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A Novel Solving Method for Multi-Objective Decision Making Problems Under Fuzzy Conditions

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Abstract

This paper proposes a satisfying optimization method for fuzzy multiple objective optimization problem. Actually, the presented method realizes the trade-off between optimization and fuzzy importance requirement. Generally, the main aim of the presented approach is to make the more important objective achieving the higher desirable satisfying degree. In practice, vagueness and imprecision of the goals, constraints and parameters in this problem make the decision-making complicated for decision makers who have to deal with the parameters to make the optimized decision. Hence, the reformulated optimization models based on goal programming is proposed for different fuzzy relations and fuzzy importance. In fact, decision makers can select the appropriate alternative considering their determinations from variety of solutions using parameter λ. Applying the proposed model, not only the satisfying results of all the objectives can be acquired, but also the fuzzy importance requirement can be simultaneously actualized. In addition, a numerical example is provided to illustrate how the model is applied. Finally, the conclusions and recommendations are presented.

Keywords:
Goal Programming
Multiple Objective Optimization
Relative Importance
Satisfying Optimization

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INTRODUCTION

In areal situation for optimization problems, many input information are not known precisely. As a result, the value of many criteria and constraints are expressed in vague terms such as "very high in quality "or" low in price” at the time of making decisions. Deterministic models cannot easily take this vagueness into account. In these cases the theory of fuzzy sets can be considered as one of the best tools for handling uncertainty. Fuzzy set theories are employed due to the presence of vagueness and imprecision of information in the linear optimization problem. In1970, Bellman and Zadeh suggested a fuzzy programming model for decision-making in fuzzy environments. Zimmermann in 1978 first used the Bellman and Zadeh method to solve fuzzy multi objective linear programming problems. It is very common in business activities, such as supplier selection, that the goals importance or weights are different for decision makers. Thus, the symmetrical models may not be appropriate for the same multi objective decision-making problem, because the objectives may not be equally important. In actual decision making situations, a major concern is that most decision problems involve multiple criteria (attributes or objectives). During the recent years, multiple objective decision making (MCDM) problem as the crucial part of multiple criteria decision making (MCDM) has become a promising field, and attracted more and more researchers (Steuer, 1986). Fuzzy methodology has been exploited for solving a decision-making problem involving a multiplicity of objectives and selection criteria for "best" compromised solution. The ‘best’ compromised solution is the one which provides the maximum satisfaction level from the membership function of the participating goals or objectives. In 1974, Tanaka et al. initially proposed the concept of fuzzy mathematical programming and in 1987, Zimmermann formulated fuzzy linear programming with several objectives. Moreover, the solution of multiple objective optimization problems is dependent upon the decision maker's preference. This can be represented by relative importance and priority (Lin, 2004; Tiwari et al., 1986; Tiwari et al., 1987) besides an explicit utility function or progressive articulation in actual decision making (Sakawa, 1987; Sakawa, 2004; Yang, 2000; Yang, 1996). In 1987, Sakawa et al. first considered several kinds of multi criteria linear optimal control problems through the application of a multi criteria simplex method. They considered a fuzzy programming approach for solving multi criteria (multi objective) linear optimal control problems by assuming that the decision maker (decision maker) may have a fuzzy goal for each of the objective functions and showed that the satisfying solution for the decision maker can be obtained through the simplex method of linear programming. In reality, however, it is difficult to specify the weights for decision maker since he or she might has vague or imprecise knowledge about these objectives, constraints and the environment in advance. For assessing the fuzzy importance of the objectives, Narasimhan (1980) has used linguistic terms, such as "very important" and "moderately important". Chen and Tsai (2001) distinguished the relative importance between objectives through determining a desirable achievement degree for each objective. That is the more important the objective, the higher the desirable achievement degree. In order to express the fuzzy importance relations in 2007, Akoz and Petrovic defined three types of fuzzy binary relations for the different linguistic terms, such as "slightly more important than", "moderately more important" and "significantly more important than". Goal Programming (GP) (Charnes and cooper, 1961; Ijiri, 1965; Narasimhan, 1980; Pal and Moitra, 2003) as the most promising methodologies for MCDM, has been utilized in real world decision making problems .GP, initially introduced by Charnes and Cooper in 1961 is used to consider all the objectives with different attainment relations in finding an acceptable solution through minimizing the deviations from the expected values. In 2008, Mavrotas et al., transformed a fuzzy programming model into an equivalent multi-objective problem to provide an integrated optimization model for energy planning, where the minimization of cost and the maximization of demand satisfaction were the objective functions. After one year, Kahraman et al. used an axiomatic design and multi-attribute decision making approaches for ranking the best renewable energy alternatives under fuzzy environment. In 2009, Cai et al. presented an integrated integer linear optimization model based on fuzzy-stochastic programming for energy management planning in order to generate decision alternatives and also helping decision makers identify proper policies regarding various economic and system-reliability constraints. In 2010, Daim et al. used a fuzzy goal programming model to accommodate changes in energy costs and future advances in technology maturity. Also Li et al. in 2010, studied an inexact fuzzy-stochastic energy model under multiple uncertainties for planning energy and environmental systems management. The model can cope with uncertainties described as fuzzy sets, interval values and probabilities distributions. A fuzzy mixed integer programming model was developed by Jinturkar and Deshmukh in 2011 for cooking and heating energy planning via considering local sources use, environmental and economic scenarios and also trade-off between them. in 2011, Kaya and Kahraman proposed a modified fuzzy multi-attribute decision approach for the selection of the best energy technology alternative. The weights of the selection criteria were determined by fuzzy pair-wise comparison matrices. In 2013, Alikhani and Azar used a combined stochastic goal programming model under fuzzy environment for gas resources quota allocation. The method draws upon the existing chance constrained programming and triangular fuzzy numbers by allowing analysis on trade-offs among objective functions and the risk of violating constraints that comprise uncertain parameters. In 2014, Sarrafha et al. presented a bi-objective mixed integer linear programming model for integrating production-distribution which aims to minimize total net costs in supply chain and transfer time of products for retailers simultaneously. The paper proposes a Pareto-based metaheuristic algorithm called multi-objective simulated annealing (MOSA) to solve the problem. In 2015, Alikhani,R. and Azar, A., proposed an integrated satisfying optimization approach based on Fuzzy Goal Programming (FGP) and Logarithmic Fuzzy Preference Programming (LFPP) in which the goal was to handle the uncertainties and vagueness of input data and different importance preferences among fuzzy constraints and weights of objectives effectively due to the above-mentioned problem. The proposed model was applied to a real case study of sustainable gas resources allocation in Iran. In 2016, Alinejad introduced a data envelopment analysis (DEA) model combined with Bootstrapping to assess performance of one of the Data mining Algorithms. The paper applied a two-step process for performance productivity analysis of insurance branches within a case study. The study analyzes the productivity of eighteen decision-making units (DMUs) using a DEA model. In 2016, Amini and Alinejad presented combined evaluation method to rank alternatives based on VIKOR and DEA with BELIEF structure under uncertainty. The paper processes a combined method, based on VIKOR and Data Envelopment Analysis (DEA) to select the units with most efficiency. VIKOR is utilized as compromise solution method. This research is a two-stage model designed to fully rank the alternatives, where each alternative has multiple inputs and outputs. The problem involves BELIEF parameters in the solution procedure.

In 2016, Tabrizi et al. presented an interactive fuzzy satisfying method for multi-objective function in reactive power market. In the method, the fuzzy goals are quantified by defining their corresponding membership functions and the decision maker is then asked to specify the desirable membership values. In 2017, Majidi et al. provided a multi-objective model for optimal operation of a battery/PV/fuel cell/grid hybrid energy system using weighted sum technique and fuzzy satisfying approach. The proposed model was solved by weighted sum technique and the best possible solution is selected by employing fuzzy satisfying approach. In 2018, Nouri et al. used a fuzzy satisfying approach for optimal performance of fuel cell-CHP-battery based micro-grid under real-time energy management. In 2019, Su and Wu applied a fuzzy multi-objective decision system for recoverable remanufacturing planning. The paper develops a novel fuzzy multi-objective decision system (FMODS) that uses the Simplex method, access database technology and the JAVA programming language for recoverable product of remanufacturing planning. In 2020, Çakır and Ulukan presented a fuzzy multi-objective decision making approach for nuclear power plant installation. Their proposed model attempts to minimize total duration time, total cost and maximize total crash time of the installation project. A numerical example was applied to demonstrate the feasibility of the proposed models to nuclear power plant installation problem. In 2020, Biuki et al. introduced a model of integrated location, routing, and inventory problem, the three key problems in optimizing a logistics system, is introduced. Considering the particular decision-making environment of such industries, a two-phase approach to incorporate the three dimensions of sustainability into supply chain practices is presented. After identifying more sustainable-oriented suppliers, a problem is formulated as a multi-objective Mixed-Integer Programming (MIP) model to assist in planning a sustainable supply chain. In 2020, Alinejad and Taherinezhad proposed a control chart recognition patterns using fuzzy rule-based system which is developed for $\bar{X}$ control charts to prioritize the control chart causes based on the accumulated evidence.

In this paper, the Pareto solutions in multi objective model are generated by transforming the multi-objective optimization problem into a single-objective problem and the optimal solution is chosen by implementing fuzzy satisfying approach. Applying the presented model, the satisfying results of all the objectives can be acquired and also, decision makers can benefit from actualization of fuzzy importance requirement simultaneously which can distinguish the proposed model.

Accordingly, we introduce a satisfying optimization method based on goal programming in this paper. It is adapted to solve the optimization problems with the above three types of fuzzy relations. Following the more important objective achieving the higher desirable satisfying degrees, fuzzy multiple objective optimization problem is reformulated. In the new model, both of all the desirable achievement degrees and the importance difference between the objectives are maximized by ranking the desirable satisfying degrees under the interaction with decision maker. The results of all the objectives are not only satisfying to decision maker, but also consistent with his or her fuzzy preference. The trade-off between optimization and importance requirement is realized. In other words, Not only the satisfying results of all the objectives and constraints can be acquired, but also the fuzzy importance requirement can be simultaneously actualized. It can be used in many real-world decision maker problems. In other words in many real-world decision maker problems, a fuzzy satisfying approach is required to satisfy decision makers and also, be consistent with his or her fuzzy preference. Hence, in the new model, both of all the desirable achievement degrees and the importance difference between the objectives are maximized by ranking the desirable satisfying degrees under the interaction with decision maker. Applying the proposed model, decision makers will be able to find the appropriate alternative considering to their intentions from various solutions by using and regulating parameter λ.

This paper is organized as follows: In Section 2, fuzzy multiple objective optimization problems are described. Section 3 is about modeling the fuzzy objective functions and fuzzy constraints using goal programming. Section 4 presents the satisfying optimization method. The optimization algorithm is provided in Section 5. The efficiency, flexibility and sensitivity of the proposed optimization approach and obtain the minimum parameter for limit case are demonstrated by the numerical examples in Section 6. Section 7 makes the conclusions.

FUZZY MULTIPLE OBJECTIVE OPTIMIZATION PROBLEM

Multiple objective optimization problem

A multiple objective optimization problem can, in general, be represented as follows:

$$\begin{cases} opt\ \left(f_1(x), f_2(x), \dots, f_k(x)\right) \\ \quad s.t:\ \ x \in G \end{cases} \qquad (1)$$

Where "opt" denotes minimization or maximization; x=(x1, x2,…, xn) is decision vector; $f_i(x), (i = 1, \dots, k)$ are multiple objectives to be optimized; and $G \subset R^n$ involves system constraints.

Fuzzy multiple objective optimization problem

Find: x

So as to satisfy:

$$f_i(x).\begin{pmatrix} \lesssim \\ \cong \\ \gtrsim \end{pmatrix}.f_i^*\ \ \mathrm{i} = 1,2, \dots, k$$

Subject to:

$$\begin{cases} \widetilde{\mathrm{G}} = \mathrm{g_r(x)}.\begin{pmatrix} \lesssim \\ \cong \\ \gtrsim \end{pmatrix}.\mathrm{b}_{\mathrm{r}}^{\sim}\ r = 1,2, \dots, h\ (\text{ fuzzy }) \\ G = g_p.\begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix}.b_p\ p = h+1, \dots, \mathrm{m(deterministic)} \end{cases} \qquad (2)$$

Where f_i^* is the perspective goal value for the objective function $f_i(x)$;'$\lesssim$ ',' $\gtrsim$ ' and ' $\cong$ ' express different fuzzy relations. For decision makers, the three types of fuzzy relations, respectively, denote that the i-th fuzzy objective is approximately less than or equal to, approximately more than or equal to, and in the vicinity of f_i^* .

For multiple objective optimization problem in fuzzy environment, many researchers used the theory of fuzzy set. There are various kinds of membership functions such as linear, exponential, hyperbolic, hyperbolic-inverse, and piecewise-linear functions (Sakawa et al., 1987). The triangle-like membership functions are usually used for the objectives and their perspective goal values in literatures (Tiwari et al., 1987; Zimmermann, 1978). The corresponding membership functions are defined for three types of fuzzy relations in this paper.

For the fuzzy relation' $\lesssim$ ', the tolerant interval for the fuzzy objective is regarded as (f_i^*, f_i^{max}). f_i^{max} is the tolerant limit for $f_i(x)$, as shown in Fig. 1.

Therefore the membership function is defined as:

$$\mu_{f_i(\mathrm{x})} = \begin{cases} 1 & \mathrm{f_i}(x) \leq f_i^* \\ 1 - \frac{\mathrm{f_i(x)} - \mathrm{f_i^*}}{\mathrm{f_i^{max}} - \mathrm{f_i^*}} & \mathrm{f_i^*} \leq \mathrm{f_i(x)} \leq \mathrm{f_i^{max}} \\ 0 & \mathrm{f_i(x)} \geq \mathrm{f_i^{max}} \end{cases} \qquad (3)$$

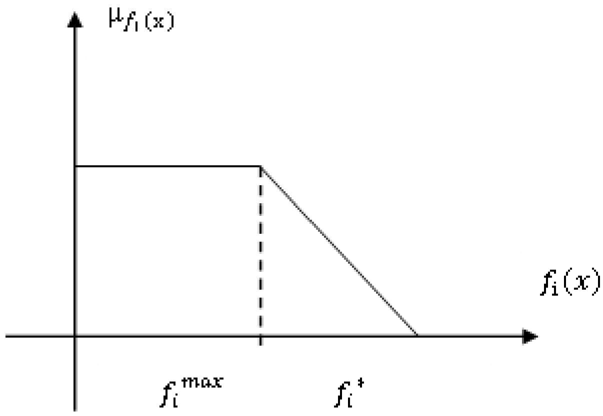

Fig. 1. Membership function $\mu_{f_i(x)}$ for fuzzy relation $'\lesssim'$

The tolerant interval for ' $\gtrsim$ ' which can be accepted by DECISIN MAKERis $(f_i^{min}, f_i^*$).Fig. 2. illustrates the graph of this fuzzy relation. The membership function can take the form:

$$\mu_{f_i(\mathrm{x})} = \begin{cases} 1 & \mathrm{f_i}(x) \geq f_i^* \\ 1 - \frac{\mathrm{f_i^*} - \mathrm{f_i(x)}}{\mathrm{f_i^*} - \mathrm{f_i^{min}}} & \mathrm{f_i}^{min} \leq \mathrm{f_i(x)} \leq \mathrm{f_i}^* \\ 0 & \mathrm{f_i(x)} \leq \mathrm{f_i^{min}} \end{cases} \qquad (4)$$

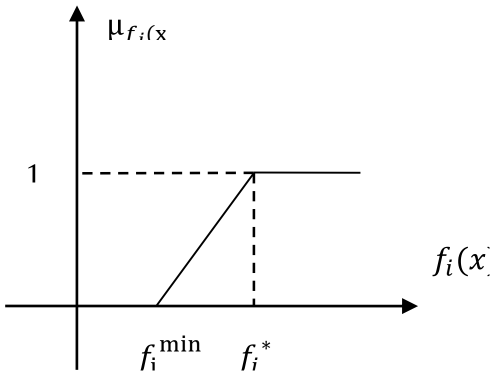

Fig. 2. Membership function $\mu_{f_i(\mathrm{x})}$ for fuzzy relation $'\gtrsim'$

(f_i^{min}, f_i^{max}) is the tolerant interval for fuzzy relation'≅' (see Fi. 3).
The membership function can be expressed as follows:

$$\mu_{f_i(\mathrm{x})} = \begin{cases} 0 & \mathrm{f_i(x)} \geq \mathrm{f_i}^{\max} \\ 1 - \frac{\mathrm{f_i(x)} - \mathrm{f_i}^*}{\mathrm{f_i}^{\max} - \mathrm{f_i}^*} & \mathrm{f_i}^* \leq \mathrm{f_i(x)} \leq \mathrm{f_i}^{\max} \\ 1 & \mathrm{f_i(x)} = \mathrm{f_i}^* \\ 1 - \frac{\mathrm{f_i}^* - \mathrm{f_i(x)}}{\mathrm{f_i}^* - \mathrm{f_i}^{\min}} & \mathrm{f_i}^{\min} \leq \mathrm{f_i(x)} \leq \mathrm{f_i}^* \\ 0 & \text{Otherwise} \end{cases} \quad (5)$$

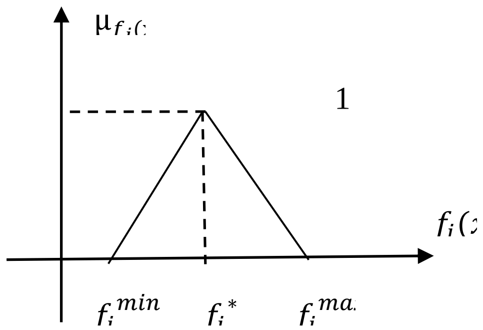

Fig. 3. Membership function $\mu_{\mathrm{f_i(x)}}$ for fuzzy relation $'\cong'$

MODELING THE FUZZY OBJECTIVE FUNCTIONS AND FUZZY CONSTRAINTS USING GOAL PROGRAMMING

For'$\leq \sim$', supposing that the values of objectives are in their tolerant ranges, the new formulation is:

$$f_i(x) - p_i = f_i^*, \mathrm{i} = 1,2,\dots,\mathrm{k}\ , (6)$$

The membership function $\mu_{f_i(\mathrm{x})}$ is converted into

$$\mu_{f_i(\mathrm{x})} = 1 - \frac{p_i}{(\mathrm{f_i}^{\max} - \mathrm{f_i^*})}\ , \qquad (7)$$

Where $p_i (p_i \geq 0)$ is positive deviational variable. And the formulation for '$\geq \sim$' is:

$$f_i(x) + n_i = f_i^*, \mathrm{i} = 1,2,\dots,\mathrm{k}\ , \qquad (8)$$

The corresponding conclusion to the membership function $\mu_{f_i(\mathrm{x})}$ is:

$$\mu_{f_i(\mathrm{x})} = 1 - \frac{n_i}{(\mathrm{f_i}^* - \mathrm{f_i}^{\min})}\ , \qquad (9)$$

and $n_i (n_i \geq 0)$ is negative deviational variable.

SATISFYING OPTIMIZATION METHOD

Satisfying optimization reformulation

The weighted additive model is widely used in vector-objective optimization problems; the basic concept is to use as ingle utility function to express the overall preference of DECISIN MAKERto draw out the relative importance of criteria (Lia and Hwang, 1994). In this case, multiplying each membership function of fuzzy goals by their corresponding weights and then adding the results together obtain a linear weighted utility function. The convex fuzzy model proposed by Bellman and Zadeh (1970), Sakawa (1987) and the weighted additive model, Tiwari et al. (1987) is:

$$\mu_{\mathrm{D}}(x) = \sum_{i=1}^{k} w_i \mu_{f_i(x)} + \sum_{r=1}^{h} \beta_r \mu_{g_r(x)}, \quad (10)$$

$$\sum_{\mathrm{i=1}}^{\mathrm{k}} \mathrm{w_i} + \sum_{\mathrm{r=1}}^{\mathrm{h}} \beta_\mathrm{r} = 1, \mathrm{w_i}, \beta_\mathrm{r} \geq 0, \qquad (11)$$

Where w_i *and* β_r are the weighting coefficients that present the relative importance among the fuzzy goals and fuzzy constraints. The following crisp single objective programming is equivalent to the above fuzzy model 2:

$$\begin{cases} Max\ \sum_{i=1}^{k} w_i\mu_{f_i}^* + \sum_{r=1}^{h} \beta_r\mu_{g_r}^* \\ S.t.: \mu_{f_i}^* \le \mu_{f_i(\mathrm{x})}, i = 1,2,\dots,k, \\ \mu_{g_r}^* \le \mu_{g_r(\mathrm{x})}, r = 1,2,\dots,h, \\ g_p(x) \le b_p, p = h+1,\dots,m, \\ \mu_{f_i}^*, \mu_{g_r}^* \in [0,1], i = 1,2,\dots,k, \\ \sum_{i=1}^{k} w_i + \sum_{r=1}^{h} \beta_r = 1, w_i, \beta_r \ge 0, \\ x \in G. \end{cases} \tag{12}$$

This model optimizes each objective as much as possible. However, DECISIN MAKERmaybe has a limited ability to specify explicit weight information. Thus the linguistic terms are generally used to denote the fuzzy importance of the objectives and constraints .They include "very important", "somewhat important", "important", "general", "unimportant", ‘‘somewhat unimportant”, "very unimportant" seven terms. For example, the objective $f_j(x)$ is "very important", and $f_q(x)$ is "somewhat important" , $j, q\epsilon\{1,2,\dots,k\}\ j \ne q$. Then it is impossible for the above model (12) to solve multiple objective optimization problem when decision maker only gives the linguistic terms information instead of the explicit weights. Therefore alternative fuzzy methods are presented. Chen and Tsai (2001) proposed the principle that the more important objective has the higher desirable achievement degree. That is:

$$\mu_{f_i(\mathrm{x})} \ge \mu_{f_i}^*, \quad \mu_{g_r(\mathrm{x})} \ge \mu_{g_r}^*, \tag{13}$$

Where $\mu_{f_i}^*$ is the desirable satisfying degree of the objective $f_i(x)$ and is given by decision maker in advance. They incorporated inequalities (13) into the fuzzy programming (12) as additional constraints. For the multiple objective optimization problems, it is ordinarily impossible to give the specific quantities of importance between the objectives when there is imprecise preference from decision maker. Thus, the additional constraints need to be incorporated to make the solution exist and satisfy the objectives and preference requirement of decision maker. In this paper, the different importance is expressed using linguistic terms. The importance difference between the corresponding objectives can be realized by comparison between their desirable satisfying degrees. Such as the objective $f_j(x)$ is "very important", and $f_q(x)$ is "somewhat important", and for constraint $g_u(x)$ is "very important", and $g_v(x)$ is "somewhat important", $j, q \in \{1,2,\dots,k\}, j \ne q\ and\ u, v \in \{1,2,\dots,h\}, u \ne v$. The crisp comparison relation is expressed by the following formulation:

$$\mu_{f_q}^* \le \mu_{f_j}^*, \quad \mu_{g_v}^* \le \mu_{g_u}^*, \tag{14}$$

Where $\mu_{f_j}^*, \mu_{f_q}^*$ are, respectively, the desirable satisfying degrees of $f_j(x), f_q(x)$ and $\mu_{g_u}^*, \mu_{g_v}^*$ are, respectively, the desirable satisfying degrees of $g_u(x)$, $g_v(x)$. In order to guarantee the feasible and satisfying solution, the ranking strategy in Li et al. (2004) is used in this paper to compare the importance among these objectives .The similar constraint to $\beta_j - \beta_{j-1} \le \gamma$ and the decision variable $\gamma(-1 \le \gamma \le 1)$ are also incorporated into (14) to form the new comparison inequality in this paper.

Then $\mu_{f_q}^* - \mu_{f_j}^* \le \gamma, \quad \mu_{g_v}^* - \mu_{g_u}^* \le \gamma,$ (15)

Where γ is called importance difference variable in this paper. By means of (15), the desirable satisfying degrees of different important objectives are divided into various levels. In this paper, our method is introduced to reformulate this problem. The trade-off between optimization and importance is realized in terms of the idea of satisfying optimization (1998) in order to guarantee feasible solution. For optimizing every objective under the aspiration value as much as possible, we refer to the additive model in Chen and Tsai (2001) and Tiwari et al. (1987). Here, all of the desirable satisfying degrees are decision variables. Thus the goal of our new model is to maximize the sum of all desirable satisfying degrees. This not only maximizes each desirable satisfying degree, but also acquires the maximum individual satisfying degree under the fuzzy importance.

The optimization model is formulated as follows:

$$\begin{cases} Max\ \frac{\sum_{i=1}^{k}\mu^*_{f_i(x)}+\sum_{r=1}^{h}\mu^*_{g_r(x)}}{k+h}-\lambda\gamma \\ \quad \text{Subject to}: \\ \mu_{f_i(x)} \geq \mu^*_{f_i}, i=1,2,\dots,k, \\ \mu^*_{f_q}-\mu^*_{f_j} \leq \gamma, q \neq j, j,q \in \{1,2,\dots,k\}, \\ \mu_{f_i(x)} \leq 1, \\ \mu_{g_r(x)} \geq \mu^*_{g_r}, r=1,2,\dots,h, \\ \mu^*_{g_v}-\mu^*_{g_u} \leq \gamma, u \neq v, u,v \in \{1,2,\dots,h\}, \\ \mu_{g_r(x)} \leq 1, \\ x \in G. \end{cases} \quad (16)$$

In optimization model 16, the aim of minimizing γ is to obtain the order of desirable satisfying degrees as far as possible. Since $\mu^*_{f_i}, \mu^*_{g_r}$ is located in the interval [0,1], γ belongs to [-1,1]. If $\gamma > 0$, the solution does not satisfy the fuzzy importance. On the contrary, the satisfying solution is acquired if $\gamma \leq 0$ and the results conform to the preference extent. Consequently, for the multiple objective optimization problems with fuzzy relation and fuzzy importance, the following symbols, respectively, express the sets including the objectives and constraints with the same linguistic term:

S_{vi}: "very important"
S_{si}: "somewhat important"
S_i: "important"
S_g: "general"
S_u: "unimportant"
S_{su}: "somewhat unimportant"
S_{vu}: "very unimportant"

Where the intersection between the different sets is empty, for a hybrid multiple objective optimization problem, including inequality and equality fuzzy relations, if $f_j(x)$ is "very important", $f_q(x)$ is "somewhat important", $g_u(x)$ is "very important", $g_v(x)$ is "somewhat important", then the problem can be described as follows: Find: x

So as to satisfy:

$$\begin{cases} f_i(x) \lesssim f_i^*, i=1,2,\dots,k_1, \\ f_l(x) \gtrsim f_l^*, l=k_1+1,\dots,k_2, \\ f_s(x) \cong f_s^*, s=k_2+1,\dots,k, \\ \quad \textit{Subject to}: \\ g_r(x) \lesssim g_r^*, r=1,2,\dots,h_1, \\ g_t(x) \gtrsim g_t^*, t=h_1+1,\dots,h_2, \\ g_p(x) \cong g_p^*, p=h_2+1,\dots,h, \\ f_j(x) \in S_{vi}, f_q(x) \in S_{si}, S_{vi}, S_{si} \subset \{1,2,\dots,k\}, S_{vi} \cap S_{si} = \emptyset, \\ g_u \in S_{vi}, g_v \in S_{si}, S_{vi}, S_{si} \subset \{1,2,\dots,h\}, S_{vi} \cap S_{si} = \emptyset, \\ x \in G. \end{cases} \quad (17)$$

Minimum λ^* for limit case

Simultaneously, some of the desirable satisfying degrees will get zero and the solution will remain identical when $\lambda > \lambda^*$. This is the limit case of distribution of importance. λ^* is the minimum parameter which can lead to this case. In this paper, we present the algorithm to find λ^*. As the parametric programming, the sensitivity analysis of linear programming is used to realize it. For the programming model (18), the following auxiliary linear programming is adapted to acquire the minimum λ.

Algorithm for minimum λ^*

Step1: Initially transform the optimization objective of the programming (18) into "max$(-\gamma)$", and acquire the solution x^*, n^*, p^*, γ^*.

Step2: Formulate auxiliary programming (19) with the above x^*, n^*, p^*, and solve it by simplex method when $\lambda = 0$.

Step3: According the obtained simplex table, determine the different optimum results with the corresponding various λ by means of sensitivity analysis. Finally acquire the minimum λ^0 when the maximum value of the objective is unaltered.

Step4: Substitute λ^0 into (18) and judge: if the result γ^0 is identical with that in step1 (γ^*), λ^0 is the final solution; if the case does not exist, γ^0 is not the solution, and go to the next step.

Step5: Using the above result, solve the following equality:

$$(\sum_{i=1}^{k}\mu^0_{f_i}+\sum_{r=1}^{h}\mu^0_{g_r})/\mathrm{k+h}-\lambda.\gamma^0 = (\sum_{i=1}^{k}\mu^*_{f_i}+\sum_{r=1}^{h}\mu^*_{g_r})/\mathrm{k+h}-\lambda.\gamma^* \quad (20)$$

The solution is λ^*. Solve (18) with λ^*, if the result is equal to γ^*, $\lambda^*$is considered as the final solution and algorithm stops; or else, increase λ^*properly and express it as λ^0, go back to step 4.

$$
\begin{cases}
\max \dfrac{\left(\sum_{i=1}^{k_1}\mu_{f_i}^* + \sum_{l=k_1+1}^{k_2}\mu_{f_l}^* + \sum_{s=k_2+1}^{k}\mu_{f_s}^* + \sum_{r=1}^{h_1}\mu_{g_r}^* + \sum_{t=h_1+1}^{h_2}\mu_{g_t}^* + \sum_{e=h_2+1}^{h}\mu_{g_e}^*\right)}{k+h} - \lambda\gamma \\
\text{S.t.: } f_i(x) + n_i - p_i = f_i^*, i = 1,2,\dots,k_1, \\
f_l(x) + n_l - p_l = f_l^*, l = k_1+1,\dots,k_2, \\
f_s(x) + n_s - p_s = f_l^*, s = k_2+1,\dots,k, \\
g_r(x) + n_r - p_r = g_r^*,\ r = 1,2,\dots,h_1, \\
g_t(x) + n_t - p_t = g_t^*, t = h_1+1,\dots,h_2, \\
g_e(x) + n_e - p_e = g_e^*, e = h_2+1,\dots,h, \\
1 - \dfrac{p_i}{(f_i^{max} - f_i^*)} \geq \mu_{f_i}^*, \\
1 - \dfrac{n_l}{(f_l^* - f_l^{min})} \geq \mu_{f_l}^*, \\
1 - \left(\dfrac{n_s}{f_s^* - f_s^{min}} + \dfrac{p_s}{f_s^{max} - f_s^*}\right) \geq \mu_{f_s}^*, \\
1 - \dfrac{p_r}{(g_r^{max} - g_r^*)} \geq \mu_{g_r}^*, \\
1 - \dfrac{n_t}{(g_t^* - g_t^{min})} \geq \mu_{g_t}^*, \\
1 - \left(\dfrac{n_e}{g_e^* - g_e^{min}} + \dfrac{p_e}{g_e^{max} - g_e^*}\right) \geq \mu_{f_e}^*, \\
\mu_{f_q}^* - \mu_{f_j}^* \leq \gamma, \\
\mu_{g_v}^* - \mu_{g_u}^* \leq \gamma, \\
n_l \leq f_l^* - f_l^{min} \quad , \quad p_i \leq f_i^{max} - f_i^*, \\
n_t \leq g_t^* - g_t^{min} \quad , \quad p_r \leq g_r^{max} - g_r^*, \\
n_s \leq f_s^* - f_s^{min} \quad , \quad p_s \leq f_s^{max} - f_s^*, \\
n_e \leq g_e^* - g_e^{min} \quad , \quad p_e \leq g_e^{max} - g_e^*, \\
n_i, p_i, n_l, p_l, n_r, p_r, n_t, p_t, n_s, p_s, n_e, p_e \geq 0, \\
\mu_{f_i}^*, \mu_{f_l}^*, g_r^*, g_t^*, \mu_{f_q}^*, \mu_{f_j}^*, \mu_{g_v}^*, \mu_{g_u}^*, \mu_{f_s}^*, \mu_{f_e}^* \geq 0, \\
n_i.p_i = 0, n_l.p_l = 0, n_r.p_r = 0, n_t.p_t = 0, n_s.p_{s=}0, n_e.p_e = 0, \\
x \in G.
\end{cases}
\tag{18}
$$

Model 18 can be taken as the general formulation to solve the multiple objective optimization problems with any type of fuzzy relation.

OPTIMIZATION ALGORITHM

According to the proposed satisfying optimization method , the following algorithm for fuzzy multiple objective optimization under fuzzy importance, fuzzy objectives and fuzzy constraints is given as follows

Step1. Formulate the proper optimization model according to the fuzzy relations and preference of decision maker expressed by the linguistic terms in original optimization problem.

Step 2. Initially solve the reformulated optimization problem with a small λ .

Step 3. Judge: if there is the importance difference variable $\gamma > 0$, go to next step .If $\gamma \leq 0$, but not satisfy decision maker, go to next step, too; otherwise optimization stop, and the satisfying solution is acquired.

Step 4. Increase λ, and solve the reformulation again, then go back to step3 and continue.

NUMERICAL EXAMPLES

Example1- (Chen and Tsai, 2001; Li et al., 2004; Tiwari et al., 1987; Li and Hu, 2009).

Find x (x_1, x_2, x_3, x_4) to satisfy

$$
\begin{cases}
f_1(x): 4x_1 + 2x_2 + 8x_3 + x_4 \tilde{\leqq} 35 \\
f_2(x): 4x_1 + 7x_2 + 6x_3 + 2x_4 \tilde{\geqq} 100 \\
f_3(x): x_1 - 6x_2 + 5x_3 + 10x_4 \tilde{\geqq} 120 \\
f_4(x): 5x_1 + 3x_2 + 2x_4 \tilde{\geqq} 70 \\
f_5(x): 4x_1 + 4x_2 + 4x_3 \tilde{\geqq} 40 \\
\text{Subject to :} \\
g_1(x): 7x_1 + 5x_2 + 3x_3 + 2x_4 \tilde{\leqq} 98 \\
g_2(x): 7x_1 + x_2 + 6x_3 + 6x_4 \tilde{\leqq} 117 \\
g_3(x): x_1 + x_2 + 2x_3 + 6x_4 \tilde{\leqq} 130 \\
g_4(x): 9x_1 + x_2 + 6x_4 \leq 105 \\
x_i \geq 0, i = 1,2,3,4
\end{cases}
\tag{21}
$$

The tolerant limits of the five fuzzy objectives are (55, 40, 70, 30, 10), respectively. The tolerant limits of the three fuzzy constraints are (118, 157, 150) and the $g_4(x)$ is a deterministic constraint. The fuzzy importance requirement is: $f_1(x)$ and $f_5(x)$ are "very important"; $f_2(x)$ is "somewhat important"; $f_4(x)$ is "important"; $f_3(x)$ is "general"; $g_1(x)$ is "very important"; $g_3(x)$ is "somewhat important"; $g_2(x)$ is "general". Firstly, according to the fuzzy relations of the objectives and constraints, the optimization model is reformulated as follows:

Solving (22) by LINGO 8.0; the different results according to different λ are listed in Table 1. From Table 1, the sum of desirable satisfying degrees and variable γ decrease monotonously with the increment of λ. The order of desirable satisfying degrees is not consistent with the given relative importance under $\gamma > 0$. Moreover we can see the solution may remain in variable in the interval of λ. decision maker considers the solution (0.9754 ,0.9440 ,0.5952 ,0.7792,0.9754, 1.000, 0.9371, 0.9686) as his or her preferred one when $\gamma \in (-0.03143, -0.1267)$. The desirable satisfying degrees of all objectives conform to decision maker's linguistic terms. The optimization result of each objective and constraint satisfies decision maker.

$$
\begin{cases}
\max \quad \frac{\sum_{i=1}^{5}\mu_{f_i}^{*}+\sum_{r=1}^{3}\mu_{g_r}^{*}}{8}-\lambda.\gamma \\
4x_1+2x_2+8x_3+x_4+n_1-p_1=35 \\
4x_1+7x_2+6x_3+2x_4+n_2-p_2=100 \\
x_1-6x_2+5x_3+10x_4+n_3-p_3=120 \\
5x_1+3x_2+2x_4+n_4-p_4=70 \\
4x_1+4x_2+4x_3+n_5-p_5=40 \\
7x_1+5x_2+3x_3+2x_4+n_6-p_6=98 \\
7x_1+x_2+6x_3+6x_4+n_7-p_7=117 \\
x_1+x_2+2x_3+6x_4+n_8-p_8=130 \\
1-\frac{p_1}{20}\geq \mu_{f_1}^{*} \\
1-\frac{n_2}{60}\geq \mu_{f_2}^{*} \\
1-\frac{n_3}{50}\geq \mu_{f_3}^{*} \\
1-\frac{n_4}{40}\geq \mu_{f_4}^{*} \\
1-\frac{n_5}{30}\geq \mu_{f_5}^{*} \\
1-\frac{p_6}{20}\geq \mu_{g_1}^{*} \\
1-\frac{p_7}{40}\geq \mu_{g_2}^{*} \\
1-\frac{p_8}{20}\geq \mu_{g_3}^{*} \\
\mu_{f_2}^{*}-\mu_{f_1}^{*}\leq \gamma \\
\mu_{f_2}^{*}-\mu_{f_5}^{*}\leq \gamma \\
\mu_{f_4}^{*}-\mu_{f_2}^{*}\leq \gamma \\
\mu_{f_3}^{*}-\mu_{f_4}^{*}\leq \gamma \\
\mu_{g_3}^{*}-\mu_{g_1}^{*}\leq \gamma \\
\mu_{g_2}^{*}-\mu_{g_3}^{*}\leq \gamma \\
p_1\leq 20\ , n_2\leq 60, n_3\leq 50, n_4\leq 40, \\
n_5\leq 30\ p_6\leq 20, p_7\leq 40, p_8\leq 20 \\
n_i\ , p_i, \mu_{f_i}^{*}\ \geq 0\ \ , i=1,\dots,5 \\
n_r, p_r, \mu_{g_r}^{*}\geq 0\ , r=1,2,3 \\
n_i.p_i=0\ , \qquad i=1,\dots,5 \\
n_r.p_r=0\ , \qquad r=1,2,3 \\
9x_1+x_2+6x_4\leq 105 \\
x_i\geq 0\ , i=1,2,3,4
\end{cases}
\tag{22}
$$

Minimum $\boldsymbol{\lambda}^{*}$

Initially transform the optimization objective of the programming (22) into "$\max(-\gamma)$", and get the following result:

$$
\begin{cases}
x^{*}=(0.0000,10.0000,0.0000,15.0000) \\
n^{*}=(0.0000,0.0000,30.0000,10.0000,0.0000,18.0000,17.0000,30.0000) \\
p^{*}=(0.0000,0.0000,0.0000.0.0000,0.0000,0.0000,0.0000,0.0000) \\
\gamma^{*}=-0.3333
\end{cases}
\tag{23}
$$

According to the above solution, the auxiliary linear programming model is formulated as:

$$
\begin{cases}
\max \quad \frac{\sum_{i=1}^{5}\mu_{f_i}^{*}+\sum_{r=1}^{3}\mu_{g_r}^{*}}{8}-\lambda.\gamma \\
\text{Subject to :} \\
1\geq \mu_{f_1}^{*} \\
1\geq \mu_{f_2}^{*} \\
0.4\geq \mu_{f_3}^{*} \\
0.75\geq \mu_{f_4}^{*} \\
1\geq \mu_{f_5}^{*} \\
1\geq \mu_{g_1}^{*} \\
1\geq \mu_{g_2}^{*} \\
1\geq \mu_{g_3}^{*} \\
\mu_{f_2}^{*}-\mu_{f_1}^{*}\leq \gamma \\
\mu_{f_2}^{*}-\mu_{f_5}^{*}\leq \gamma \\
\mu_{f_4}^{*}-\mu_{f_2}^{*}\leq \gamma \\
\mu_{f_3}^{*}-\mu_{f_4}^{*}\leq \gamma \\
\mu_{g_3}^{*}-\mu_{g_1}^{*}\leq \gamma \\
\mu_{g_2}^{*}-\mu_{g_3}^{*}\leq \gamma \\
\gamma\leq 0
\end{cases}
\tag{24}
$$

Let λ =0, the linear programming software LINGO 8.0 is used to solve the above auxiliary programming. According to the sensitivity analysis of linear programming, the solution is optimal and identical when all the reduced costs are greater than or equal to 0. Therefore the solution remains constant and γ attains minimum when $\lambda\geq 0$ after several pivot iterations of simplex method. Substitute $\lambda=0$ into (22) and solve it. get the following result in Table 3.

For computing the minimum λ , solve the following equality:

$$
\left(\sum_{i=1}^{k}\mu_{f_i}^{0}+\sum_{r=1}^{h}\mu_{g_r}^{0}\right)/k+h-\lambda.\gamma^{0}=\left(\sum_{i=1}^{k}\mu_{f_i}^{*}+\sum_{r=1}^{h}\mu_{g_r}^{*}\right)/k+h-\lambda.\gamma^{*} \tag{25}
$$

The result is $\lambda=1.14$. Substitute λ =1.14 into (22) and solve it. The results are presented in Table 4.

Table 1**:** Optimization results with different λ for Example1

λ	Sum of desirable Satisfying degrees	Importance difference (γ)	Desirable satisfying degree $(\mu^*_{f_1},\mu^*_{f_2},\mu^*_{f_3},\mu^*_{f_4},\mu^*_{f_5},\mu^*_{g_1},\mu^*_{g_2},\mu^*_{g_3})$	Solution (x)
0.05	7.3252	0.02460	(0.9754,1.0000,0.5952,0.7792,0.9754,1.0000,1.0000,1.0000)	(0.0000,9.8137,0.000,15.8644)
0.3	7.3006	0.0000	(0.9754,0.9754,0.5952,0.7792,0.9754,1.000,1.000.1.000)	(0.0000,9.8137,0.000,15.8644)
0.5	7.1749	-0.03143	(0.9754,0.9440,0.5952,0.7792,0.9754,1.000,0.9371,0.9686)	(0.0000,9.8137,0.000,15.8644)
0.8	6.7364	-0.1267	(0.9754,0.8487,0.5952,0.7220,0.9754,1.0000,0.7465,0.8732)	(0.0000,9.8137,0.0000,15.8644)
1.0	5.8	-0.2000	(1.0000,0.8000,0.4000,0.6000,1.0000,1.0000,0.6000,0.40000)	(0.0000,10.0000,0.0000,15.0000)
1.5	5	-0.3333	(1.0000,0.6667,0.0000,0.3333,1.0000,1.0000,0.3333,0.6667)	(0.0000,10.0000,0.0000,15.0000)

Table 2: Optimization results of desirable Satisfying degrees for Example 1

$\mu^*_{f_1}$	$\mu^*_{f_2}$	$\mu^*_{f_3}$	$\mu^*_{f_4}$	$\mu^*_{f_5}$	$\mu^*_{g_1}$	$\mu^*_{g_2}$	$\mu^*_{g_3}$
1.0000	0.6667	0.0000	0.3333	1.0000	0.6667	0.0000	0.3333

Table 3: Optimization results of desirable Satisfying degrees with $\lambda = 0$ for Example 1

$\mu^0_{f_1}$	$\mu^0_{f_2}$	$\mu^0_{f_3}$	$\mu^0_{f_4}$	$\mu^0_{f_5}$	$\mu^0_{g_1}$	$\mu^0_{g_2}$	$\mu^0_{g_3}$
0.9812	1.0000	0.6050	0.7750	0.9670	1.0000	1.0000	1.0000

Table :4 Optimization results of desirable satisfying degrees with $\lambda = 1.14$ for Example 1

$\mu^*_{f_1}$	$\mu^*_{f_2}$	$\mu^*_{f_3}$	$\mu^*_{f_4}$	$\mu^*_{f_5}$	$\mu^*_{g_1}$	$\mu^*_{g_2}$	$\mu^*_{g_3}$
1.0000	0.6667	0.0000	0.3333	1.0000	1.0000	0.3333	0.6667

CONCLUSION

Based on (Li et al., 2004; Li and Hu, 2009), this paper presents the satisfying optimization method for fuzzy multiple objective optimization problems. This method realizes the trade-off between optimization and fuzzy importance requirement. Decision maker can find the appropriate alternative according to his intention from various solutions by regulating parameter λ. The results of the examples show its efficiency, flexibility and sensitivity for the optimization problems with three types of fuzzy relations. For different fuzzy relations and fuzzy importance, the reformulated optimization models based on goal programming is proposed. Not only the satisfying results of all the objectives and constraints can be acquired, but also the fuzzy importance requirement can be simultaneously actualized. It can be used in many real-world decision maker problems.

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