (b, 3), (c, 1)\}$. Then f is weakly e^*-open, but it is not e^*-open.

Theorem 2.5 Let (X, τ) be a regular space. Then $f : X \to Y$ is weakly e^*-open if and only if f is e^*-open.

Proof. *Sufficiency.* Obvious.
Necessity. Let U be any open subset of X.

$$\left.\begin{array}{r} x \in U \in O(X) \\ (X,\tau) \text{ is regular} \end{array}\right\} \Rightarrow (\exists V_x \in O(X, x))(cl(V_x) \subseteq U)$$

$$\Rightarrow U = \bigcup\{V_x | x \in U\} = \bigcup\{cl(V_x) | x \in U\}$$

$$\left.\begin{array}{r} \Rightarrow f[U] = f[\bigcup\{V_x | x \in U\}] = \bigcup\{f[V_x] | x \in U\} = \bigcup\{f[cl(V_x)] | x \in U\} \\ f \text{ is weakly } e^*\text{-open} \end{array}\right\} \Rightarrow$$

$$\Rightarrow f[U] \subseteq \bigcup\{e^*\text{-}int(f[cl(V_x)]) | x \in U\} \overset{\text{Lemma 1.5}}{\subseteq} e^*\text{-}int\{f[\bigcup cl(V_x)] | x \in U\} = e^*\text{-}int(f[U])$$

Then we have $e^*\text{-}int(f[U]) = f[U]$. Therefore $f[U] \in e^*O(Y)$. ■

Theorem 2.6 Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:
(1) f is weakly e^*-open,
(2) $f[\theta\text{-}int(A)] \subseteq e^*\text{-}int(f[A])$ for every subset A of X,
(3) $\theta\text{-}int(f^{-1}[B]) \subseteq f^{-1}[e^*\text{-}int(B)]$ for every subset B of Y,
(4) $f^{-1}[e^*\text{-}cl(B)] \subseteq \theta\text{-}cl(f^{-1}[B])$ for every subset B of Y,
(5) For each $x \in X$ and each open subset U of X containing x, there exists an e^*-open set V containing $f(x)$ such that $V \subseteq f[cl(U)]$,

(6) $f[int(F)] \subseteq e^*\text{-}int(f[F])$ for each closed subset F of X,
(7) $f[int(cl(U))] \subseteq e^*\text{-}int(f[cl(U)])$ for each open subset U of X,
(8) $f[U] \subseteq e^*\text{-}int(f[cl(U)])$ for each preopen subset U of X,
(9) $f[U] \subseteq e^*\text{-}int(f[cl(U)])$ for each α-open subset U of X.

Proof. $(1) \Rightarrow (2)$: Let A be any subset of X and $y \in f[\theta\text{-}int(A)]$.

$$y \in f[\theta\text{-}int(A)] \Rightarrow (\exists x \in \theta\text{-}int(A))(y = f(x)) \Rightarrow (\exists U \in O(X,x))(U \subseteq cl(U) \subseteq A)$$

$$\left.\begin{array}{r}\Rightarrow (\exists U \in O(X,x))(y = f(x) \in f[U] \subseteq f[cl(U)] \subseteq f[A]) \\ \text{Hypothesis}\end{array}\right\} \Rightarrow$$

$$\Rightarrow y = f(x) \in f[U] \subseteq e^*\text{-}int(f[cl(U)]) \subseteq e^*\text{-}int(f[A])$$

$$\Rightarrow y \in e^*\text{-}int(f[A]).$$

$(2) \Rightarrow (3)$: Let B be any subset of Y.

$$\left.\begin{array}{r}B \subseteq Y \Rightarrow f^{-1}[B] \subseteq X \\ \text{Hypothesis}\end{array}\right\} \Rightarrow f[\theta\text{-}int(f^{-1}[B])] \subseteq e^*\text{-}int(f[f^{-1}[B]]) \subseteq e^*\text{-}int(B)$$

$$\Rightarrow \theta\text{-}int(f^{-1}[B]) \subseteq f^{-1}[e^*\text{-}int(B)].$$

$(3) \Rightarrow (4)$: Let B be any subset of Y.

$$\left.\begin{array}{r}B \subseteq Y \Rightarrow Y \setminus B \subseteq Y \\ \text{Hypothesis}\end{array}\right\} \Rightarrow \theta\text{-}int(f^{-1}[Y \setminus B]) \subseteq f^{-1}[e^*\text{-}int(Y \setminus B)]$$

$$\Rightarrow \theta\text{-}int(X \setminus f^{-1}[B]) = X \setminus \theta\text{-}cl(f^{-1}[B]) \subseteq X \setminus f^{-1}[e^*\text{-}cl(B)] = f^{-1}[Y \setminus e^*\text{-}cl(B)]$$

$$\Rightarrow f^{-1}[e^*\text{-}cl(B)] \subseteq \theta\text{-}cl(f^{-1}[B]).$$

$(4) \Rightarrow (5)$: Let $x \in X$ and $U \in O(X,x)$.

$$\left.\begin{array}{r}U \in O(X,x) \Rightarrow f(x) \in f[cl(U)] \subseteq Y \Rightarrow f(x) \notin Y \setminus f[cl(U)] \subseteq Y \\ \text{Hypothesis}\end{array}\right\} \Rightarrow$$

$$\Rightarrow f^{-1}[e^*\text{-}cl(Y \setminus f[cl(U)])] \subseteq \theta\text{-}cl(f^{-1}[Y \setminus f[cl(U)]])$$

$$\Rightarrow X \setminus f^{-1}[e^*\text{-}int(f[cl(U)])] \subseteq X \setminus \theta\text{-}int(f^{-1}[f[cl(U)]])$$

$$\Rightarrow \theta\text{-}int(cl(U)) \subseteq \theta\text{-}int(f^{-1}[f[cl(U)]]) \subseteq f^{-1}[e^*\text{-}int(f[cl(U)])]$$

$$\left.\begin{array}{r}\Rightarrow f(x) \in f[int(cl(U))] = f[\theta\text{-}int(cl(U))] \subseteq e^*\text{-}int(f[cl(U)]) \\ V := e^*\text{-}int(f[cl(U)])\end{array}\right\} \Rightarrow$$

$$\Rightarrow (V \in e^*O(Y, f(x)))(V \subseteq f[cl(U)]).$$

$(5) \Rightarrow (6)$: Let $F \in C(X,\tau)$ and $y \in f[int(F)]$.

$$\left.\begin{array}{r}y \in f[int(F)] \Rightarrow (\exists x \in int(F))(y = f(x)) \Rightarrow int(F) \in O(X,x) \\ \text{Hypothesis}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists V \in e^*O(Y, f(x)))(V \subseteq f[cl(int(F))]) \\ F \in C(X,\tau)\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\exists V \in e^*O(Y, f(x)))(V \subseteq f[cl(int(cl(F)))] \subseteq f[F])$$

$$\Rightarrow (\exists V \in e^*O(Y, f(x)))(e^*\text{-}int(V) = V \subseteq e^*\text{-}int(f[F]))$$

$$\Rightarrow y = f(x) \in e^*\text{-}int(f[F]).$$

$(6) \Rightarrow (7)$: Let $U \in O(X)$.

$$\left.\begin{array}{r}U \in O(X) \Rightarrow cl(U) \in C(X,\tau) \\ \text{Hypothesis}\end{array}\right\} \Rightarrow f[int(cl(U))] \subseteq e^*\text{-}int(f[cl(U)]).$$

$(7) \Rightarrow (8)$: Let $U \in PO(X)$.

$$\left.\begin{array}{r}U \in PO(X) \Rightarrow (U \subseteq int(cl(U)))(int(cl(U)) \in O(X)) \\ \text{Hypothesis}\end{array}\right\} \Rightarrow$$

$\Rightarrow f[U] \subseteq f[int(cl(U))] \subseteq e^*\text{-}int(f[cl(int(cl(U)))]) \subseteq e^*\text{-}int(f[cl(U)])$.

(8) $\Rightarrow$ (9) : It is obvious the fact that every α-open set is preopen.
(9) $\Rightarrow$ (1) : It is obvious the fact that every open set is α-open. ■

Theorem 2.7 For a bijective function $f : (X, \tau) \to (Y, \sigma)$, the followings are equivalent:
(1) f is weakly e^*-open,
(2) $e^*\text{-}cl(f[U]) \subseteq f[cl(U)]$ for each open subset U of X,
(3) $e^*\text{-}cl(f[int(F)]) \subseteq f[F]$ for each closed subset F of X.

Proof. (1) $\Rightarrow$ (2) : Let $U \in O(X)$.

$$\left.\begin{array}{r} U \in O(X) \Rightarrow int(X \setminus U) \in O(X) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r} \Rightarrow f[int(X \setminus U)] \subseteq e^*\text{-}int(f[cl(int(X \setminus U))]) \\ f \text{ is bijective} \end{array}\right\} \Rightarrow$$

$\Rightarrow Y \setminus f[cl(U)] \subseteq Y \setminus e^*\text{-}cl(f[int(cl(U))])$
$\Rightarrow e^*\text{-}cl(f[U]) \subseteq e^*\text{-}cl(f[int(cl(U))]) \subseteq f[cl(U)]$.

(2) $\Rightarrow$ (3) : Let $F \in C(X, \tau)$.

$$\left.\begin{array}{r} F \in C(X, \tau) \Rightarrow int(F) \in O(X) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow$$

$\Rightarrow e^*\text{-}cl(f[int(F)]) \subseteq f[cl(int(F))] \subseteq f[cl(F)] = f[F]$.

(3) $\Rightarrow$ (1) : Let $U \in O(X)$.

$$\left.\begin{array}{r} U \in O(X) \Rightarrow X \setminus U \in C(X, \tau) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow \left.\begin{array}{r} e^*\text{-}cl(f[int(X \setminus U)]) \subseteq f[X \setminus U] \\ f \text{ is bijective} \end{array}\right\} \Rightarrow$$

$\Rightarrow Y \setminus e^*\text{-}int(f[cl(U)]) \subseteq Y \setminus f[U] \Rightarrow f[U] \subseteq e^*\text{-}int(f[cl(U)])$. ■

Lemma 2.8 [24] If $f : (X, \tau) \to (Y, \sigma)$ is continuous, then $f[cl(A)] \subseteq cl(f[A])$ for any subset A of X.

Theorem 2.9 If $f : (X, \tau) \to (Y, \sigma)$ is weakly e^*-open and continuous, then f is e^*-open.

Proof. Let U be any open subset of X.

$$\left.\begin{array}{r} U \in O(X) \\ f \text{ is weakly } e^*\text{-open} \end{array}\right\} \Rightarrow \left.\begin{array}{r} f[U] \subseteq e^*\text{-}int(f[cl(U)]) \\ f \text{ is continuous} \end{array}\right\} \Rightarrow$$

$\Rightarrow f[U] \subseteq e^*\text{-}int(f[cl(U)]) \subseteq e^*\text{-}int(cl(f[U])) \subseteq e^*\text{-}int(\delta\text{-}cl(f[U])) \subseteq cl(int(\delta\text{-}cl(f[U])))$
$\Rightarrow f[U] \in e^*O(Y)$. ■

Corollary 2.10 If $f : (X, \tau) \to (Y, \sigma)$ is weakly e^*-open and strongly continuous, then f is e^*-open.

Proof. From Theorem 2.9, it is obvious since every strong continuity implies continuity. ■

The following example shows that strong continuity is not decomposition of e^*-openness. Namely, an e^*-open function need not be strongly continuous.

***Example* 2.11** Let $X := \{a, b, c\}$ and $\tau := \{\emptyset, X\}$. Then the identity function $f : (X, \tau) \to (X, \tau)$ is an e^*-open function but it is not strongly continuous.

Theorem 2.12 If (Y, σ) is a regular space, $f : (X, \tau) \to (Y, \sigma)$ is weakly e^*-open and relatively weakly open, then f is e^*-open.

Proof. Let U be any open subset of X.

$$\left.\begin{array}{r} U \in O(X) \\ f \text{ is relatively weakly open} \end{array}\right\} \Rightarrow f[U] \in \sigma_{f[cl(U)]}$$

$\Rightarrow (\exists V \in \sigma)(f[U] = f[cl(U)] \cap V) \ldots (1)$

$$\left.\begin{array}{r} U \in O(X) \\ f \text{ is weakly } e^*\text{-open} \end{array}\right\} \Rightarrow f[U] \subseteq e^*\text{-}int(f[cl(U)]) \ldots (2)$$

$(1), (2) \Rightarrow f[U] = f[U] \cap V \subseteq e^*\text{-}int(f[cl(U)]) \cap V \subseteq f[cl(U)] \cap V = f[U]$

$$\left.\begin{array}{r} \Rightarrow e^*\text{-}int(f[cl(U)]) \cap V = f[U] \\ \text{Lemma 1.3 and Lemma 1.4} \end{array}\right\} \Rightarrow f[U] \in e^*O(Y).$$ ■

Definition 2.13 A function $f : (X, \tau) \to (Y, \sigma)$ is called contra e^*-closed if $f[U]$ is e^*-open in Y for each closed set U of X.

Theorem 2.14 If $f : (X, \tau) \to (Y, \sigma)$ is contra e^*-closed, then f is weakly e^*-open.

Proof. Let U be any open subset of X.

$$\left.\begin{array}{r} U \in O(X) \Rightarrow cl(U) \in C(X, \tau) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow f[cl(U)] \in e^*O(Y)$$

$\Rightarrow f[U] \subseteq f[cl(U)] = e^*\text{-}int(f[cl(U)])$. ■

Remark 2 *A weakly e^*-open function need not be contra e^*-closed as shown by the following example.*

***Example* 2.15** Let $X := \{a, b, c\}$, $\tau_1 := \{\emptyset, X, \{a\}, \{a, c\}\}$ and $Y := \{1, 2, 3\}$, $\tau_2 := \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$. Define the function $f : (X, \tau_1) \to (Y, \tau_2)$ by $f := \{(a, 1), (b, 3), (c, 1)\}$. Then f is weakly e^*-open, but it is not contra e^*-closed.

Definition 2.16 A function $f : (X, \tau) \to (Y, \sigma)$ is called complementary weakly e^*-open (briefly $c.w.e^*.o.$) if for each open set U of X, $f[Fr(U)]$ is e^*-closed in Y, where $Fr(U)$ denotes the frontier of U.

Remark 3 *The notions of complementary weakly e^*-open function and weakly e^*-open function are independent as shown by the following examples.*

***Example* 2.17** Let $X := \{1, 2, 3\}$, $\tau_1 := \{\emptyset, X, \{1\}, \{1, 3\}\}$ and $Y := \{a, b, c, d\}$, $\tau_2 := \{\emptyset, Y, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Define the function $f : (X, \tau_1) \to (Y, \tau_2)$ by $f := \{(1, a), (2, c), (3, d)\}$. Then f is weakly e^*-open, but it is not $c.w.e^*.o.$

***Example* 2.18** Let $X := \{a, b, c\}$, $\tau_1 := \{\emptyset, X, \{a\}, \{b, c\}\}$ and $Y := \{1, 2, 3\}$, $\tau_2 := \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$. Define the function $f : (X, \tau_1) \to (Y, \tau_2)$ by $f := \{(a, 1), (b, 3), (c, 3)\}$. Then f is $c.w.e^*.o.$, but it is not weakly e^*-open.

Theorem 2.19 Let $e^*O(X)$ closed under intersections. If $f : (X, \tau) \to (Y, \sigma)$ is bijective $c.w.e^*.o.$ and weakly e^*-open, then f is e^*-open.

Proof. Let U be any open subset of X and $x \in U$.

$$\left.\begin{array}{r} x \in U \in O(X) \\ f \text{ is weakly } e^*\text{-open} \end{array}\right\} \overset{\text{Theorem 2.6(5)}}{\Rightarrow}$$

$\Rightarrow (\exists V \in e^*O(Y, f(x)))(V \subseteq f[cl(U)])(x \notin Fr(U) = cl(U) \setminus U)$

$$\left.\begin{array}{r} \Rightarrow (y = f(x) \notin f[Fr(U)])(y \in V \setminus f[Fr(U)])(V_y := V \setminus f[Fr(U)]) \\ f \text{ is } c.w.e^*.o. \end{array}\right\} \Rightarrow$$

$\Rightarrow y \in V_y \in e^*O(Y)$

$$\left.\begin{array}{r}\Rightarrow y \notin f[Fr(U)] = f[cl(U) \setminus U] \overset{f \text{ is bijective}}{=} f[cl(U)] \setminus f[U] \Rightarrow y \in f[U] \\ f[U] = \bigcup\{V_y | (V_y \in e^*O(Y))(y \in f[U])\}\end{array}\right\} \Rightarrow$$
$\Rightarrow f[U] \in e^*O(Y)$. ■

Theorem 2.20 If $f : (X, \tau) \to (Y, \sigma)$ is an *a.o.S.* function, then f is weakly e^*-open.

Proof. Let U be any open subset of X.
$$\left.\begin{array}{r}U \in O(X) \Rightarrow U \subseteq int(cl(U)) \in RO(X) \\ f \text{ is } a.o.S.\end{array}\right\} \Rightarrow f[U] \subseteq f[int(cl(U))] \in O(Y)$$
$\Rightarrow f[U] \subseteq f[int(cl(U))] = int(f[int(cl(U))])$
$\Rightarrow f[U] \subseteq f[int(cl(U))] \subseteq int(f[cl(U)]) \subseteq e^*\text{-}int(f[cl(U)])$. ■

Remark 4 *The converse of Theorem 2.20 is not true in general. Namely, a weakly e^*-open function need not be a.o.S.*

***Example* 2.21** Let $X := \{a, b, c\}$, $\tau_1 := \{\emptyset, X, \{a\}, \{b, c\}\}$ and $Y := \{1, 2, 3\}$, $\tau_2 := \{\emptyset, Y, \{1\}, \{1, 3\}\}$. Define the function $f : (X, \tau_1) \to (Y, \tau_2)$ by $f := \{(a, 1), (b, 1), (c, 2)\}$. Then f is weakly e^*-open function, but it is not *a.o.S.*

Theorem 2.22 If $f : (X, \tau) \to (Y, \sigma)$ is bijective weakly e^*-open of a space X onto an e^*-connected space Y, then X is connected.

Proof. Let f be a bijective weakly e^*-open of a space X onto an e^*-connected space Y and suppose that X is not connected.
$$\left.\begin{array}{r}X \text{ is not connected} \Rightarrow (\exists U_1, U_2 \in O(X) \setminus \{\emptyset\})(U_1 \cap U_2 = \emptyset)(U_1 \cup U_2 = X) \\ f \text{ is bijective weakly } e^*\text{-open}\end{array}\right\} \Rightarrow$$
$\Rightarrow (f[U_i] \neq \emptyset)(\bigcap_i f[U_i] = \emptyset)(\bigcup_i f[U_i] = Y)(f[U_i] \subseteq e^*\text{-}int(f[cl(U_i)]) = e^*\text{-}int(f[U_i]))(i = 1, 2)$
$\Rightarrow (f[U_i] \neq \emptyset)(\bigcap_i f[U_i] = \emptyset)(\bigcup_i f[U_i] = Y)(f[U_i] = e^*\text{-}int(f[U_i]))(i = 1, 2)$
$\Rightarrow (f[U_i] \in e^*O(Y) \setminus \{\emptyset\})(\bigcap_i f[U_i] = \emptyset)(\bigcup_i f[U_i] = Y)(i = 1, 2)$
This means Y is not e^*-connected which is a contradiction. ■

Definition 2.23 [17] A space X is called hyperconnected if every nonempty open subset of X is dense in X.

Theorem 2.24 If X is a hyperconnected space, then a function $f : (X, \tau) \to (Y, \sigma)$ is weakly e^*-open if and only if $f[X]$ is e^*-open in Y.

Proof. *Sufficiency.* Obvious.
Necessity. Let U be any open subset of X.
$$\left.\begin{array}{r}U \in O(X) \\ X \text{ is hyperconnected}\end{array}\right\} \Rightarrow \left.\begin{array}{r}cl(U) = X \Rightarrow e^*\text{-}int(f[cl(U)]) = e^*\text{-}int(f[X]) \\ f[X] \text{ is } e^*\text{-open}\end{array}\right\} \Rightarrow$$
$\Rightarrow f[U] \subseteq f[X] = e^*\text{-}int(f[X]) = e^*\text{-}int(f[cl(U)])$. ■

3. Weakly e*-Closed Functions

Definition 3.1 A function $f : (X, \tau) \to (Y, \sigma)$ is called weakly e^*-closed if for each closed set F of X such that $e^*\text{-}cl(f[int(F)]) \subseteq f[F]$.

The implications between weakly e^*-closed functions and other types of closed functions are given by the following diagram.

$$\begin{array}{ccccccc}
 & & \begin{array}{c}\text{preclosed}\\ \text{(preopen)}\end{array} & & \begin{array}{c}e^*\text{-closed}\\ (e^*\text{-open})\end{array} & \rightarrow & \begin{array}{c}\text{weakly } e^*\text{-closed}\\ (\text{weakly } e^*\text{-open})\end{array} \\
 & & \uparrow & \searrow & \uparrow & & \uparrow \\
\begin{array}{c}\text{closed functions}\\ \text{(open functions)}\end{array} & \rightarrow & \begin{array}{c}\alpha\text{-closed}\\ (\alpha\text{-open})\end{array} & \rightarrow & \begin{array}{c}\beta\text{-closed}\\ (\beta\text{-open})\end{array} & \rightarrow & \begin{array}{c}\text{weakly } \beta\text{-closed}\\ (\text{weakly } \beta\text{-open})\end{array} \\
 & & \downarrow & \nearrow & & & \\
 & & \begin{array}{c}\text{semiclosed}\\ \text{(semiopen)}\end{array} & & & &
\end{array}$$

The converses of these implications are not true in general as shown by the following examples.

***Example* 3.2**
(i) Let $f : (X, \tau) \to (Y, \sigma)$ be the function in Example 2.3. f is weakly e^*-closed (e^*-closed) which is not weakly β-closed (β-closed).
(ii) Let $X := \{a, b, c\}$, $\tau_1 := \{\emptyset, X, \{b\}, \{b, c\}\}$ and $Y := \{1, 2, 3\}$, $\tau_2 := \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$. Define the function $f : (X, \tau_1) \to (Y, \tau_2)$ by $f := \{(a, 1), (b, 3), (c, 2)\}$. Then f is weakly e^*-closed, but it is not e^*-closed.

Theorem 3.3 Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then the following statements are equivalent :
(1) f is weakly e^*-closed,
(2) $e^*\text{-}cl(f[U]) \subseteq f[cl(U)]$ for each open subset U of X,
(3) $e^*\text{-}cl(f[U]) \subseteq f[cl(U)]$ for each regular open subset U of X,
(4) For each subset F in Y and each open subset U in X with $f^{-1}[F] \subseteq U$, there exists an e^*-open set A in Y with $F \subseteq A$ and $f^{-1}[A] \subseteq cl(U)$,
(5) For each $y \in Y$ and each open subset U in X with $f^{-1}(y) \subseteq U$, there exists an e^*-open set A in Y containing y and $f^{-1}[A] \subseteq cl(U)$,
(6) $e^*\text{-}cl(f[int(cl(U))]) \subseteq f[cl(U)]$ for each open set U of X,
(7) $e^*\text{-}cl(f[int(\theta\text{-}cl(U))]) \subseteq f[\theta\text{-}cl(U)]$ for each open set U of X.

Proof. (1) $\Rightarrow$ (2) : Let U be any open subset of X.

$$\left.\begin{array}{r} U \in O(X) \Rightarrow e^*\text{-}cl(f[U]) = e^*\text{-}cl(f[int(U)]) \subseteq e^*\text{-}cl(f[int(cl(U))]) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow$$
$$\Rightarrow e^*\text{-}cl(f[U]) \subseteq e^*\text{-}cl(f[int(cl(U))]) \subseteq f[cl(U)].$$

(2) $\Rightarrow$ (3) : Straightforward.

(3) $\Rightarrow$ (4) : Let F be any set in Y and U be any open set in X such that $f^{-1}[F] \subseteq U$.

$$\left.\begin{array}{r} f^{-1}[F] \subseteq U \subseteq cl(U) \Rightarrow f^{-1}[F] \cap cl(X \setminus cl(U)) = \emptyset \\ f \text{ is bijective} \end{array}\right\} \Rightarrow$$
$$\left.\begin{array}{r} \Rightarrow F \cap f[cl(X \setminus cl(U))] = \emptyset \\ X \setminus cl(U) \in RO(X) \end{array}\right\} \overset{\text{Hypothesis}}{\Rightarrow} F \cap e^*\text{-}cl(f[X \setminus cl(U)]) = \emptyset$$
$$\left.\begin{array}{r} \Rightarrow F \subseteq Y \setminus e^*\text{-}cl(f[X \setminus cl(U)]) \\ A := Y \setminus e^*\text{-}cl(f[X \setminus cl(U)]) \end{array}\right\} \Rightarrow$$
$$\Rightarrow (A \in e^*O(Y))(F \subseteq A)(f^{-1}[A] \subseteq X \setminus f^{-1}[f[X \setminus cl(U)]] = cl(U)).$$

$(4) \Rightarrow (5)$: Let $y \in Y$ and U be any open set in X such that $f^{-1}(y) \subseteq U$.

$$\left.\begin{array}{r} y \in Y \Rightarrow \{y\} \subseteq Y \\ U \in O(X) \\ f^{-1}(y) \subseteq U \Rightarrow f^{-1}[\{y\}] \subseteq U \end{array}\right\} \overset{\text{Hypothesis}}{\Rightarrow}$$
$$\Rightarrow (\exists A \in e^*O(Y))(\{y\} \subseteq A)(f^{-1}[A] \subseteq cl(U))$$
$$\Rightarrow (\exists A \in e^*O(Y,y))(f^{-1}[A] \subseteq cl(U)).$$

$(5) \Rightarrow (6)$: Let U be any open subset of X and $y \notin f[cl(U)]$.

$$\left.\begin{array}{r} y \notin f[cl(U)] \Rightarrow y \in Y \setminus f[cl(U)] \Rightarrow f^{-1}(y) \subseteq X \setminus cl(U) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow$$
$$\left.\begin{array}{r} \Rightarrow (\exists A \in e^*O(Y,y))(f^{-1}[A] \subseteq cl(X \setminus cl(U))) \\ f \text{ is bijective} \end{array}\right\} \Rightarrow$$
$$\Rightarrow (\exists A \in e^*O(Y,y))(A \subseteq f[cl(X \setminus cl(U))] = Y \setminus f[int(cl(U))])$$
$$\Rightarrow (\exists A \in e^*O(Y,y))(A \cap f[int(cl(U))] = \emptyset)$$
$$\Rightarrow y \notin e^*\text{-}cl(f[int(cl(U))]).$$

$(6) \Rightarrow (7)$: Let U be any open subset of X.

$$\left.\begin{array}{r} U \in O(X) \Rightarrow \theta\text{-}cl(U) = cl(U) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow \ e^*\text{-}cl(f[int(\theta\text{-}cl(U))]) \subseteq f[\theta\text{-}cl(U)].$$

$(7) \Rightarrow (1)$: Let F be any closed subset of X.

$$\left.\begin{array}{r} F \in C(X,\tau) \Rightarrow int(F) \in O(X) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow e^*\text{-}cl(f[int(\theta\text{-}cl(int(F)))]) \subseteq f[\theta\text{-}cl(int(F))]$$
$$\Rightarrow e^*\text{-}cl(f[int(F)]) \subseteq e^*\text{-}cl(f[int(\theta\text{-}cl(int(F)))]) \subseteq f[cl(int(F))] \subseteq f[F].$$ ■

Remark 5 *By Theorem 2.7. if $f : (X,\tau) \to (Y,\sigma)$ is a bijective function, then f is weakly e^*-open if and only if f is weakly e^*-closed.*

Now we look into conditions under which weakly e^*-closed functions are e^*-closed.

Theorem 3.4 If $f : (X,\tau) \to (Y,\sigma)$ is bijective weakly e^*-closed and if for each closed subset F of X and each fiber $f^{-1}(y) \subseteq X \setminus F$ there exists an open U of X such that $f^{-1}(y) \subseteq U \subseteq cl(U) \subseteq X \setminus F$. Then f is e^*-closed.

Proof. Let F be any closed subset of X and let $y \notin f[F]$.

$$\left.\begin{array}{r} y \notin f[F] \Rightarrow y \in Y \setminus f[F] \\ f \text{ is bijective} \end{array}\right\} \Rightarrow \left.\begin{array}{r} f^{-1}(y) \subseteq X \setminus F \\ \text{Hypothesis} \end{array}\right\} \Rightarrow$$
$$\left.\begin{array}{r} \Rightarrow (\exists U \in O(X))(f^{-1}(y) \subseteq U \subseteq cl(U) \subseteq X \setminus F) \\ f \text{ is weakly } e^*\text{-closed} \end{array}\right\} \overset{\text{Theorem 3.3(5)}}{\Rightarrow}$$
$$\Rightarrow (\exists A \in e^*O(Y,y))(f^{-1}[A] \subseteq cl(U) \subseteq X \setminus F)$$
$$\left.\begin{array}{r} \Rightarrow (\exists A \in e^*O(Y,y))(f^{-1}[A] \cap F = \emptyset) \\ f \text{ is bijective} \end{array}\right\} \Rightarrow (\exists A \in e^*O(Y,y))(A \cap f[F] = \emptyset)$$
$$\Rightarrow y \notin e^*\text{-}cl(f[F])$$

Then we have $e^*\text{-}cl(f[F]) = f[F]$. Therefore $f[F] \in e^*C(Y)$. ■

Theorem 3.5 If $f : (X,\tau) \to (Y,\sigma)$ is contra-open, then f is weakly e^*-closed.

Proof. Let F be any closed subset of X.

$$\left.\begin{array}{r} F \in C(X,\tau) \Rightarrow int(F) \in O(X) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow f[int(F)] \in C(Y,\sigma)$$

$\Rightarrow e^*\text{-}cl(f[int(F)]) \subseteq cl(f[int(F)]) = f[int(F)] \subseteq f[F]$. ■

Theorem 3.6 If $f : (X, \tau) \to (Y, \sigma)$ is bijective weakly e^*-closed, then for every subset F of Y and every open set U in X with $f^{-1}[F] \subseteq U$, there exists an e^*-closed set B in Y such that $F \subseteq B$ and $f^{-1}[B] \subseteq cl(U)$.

Proof. Let F be any set in Y and U be an open set in X such that $f^{-1}[F] \subseteq U$.

$$\left.\begin{array}{r}(F \subseteq Y)(U \in O(X))(f^{-1}[F] \subseteq U) \\ B := \ e^*\text{-}cl(f[int(cl(U))])\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (B \in e^*C(Y))(F \subseteq f[U] \subseteq f[int(cl(U))] \subseteq e^*\text{-}cl(f[int(cl(U))]) = B) \\ f \text{ is weakly } e^*\text{-closed}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (B \in e^*C(Y))(F \subseteq B)(B = e^*\text{-}cl(f[int(cl(U))]) \subseteq f[cl(U)]) \\ f \text{ is bijective}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (B \in e^*C(Y))(F \subseteq B)(f^{-1}[B] \subseteq cl(U)).$$ ■

Corollary 3.7 If $f : (X, \tau) \to (Y, \sigma)$ is bijective weakly e^*-closed, then for each $y \in Y$ and each open set U of X with $f^{-1}(y) \subseteq U$, there exists an e^*-closed set B in Y containing y such that $f^{-1}[B] \subseteq cl(U)$.

Definition 3.8 [20] A set F in a space X is called θ-compact if for each cover Ω of F by open U in X, there is a finite family $U_1, \ldots, U_n \in \Omega$ such that $F \subseteq int(\bigcup\{cl(U_i) | i = 1, 2, ..., n\})$.

Theorem 3.9 If $f : (X, \tau) \to (Y, \sigma)$ is bijective weakly e^*-closed with all fibers θ-closed, then $f[F]$ is e^*-closed for each θ-compact F in X.

Proof. Let F be θ-compact and $y \notin f[F]$.

$$\left.\begin{array}{r}y \notin f[F] \overset{f \text{ is bijective}}{\Rightarrow} f^{-1}(y) \cap F = \emptyset \\ (\forall y \in Y)(f^{-1}(y) \in \theta C(X))\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\left.\begin{array}{r}\Rightarrow (\forall x \in F)(\exists U_x \in \mathcal{U}(x))(cl(U_x) \cap f^{-1}(y) = \emptyset) \\ \mathcal{A} := \{U_x | x \in F\}\end{array}\right\} \Rightarrow (\mathcal{A} \subseteq \tau)(F \subseteq \bigcup \mathcal{A}) \\ F \text{ is } \theta\text{-compact}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(F \subseteq int(\bigcup\{cl(U) | U \in \mathcal{A}^*\})) \\ (A := \bigcup\{cl(U) | U \in \mathcal{A}^*\})(f \text{ is weakly } e^*\text{-closed})\end{array}\right\} \overset{\text{Theorem 3.3(5)}}{\Rightarrow}$$

$$\left.\begin{array}{r}\Rightarrow (\exists B \in e^*O(Y, y))(f^{-1}(y) \subseteq f^{-1}[B] \subseteq cl(X \setminus A) = X \setminus int(A) \subseteq X \setminus F) \\ f \text{ is bijective}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\exists B \in e^*O(Y, y))(B \cap f[F] = \emptyset)$$

$$\Rightarrow y \notin e^*\text{-}cl(f[F])$$

Then we have $e^*\text{-}cl(f[F]) = f[F]$. Therefore $f[F] \in e^*C(Y)$. ■

Definition 3.10 Two nonempty subsets A and B in X are strongly separated [20], if there exist open sets U and V in X with $A \subseteq U$ and $B \subseteq V$ and $cl(U) \cap cl(V) = \emptyset$. If A and B are singleton sets we may speak of points being strongly separated. We will use the fact that in a normal space, disjoint closed sets are strongly separated.

Definition 3.11 A space X is called e^*-T_2 [10, 11], if for every pair of distinct points x and y in X, there exist two e^*-open sets U and V such that $x \in U$ and $y \in V$ and $U \cap V = \emptyset$.

Theorem 3.12 If $f : (X, \tau) \to (Y, \sigma)$ is surjection weakly e^*-closed and all pairs of disjoint fibers are strongly separated, then Y is e^*-T_2.

Proof. Let $y, z \in Y$ and $y \neq z$.

$$\left.\begin{array}{r}(y,z\in Y)(y\neq z)\\(\forall y,z\in Y)(f^{-1}(y),f^{-1}(z)\text{ strongly seperated})\end{array}\right\}\Rightarrow$$
$$\Rightarrow \left.\begin{array}{r}(\exists U\in O(X,f^{-1}(y)))(\exists V\in O(X,f^{-1}(z)))(cl(U)\cap cl(V)=\emptyset)\\ f\text{ is weakly }e^*\text{-closed}\end{array}\right\}\overset{\text{Theorem 3.3 (5)}}{\Rightarrow}$$
$$\Rightarrow (\exists A\in e^*O(Y,y))(\exists B\in e^*O(Y,z))(f^{-1}[A]\cap f^{-1}[B]\subseteq cl(U)\cap cl(V)=\emptyset)$$
$$\Rightarrow \left.\begin{array}{r}(\exists A\in e^*O(Y,y))(\exists B\in e^*O(Y,z))(f^{-1}[A]\cap f^{-1}[B]=\emptyset)\\ f\text{ is surjective}\end{array}\right\}\Rightarrow$$
$$\Rightarrow (\exists A\in e^*O(Y,y))(\exists B\in e^*O(Y,z))(A\cap B=\emptyset).$$ ■

Corollary 3.13 If $f:(X,\tau)\to(Y,\sigma)$ is surjection weakly e^*-closed with all fibers closed and X is normal, then Y is e^*-T_2.

Corollary 3.14 If $f:(X,\tau)\to(Y,\sigma)$ is continuous surjection weakly e^*-closed with X compact T_2 space and Y a T_1 space, then Y is compact e^*-T_2 space.

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