Poisson Process with Fuzzy Parameter

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Article Info

Keywords
Fuzzy numbers
Poisson
Process
Integer valed
Parametric form.

Article History
Received: 2021 April 26
Accepted: 2021 June 11

Abstract

This paper, introduces the fuzzy poisson process that plays an important role in the fuzzy sets and systems; specially, The spatial Poisson point process features prominently in spatial statistics, stochastic geometry, and continuum percolation theory. This point process is applied in various physical sciences such as a model developed for alpha particles being detected. In recent years, it has been frequently used to model seemingly disordered spatial configurations of certain wireless communication networks. For example, models for cellular or mobile phone networks have been developed where it is assumed the phone network transmitters, known as base stations, are positioned according to a homogeneous Poisson point process.

1 Introduction

Since fuzzy set theory was initiated by Zadeh [8] in 1965, it has become a hot point of study in theory and applications to control theory, optimizations, intelligent systems, inferential statistics of the parametric and non-parametric, information sciences, and so on. Fuzzy sets were characteristic with their membership functions by Zadeh. The term fuzzy variables was first introduced by Kaufmann [4] in 1975, and then appeared in zadeh [9] and Nahmias [6] as fuzzy sets of real numbers. Possibility theory was presented by Zadeh [10], and developed by many reserchers such as Dubois and Prade [2]. In order to deal with the mathematics of fuzzy variables, fuzzy variables may be defined as function from possibility space to the set of real numbers. Li and et. have introduced a type Fuzzy Random Homogeneous Poisson Process [7].

In this paper, we will use the fuzzy variable as an interval rate instead of the point rate in the poisson process and then the fuzzy poisson process is introduced. This process can apply in queueing systems and many others systems operate on customer, e.g electron, photon, tumor cell, telephone call, data packet and etc. Therefore, the fuzzy poisson process can be as a prominent stochastic process, mainly because it frequently appears in a wealth of physical phenomena and because it is relatively simple to analyze.

2 Preliminaries

The basic definitions of a fuzzy number are given in [2, 8, 11] as follows **Definition 1.** Let $\mathbb{R}$ be the set of all real numbers. A fuzzy number is a fuzzy set like $A : \mathbb{R} \to I = [0, 1]$ that satisfies:

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1. A is upper semicontinuous ,
2. $A(x) = 0$ outside some interval $[a, d]$,
3. There are real numbers b, c such that $a \leq b \leq c \leq d$ and
   3.1 $A(x)$ is increasing on $[a, b]$,
   3.2 $A(x)$ is decreasing on $[c, d]$,
   3.3 $A(x) = 1, b \leq x \leq c.$

The membership function A can be expressed as

$$A(x) = \begin{cases} A_L(x) & x \in [a, b], \\ 1 & x \in [b, c], \\ A_R(x) & x \in [c, d], \\ 0 & otherwise, \end{cases} \qquad (I)$$

where $A_L : [a, b] \to [0, 1]$ and $A_R : [c, d] \to [0, 1]$ are left and right membership functions of fuzzy number A. A_L is real valued function that is increasing and right continues and A_R is a real valued function that is decreasing and left continues. Each fuzzy number A described by (I) has the following α-level sets $(\alpha - cuts)$:
$A_\alpha = [A_L^{-1}(\alpha), A_R^{-1}(\alpha)] = [\underline{a}(\alpha), \overline{a}(\alpha)]$ where $\underline{a}(\alpha), \overline{a}(\alpha) \in \mathbb{R}$, $\alpha \in [0, 1]$.
An equivalent parametric is
Definition 2. A fuzzy number A in parametric form is a pair $[\underline{a}(\alpha), \overline{a}(\alpha)]$ of function $\underline{a}(\alpha), \overline{a}(\alpha), 0 \leq \alpha \leq 1$, which satisfies the following requirements:

1. $\underline{a}(\alpha)$ is a bounded increasing left continuous function,
2. $\overline{a}(\alpha)$ is a bounded decreasing left continuous function,
3. $\underline{a}(\alpha) \leq \overline{a}(\alpha), 0 \leq \alpha \leq 1.$

We denote this family of fuzzy number by $\mathcal{F}$. A popular fuzzy number is the trapezoidal fuzzy number $A = (a, b, c, d)$ with membership function $A(x)$ that is defined as follows

$$A(x) = \begin{cases} \dfrac{x-a}{b-a} & x \in [a, b], \\ 1 & x \in [b, c], \\ \dfrac{d-x}{d-c} & x \in [c, d], \\ 0 & Otherwise. \end{cases}$$

Its α-level sets is:

$$[A]_\alpha = [\underline{a}(\alpha), \overline{a}(\alpha)] = [a + (b-a)\alpha, d - (d-c)\alpha].$$

Note that if $b = c$, then $A(x)$ is membership function of triangular fuzzy number $A = (a, b, d)$. Support function is defined as follows:

$$supp(A) = \overline{\{x | A(x) > 0\}}$$

which $\overline{\{x|A(x) > 0\}}$ is closure of set $\{x|A(x) > 0\}$. The addition and scalar multiplication of fuzzy numbers are defined by Zadeh's extension principle [10] and can be equivalently represented in [2, 8, 11] as follows:
For arbitrary $[A]_\alpha = [\underline{a}(\alpha), \overline{a}(\alpha)]$, $[B]_\alpha = [\underline{b}(\alpha), \overline{b}(\alpha)]$ and $k > 0$ we define addition $(A + B)$ and multiplication by scaler k as:

$$(\underline{a+b})(\alpha) = \underline{a}(\alpha) + \underline{b}(\alpha), \ \ (\overline{a+b})(\alpha) = \overline{a}(\alpha) + \overline{b}(\alpha), \tag{2.1}$$

$$(\underline{ka})(\alpha) = k\underline{a}(\alpha), \ \ (\overline{ka})(\alpha) = k\overline{a}(\alpha). \tag{2.2}$$

Definition 3. A Poisson process with parameter or rate $\lambda > 0$ is an integer-valued, continuous time stochastic process $\{N(t), t > 0\}$ satisfying

(***i***) $N(0) = 0$.

(***ii***) for all $t_0 < t_1 < ... < t_n$, the increment $N(t_1) - N(t_0), N(t_2) - N(t_1), ..., N(t_n) - N(t_{n-1})$ are independent random variables.

(***iii***) for $t > 0$ and non-negative integers k, the increments have the Poisso distribution

$$Pr[N(t+s) - N(s) = k] = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \tag{2.3}$$

It is convenient to view the Poisson process $N(t)$as a special counting process, where the number of event in any interval of length t is specified via condition (**iii**). From this definition, a number of properties can be derived:

(a) Condition (**iii**) implies that the increments are stationary because the right-hand side dose not on s. In other words, the increments only depend on the length of the interval t and not on the time s when the interval begins. Further, the mean $E[N(t+s) - N(s)] = \lambda t$ and because the increments are stationary, this holds for any value of s. In particular with $s = 0$ and condition (**i**), the expected number of events in a time interval with length t is $E[N(t)] = \lambda t$, and this relation explains why λ is called the rate of the Poisson process, namely, the derivative over time t or the number of events per time unit.

(b) The probability that exactly one event occurs in an arbitrarily small time interval of length h fellows from condition (iii) as

$$Pr[N(s+h) - N(s) = 1] = \lambda h + o(h) \tag{2.4}$$

while the probability that no even occurs in an arbitrarily small time interval of length h is

$$Pr[N(s+h) - N(s) = 1] = 1 - \lambda h + o(h) \tag{2.5}$$

Similarly, the probability that more than one event occers in an arbitrarily small time interval of length h is

$$Pr[N(s+h) - N(s) > 1] = o(h) \tag{2.6}$$

where $\lim_{h\to 0} \frac{o(h)}{h} = 0$.

Theorem 1. [5] A counting process $N(t)$ that satisfies the condition (i) $N(0) = 0$, (ii) the process $N(t)$ has stationary and independent increments, (**iii**) $Pr[N(h) = 1] = \lambda h + o(h)$ and (iv) $Pr[N(h) > 1] = o(h)$ is a Poisson process with rate $\lambda > 0$.

3 Fuzzy poisson process

In this section we introduce the fuzzy poisson process, In fact we will suppose that the parameter $\lambda > 0$ is a positive fuzzy number as $\lambda(\alpha) = [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)] = [\underline{\lambda}, \overline{\lambda}], \alpha \in [0, 1]$.

Definition 4. A fuzzy poisson process with fuzzy parameter or rate $\lambda(\alpha) = [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)]$ is an interval-valued, continuous time stochastic process $\{\tilde{N}(t), t > 0\}$ satisfying

(*i*) $\tilde{N}(0) = 0$.

(*ii*) for all $t_0 < t_1 < ... < t_n$, the increment $N(t_1) - N(t_0), N(t_2) - N(t_1), ..., N(t_n) - N(t_{n-1})$ are independent random variables.

(*iii*) for $t > 0$ and non-negative integers k, the increments have the fuzzy poisso distribution

$$\tilde{Pr}_\alpha[N(t+s) - N(s) = k] = [\underline{P}r_\alpha, \overline{P}r_\alpha] \tag{3.1}$$

where for any $\alpha \in [0,1]$

$$\underline{P}r_\alpha = \frac{(\underline{\lambda}(\alpha)t)^k e^{-\overline{\lambda}(\alpha)t}}{k!}, \overline{P}r_\alpha = \frac{(\overline{\lambda}(\alpha)t)^k e^{-\underline{\lambda}(\alpha)t}}{k!} \tag{3.2}$$

note that

$$\underline{P}r_\alpha = min\{\tilde{Pr}_\alpha[N(t+s) - N(s) = k] \mid \alpha \in [0,1]\},$$

$$\overline{P}r_\alpha = max\{\tilde{Pr}_\alpha[N(t+s) - N(s) = k] \mid \alpha \in [0,1]\}$$

It is convenient to view the fuzzy poisson process $N(t)$as a special interval process, where the number of event in any interval of length t is specified via condition (iii). From this definition, a number of properties can be derived:

(á) Condition (iii) implies that the increments are stationary because the right-hand side dose not on s. In other words, the increments only depend on the length of the interval t and not on the time s when the interval begins. Further, the interval-valued mean $E[N(t+s) - N(s)] = [\underline{\lambda}(\alpha)t, \overline{\lambda}(\alpha)t]$ and because the increments are stationary, this holds for any value of s. In particular with $s = 0$ and condition (i), the expected number of events in a time interval with length t is $E[N(t)] = [\underline{\lambda}(\alpha)t, \overline{\lambda}(\alpha)t]$, and this interval shows that the interval $[\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)]$ is called the interval rate of the fuzzy poisson process, namely, the derivative over time t or the number of events per time unit in an interval.

(b´) The probability that exactly one event occurs in an arbitrarily small time interval of length h fellows from condition (iii) as

$$\widetilde{Pr}[N(s+h) - N(s) = 1] = [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)]h + o(h) \tag{3.3}$$

while the probability that no even occurs in an arbitrarily small time interval of length h is

$$\widetilde{Pr}[N(s+h) - N(s) = 0] = 1 - [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)]h + o(h) \tag{3.4}$$

Similarly, the probability that more than one event occers in an arbitrarily small time interval of length h is

$$\widetilde{Pr}[N(s+h) - N(s) > 1] = o(h) \tag{3.5}$$

where $\lim_{h\to 0} \frac{o(h)}{h} = 0$.

Theorem 2. A counting process $N(t)$ that satisfies the condition (i) $N(0) = 0$, (ii) the process $N(t)$ has stationary and independent increments, (iii) $\widetilde{Pr}[N(h) = 1] = [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)]h + o(h)$ and (iv) $\widetilde{Pr}[N(h) > 1] = o(h)$ is a fuzzy poisson process with interval rate $[\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)]$.

Prof: We must show conditions (iii) and (iv) are equal to condition (iii) in the definition of the fuzzy poisson process. Denote $\widetilde{P}_n(t) = \widetilde{P}r[N(h) = n]$ and consider first the case $n = 0$, then

$$\widetilde{P}_0(t+h) = \widetilde{P}r[N(t+h) = 0] = \widetilde{P}r[N(t+h) - N(t) = 0, N(t) = 0] \tag{3.6}$$

Invoking independence via (ii)

$$\widetilde{P}_0(t+h) = \widetilde{P}r[N(t+h) - N(t) = 0]\widetilde{P}r[N(t) = 0] \tag{3.7}$$

By definition, $\widetilde{P}_0(t) = \widetilde{P}r[N(t) = 0]$ and from (iii), (iv) and the fact $\sum_{k=0}^{\infty} \widetilde{P}r[N(h) = k] = \widetilde{1} = [1, 1]$, it follows that

$$\widetilde{Pr}[N(h) = 0] = \widetilde{1} - [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)]h + o(h) \tag{3.8}$$

Combing these with the stationarity in (ii), we obtain

$$\widetilde{P}_0(t+h) = \widetilde{P}_0(t)(\widetilde{1} - [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)]h + o(h))$$

or

$$\begin{aligned} \tfrac{\underline{P}_0(t+h) - \underline{P}_0(t)}{h} &= -\overline{\lambda}\underline{P}_0(t) + \tfrac{o(h)}{h} \\ \tfrac{\overline{P}_0(t+h) - \overline{P}_0(t)}{h} &= -\underline{\lambda}\overline{P}_0(t) + \tfrac{o(h)}{h} \end{aligned}$$

from which, in the limit $h \to 0$, the differential equations

$$\begin{aligned} \underline{P}_0^{'}(t) &= -\overline{\lambda}\underline{P}_0(t) \\ \overline{P}_0^{'}(t) &= -\underline{\lambda}\overline{P}_0(t) \end{aligned}$$

are immediate. The solution is $\underline{P}_0(t) = C_1 e^{-\overline{\lambda}t}$ and the integration constant C_1 follows from (i) and $\underline{P}_0(0) = \underline{P}r[N(0) = 0] = 1$ as $C_1 = 1$. Similarly, $\overline{P}_0(t) = e^{-\underline{\lambda}t}$. These establish condition (iii) in the definition of fuzzy poisson process for $k = 0$.

The verification for $n > 0$ is more involved. Applying the law of total probability $\widetilde{P}_n(t+h) = \widetilde{P}r[N(t+h) = n]$

$= \sum_{k=0}^{n} \widetilde{P}r[N(t+h) - N(t) = k | N(t) = n-k]\widetilde{P}r[N(t) = n-k]$ By independence (ii),

$$\widetilde{P}r[N(t+h) - N(t) = k | N(t) = n-k]\widetilde{P}r[N(t) = n-k] = \widetilde{P}r[N(t+h) - N(t) = k]$$

and by definition $\widetilde{P}r[N(t) = n-k] = \widetilde{P}_{n-k}(t)$, we have

$$\widetilde{P}_n(t+h) = \sum_{k=0}^{n} \widetilde{P}r[N(t+h) - N(t) = k | N(t) = n-k]\widetilde{P}_{n-k}(t)$$

By the stationarity (ii)

$$\widetilde{P}r[N(t+h) - N(t) = k] = \widetilde{P}r[N(h) - N(0) = k]$$

we obtain using (i)

$$\widetilde{P}_n(t+h) = \sum_{k=0}^{n} \widetilde{P}r[N(h)=k]\widetilde{P}_{n-k}(t)$$

while (v) and (iii) suggest to write the sum as

$$\widetilde{P}_n(t+h) = \widetilde{P}_n(t)\widetilde{P}r[N(h)=0] + \widetilde{P}_{n-1}(t)\widetilde{P}r[N(h)=1] + \sum_{k=2}^{n} \widetilde{P}_{n-k}(t)\widetilde{P}r[N(h)=k]$$

Since $\widetilde{P}_n(t) \leq \widetilde{1}$ and using (iv),

$$\sum_{k=2}^{n} \widetilde{P}_{n-k}(t)\widetilde{P}r[N(h)=k] \leq \sum_{k=2}^{n} \widetilde{P}r[N(h)=k] = \widetilde{P}r[N(h)>1] = o(h)$$

we arrive with (v),(iii) at

$$\widetilde{P}_n(t+h) = \widetilde{P}_n(t)(\widetilde{1} - [\underline{\lambda}, \overline{\lambda}]h + o(h)) + \widetilde{P}_{n-1}(t)([\underline{\lambda}, \overline{\lambda}]h + o(h)) + o(h)$$

or

$$\begin{aligned} \tfrac{\underline{P}_n(t+h)-\underline{P}_n(t)}{h} &= -\overline{\lambda}\,\underline{P}_n(t) + \underline{\lambda}\,\underline{P}_{n-1}(t) + \tfrac{o(h)}{h} \\ \tfrac{\overline{P}_n(t+h)-\overline{P}_n(t)}{h} &= -\underline{\lambda}\,\overline{P}_n(t) + \overline{\lambda}\,\overline{P}_{n-1}(t) + \tfrac{o(h)}{h} \end{aligned}$$

which leads, after taking the limit $h \to 0$, to the differential equations

$$\begin{aligned} \underline{P}'_n(t) &= -\overline{\lambda}\underline{P}_n(t) + \underline{\lambda}\,\underline{P}_{n-1}(t) \\ \overline{P}'_0(t) &= -\underline{\lambda}\overline{P}_0(t) + \overline{\lambda}\,\overline{P}_{n-1}(t) \end{aligned}$$

with initial conditions $\underline{P}_n(0) = \underline{P}r[N(0)=n] = 1_{n=0}$ and $\overline{P}_n(0) = \overline{P}r[N(0)=n] = 1_{n=0}$. These differential equations are rewritten as

$$\frac{d}{dt}[e^{\overline{\lambda}t}\underline{P}_n(t)] = \underline{\lambda}e^{\overline{\lambda}t}\underline{P}_{n-1}(t) \tag{3.9}$$

$$\frac{d}{dt}[e^{\underline{\lambda}t}\overline{P}_n(t)] = \overline{\lambda}e^{\underline{\lambda}t}\overline{P}_{n-1}(t) \tag{3.10}$$

In case $n=1$, the differential equation reduces with $\underline{P}_0(t) = e^{-\overline{\lambda}t}$ to $\frac{d}{dt}[e^{\underline{\lambda}t}\overline{P}_1(t)] = \lambda$. The general solution is $e^{\underline{\lambda}t}\overline{P}_1(t)] = \lambda t + C_1$. and, from the initial condition $\underline{P}_1(0) = 0$, we have $C_1 = 0$ and $\underline{P}_1(t) = \underline{\lambda}te^{-\overline{\lambda}t}$. The general solution to (15) is proved by induction. Assume that $\underline{P}_n(t) = \frac{(\underline{\lambda}t)^n e^{-\overline{\lambda}t}}{n!}$ holds for n, then the case $n+1$ follows from (15) as

$$\frac{d}{dt}[e^{\overline{\lambda}t}\underline{P}_{n+1}(t)] = \frac{\underline{\lambda}(\underline{\lambda}t)^n}{n!}$$

and integrating from 0 to t using $\underline{P}_{n+1}(0) = 0$, yields

$$\underline{P}_{n+1}(t) = \frac{(\underline{\lambda} t)^{n+1} e^{-\overline{\lambda} t}}{(n+1)!} \tag{3.11}$$

Similarly from (16) yields

$$\overline{P}_{n+1}(t) = \frac{(\overline{\lambda} t)^{n+1} e^{-\underline{\lambda} t}}{(n+1)!} \tag{3.12}$$

which establishes the induction and finalizes the prof of the theorem.□

The next theorem has very important applications since it relates the number of events in non-overlapping intervals to the interval time between these events.

Theorem 3. Let$\{N(t); t \geq 0\}$ be a fuzzy poisson process with interval rate $\lambda(\alpha) = [\underline{\lambda}(\alpha), \overline{\lambda}(\alpha)] = [\underline{\lambda}, \overline{\lambda}] > 0$ and denote by $t_0 < t_1 < t_2 < \ldots$ the successive occurrence times of events. Then the interrarrival times $\tau_n = t_n - t_{n-1}$ are independent identically distributed exponential random fuzzy variable with interval mean

$$IE(\tau_n) = \frac{1}{[\underline{\lambda}, \overline{\lambda}]} = [\frac{1}{\overline{\lambda}}, \frac{1}{\underline{\lambda}}]$$

Proof: For any $h \geq 0$ and any $n \geq 1$, the event $\{\tau_n > h\}$ is equivalent to the event $\{N(t_{n-1}) - N(t_{n-1}) = 0\}$. Indeed, the $n-$th interval time τ_n can only be longer than h time units if and only if the $n-$th event has not yet occurred h time units after the occurrence of the $(n-1)-$th event at t_{n-1}. Since the fuzzy poisson process has independent increments (condition (ii) in the definition of the fuzzy poisson process), changes in the value of the process in non-overlapping time intervals are independent. By the equivalence in events, this implies that the set of interarrival times τ_n are independent random fuzzy variables. Further, by the stationarity of the fuzzy poisson process

$$\begin{aligned}\widetilde{P}r[\tau_n > h] &= \widetilde{P}r[N(t_{n-1}+h) - N(t_{n-1}) = 0] \\ &= [min\{e^{-\underline{\lambda} h}, e^{-\overline{\lambda} h}\}, max\{e^{-\underline{\lambda} h}, e^{-\overline{\lambda} h}\}] = [e^{-\overline{\lambda} h}, e^{-\underline{\lambda} h}]\end{aligned} \tag{3.13}$$

which implies that any interarrival time has an identical, fuzzy exponential distribution,

$$\widetilde{F}_{\tau_n}(x) = \widetilde{P}r[\tau_n \leq x] = [\underline{P}r(\tau_n \leq x), \overline{P}r(\tau_n \leq x)] = [\underline{F}_{\tau_n}(x), \overline{F}_{\tau_n}(x)] =$$

$$[min\{1 - e^{-\underline{\lambda} x}, 1 - e^{-\overline{\lambda} x}\}, max\{1 - e^{-\underline{\lambda} x}, 1 - e^{-\overline{\lambda} x}\}] = [1 - e^{-\underline{\lambda} x}, 1 - e^{-\overline{\lambda} x}] \tag{3.14}$$

This proves the theorem. □

Example. A conversation in a wireless ad-hoc network is severely disturbed by interference signals according to a fuzzy poisson process with the interval rate $\lambda(\alpha) = [0.05 + 0.05\alpha, 0.15 - 0.05\alpha]$ per minute. We obtain the probability that no interference signals occur within the first two minutes of the conversation. Let $N(t)$ denote the fuzzy poisson interference process, then $\widetilde{P}r[N(2) = 0]$ needs to be computed. Hence, for any $\alpha \in [0, 1]$ we have

$$\widetilde{P}r_\alpha[N(2) = 0] = \widetilde{P}r_\alpha[N(2) - N(0) = 0] = [e^{-2(0.15-0.05\alpha)}, e^{2(0.05+0.05\alpha)}]$$

Also, given that the first two minutes are free of disturbing effects, the probability that in the next minute precisely

1 interfering signal disturbs the conversation is

$$\widetilde{Pr}_\alpha[N(3)-N(2)=1]=[(0.05+0.05\alpha)e^{-(0.15-0.05\alpha)},(0.15-0.05\alpha)e^{-(0.05+0.05\alpha)}]$$

Note that the events during two non-overlapping intervals of a fuzzy poisson process are independent. Thus the event $\{N(2)-N(0)=0\}$ is independent from the event $\{N(3)-N(2)=1\}$ which means that the asked condition probability

$$\widetilde{Pr}_\alpha[N(3)-N(2)=1|N(2)-N(0)=0]=\widetilde{Pr}_\alpha[N(3)-N(2)=1].$$

α	$\widetilde{Pr}_\alpha[N(2)-N(0)=0]$	$\widetilde{Pr}_\alpha[N(3)-N(2)=1]$
$\alpha=0$	[0.0741 0.0905]	[0.0430 0.1427]
$\alpha=0.2$	[0.7558 0.8869]	[0.0523 0.1318]
$\alpha=0.4$	[0.7711 0.8693]	[0.0615 0.1212]
$\alpha=0.5$	[0.7788 0.8607]	[0.0662 0.1160]
$\alpha=0.7$	[0.7945 0.8437]	[0.0758 0.1056]
$\alpha=0.9$	[0.8106 0.8269]	[0.0855 0.0955]
$\alpha=1$	[0.8187 0.8187]	[0.0905 0.0905]

Table 1: Interval probabilities for events

4 Conclusion

the paper introduced the poisson process with the fuzzy parameter that is called the fuzzy poisson process. This fuzzy process plays an important role in a wealth of physics phenomena. We also have stated the two theorems for the fuzzy poisson process and its relation with an exponential distribution with the interval parameter. One can this method develop on the nonhomogeneous poisson process and its properties mention in the future researches.

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