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Research Article

Modelling Malaysia Stock Markets Using GARCH, EGARCH and Copula Models

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Abstract

Copula is a favored method used to measure dependency for financial data due to its flexibility. Yet, studies about dependence structure between bivariate data especially by using time-varying copula approach is very limited. Hence, this paper will examine the dependency between KLCI-FBMHS pair by considering static and time-varying copula. Traditionally, ARCH model is used to measure the volatility. However, it failed to capture stylized facts that usually exist in financial data such as the volatility clustering and leverage effect. Thus, the study also investigates the effect of different marginal models (GARCH and EGARCH) towards dependence structure and parameter estimations. Generally, the findings reveal that KCLI-FBMHS pair have strong dependency. In addition, this study highlight that ARMA(1,0)-GARCH(1,1) and ARMA(1,0)-EGARCH(1,1) with student t distribution are well-fitted to both (KLCI and FBMHS) series, the KLCI-FBMHS pair have similar dependence structure for both static and dynamic copula models.

***Keywords*:** Time-Varying Copula, GARCH, EGARCH, KLCI-FBMHS

1.Introduction

There are several methods for estimating copula such as exact maximum likelihood (EML), inference functions of margins (IFM), and canonical maximum likelihood (CML). To estimate copula parameters, the IFM and the CML approach involving two main steps; identifying the best marginal distribution (input model) and estimate the dependence parameter. The CML method was introduced by Genest, Ghoudi, and Rivest (1995) and is categorized as semi-parametric procedure because it uses empirical cumulative distribution function to model the margins. By using the EML approach, marginals and copula parameters can be estimated simultaneously. However, Joe (1997) recommend that the IFM is more efficient as compared to the EML method due to its flexibility especially when dealing with complex marginal distribution. Therefore, this paper will use the IFM method to measure the dependency between Islamic (FBMHS) and conventional (KLCI) index.
ARCH model is proposed by Engle (1982) and is used to measure the volatility of time series financial data. However, ARCH model is unable to deal with volatility clustering. Due to that issue, the standard GARCH model is presented in 1986. For the following years, the extended GARCH model such as Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), Integrated GARCH (IGARCH), and Glosten, Jagannathan and Runkle GARCH (GJR-GARCH) have been introduced since the standard GARCH model is unable to capture some stylized facts such as asymmetric distributions and effects leverage. Thus, this study will employ GARCH and extended GARCH (EGARCH) models with normal and non-normal distributions for modelling the volatility of univariate data.
Past researchers usually used the Pearson linear correlation for measuring the dependency of bivariate data. However, this method is inappropriate for non-normal distribution including financial data. This problem has been proven by some authors in their articles including Embrechts, McNeil, and Straumann (2002), McNeil, Frey, and Embrechts (2005), and Rachev, Stein, and Sun (2009). Furthermore, an alternative approach which is known as the cointegration analysis has some limitation, i.e the method is not robust enough (Liew and Wu, 2013). Hence, researchers started to use copula models to measure the dependency due to its flexibility and its ability to deal with any kind of distributions. Therefore, this study will apply static copulas and time-varying copulas approach for measuring the co-movements between KLCI-FBMHS data.
There are several motivations that trigger this study. Firstly, as mentioned by Ning (2010), the inaccurate marginal model will lead to incorrect specification of copula model. Hence this study will examine the results of dependence structure and its parameter estimations by using different marginal model. Secondly, Hammoudeh et al. (2014) claims that financial disaster and changing of business cycle causes the

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dynamic dependence to be more appropriate compared to its constant version. Therefore, we will investigate either Malaysia stock markets will experience the same phenomena or not. Thirdly, this study utilized the recent data of KLCI and FBMHS index covering the period of 2007 until 2018. Thus, the current trend of Malaysia indices will be discussed as well. Lastly, by choosing KLCI and FBMHS as the sample data, the dependency among Islamic and conventional indices for Malaysia cases can be examined as well.

The rest of this article is arranged as follows; Section 2 describes past studies, Section 3 briefly discuss the methodology for marginal modelling and dependence estimation, findings for this study are reported in Section 4, and our conclusions are provided in final part of this paper.

2. Literature Review

This article will investigate the specification of the input model, also known as the marginal model, which is the underlying factor in estimating the parameters of copula. According to Ning (2010), the wrong specification of the marginal model will lead to the inaccurate estimation of copula model. However, there is no consensus of which marginal model can provide accurate estimation. Therefore, the aim of this study is to investigate how various specifications of the marginal models represented by GARCH and EGARCH models would affect the copula model selection and parameter estimations. Past studies commonly used GARCH model to fit the marginal distribution such as Shams and K. Haghighi (2013), Razak and Ismail (2014), Shams and Zarshenas (2014), Chen and Khashanah (2016), and Aminuddin, Razak, and Ismail (2018). In addition, Rahman, Omar, and Kassim (2015) modelled the volatility of Malaysian bond by using TGARCH and EGARCH models. Sukcharoen et al. (2014) used the GJR-GARCH model and claim that it is more appropriate than the standard GARCH model due to the negative skewness characteristics for most of the return series. Dajcman (2013) found that the APGARCH (1,1) is chosen as the best model for France, U.K., and Italy returns series, the GARCH (1,2) for Croatia and Germany stock market, and the EGARCH (1,1) for FTSE-MIB pair. This prove that different GARCH family will be chosen as the best marginal model for different series of data.

Recently, the copula approach become more popular and is widely used especially in financial studies for analyzing dependency. Several studies that used copula approach were actively carried out by researchers and lead to the creation of dynamic copula methods. Aussenegg and Cech (2008), for example, has examined the ability of time-varying Gaussan and time-varying Student *t* copula to anticipate the co-movements between Eurostoxx Index and Dow Jones Industrial Index by using daily closing price data. Manner and Reznikova (2011) used Monte Carlo approach and compared several dynamic copula models. In another study, Kara and Kemaloglu (2016) found that the tDCC copula is selected for USD-EUR currency data when they used dynamic copula approach. Nevertheless, when static copula technique is used on the same data, the Gaussan copula is selected as the best model. Consequently, this study will be focusing on both static and time-varying copula to model the dependency between sample data.

Lately, Islamic investment started to be an alternative platform for investors especially during the period of financial disaster. According to Hakim and Rashidian (2002), the Dow Jones Islamic Market Index (DJIM) started to get an attention of Muslims investors worldwide since it is being introduced in the United States in 1999. Based on past articles (Hammoudeh et al. 2014; Razak, Ismail, and Aridi 2016), the Islamic index is said to behave similarly with their conventional counterpart. However, studies conducted by Jawadi, Jawadi, and Louhichi (2014) proved that the Islamic portfolio index showed good achievement as compared to the conventional portfolio index during the period of economic crisis occurrence. In another study, Mensi et al. (2016) claimed that the Islamic index is able to minimize risk since it can provide better diversifications. Hence, this study will use the Islamic (FBMHS) and the conventional (KLCI) stock markets for Malaysia indices as a case study.

This article will investigate the impact of different marginal models (GARCH and EGARCH) on the parameter estimation of both marginal and copula models for KLCI-FBMHS pair. Instead of static copula model, this study also considers time-varying copula approach. There are three important things that will be determined throughout this study: either GARCH model will outperform EGARCH model or vice versa, different input model will result in different or similar dependency outcomes, and either diversification between Malaysia indices represented by KLCI (conventional) and FBMHS (Islamic) is necessary or not.

3. Methodology / Materials

This paper employs the daily returns data of Malaysia indices which is represented by the FTSE Bursa Malaysia Kuala Lumpur Composite Index (KLCI) and the FTSE Bursa Malaysia Hijrah Syariah Index (FBMHS) covering the period of 21 May 2007 until 28 September 2018. The closing price data is sourced from Bloomberg terminal and consists of 2796 observations. The price series of conventional (KLCI) and Islamic (FBMHS) stock markets are converted into returns series due to the stationary issue by using equation $R_t = \log P_t - \log P_{t-1}$. The financial crisis which marked over the study period is the 2008 Global Financial Crisis (GFC).

As proposed by Joe (1997), this study imposed the inference function of margins (IFM) approach to measure the dependency between KLCI-FBMHS pair which involve several steps. Firstly, various marginal models are fitted to each time series and the best marginal model will be chosen. Next, the standardized residuals for the best volatility model of each GARCH category (ARMA-GARCH and ARMA-EGARCH) are converted into pseudo observations [0,1]. Finally, the static and time-varying copula approaches are employed to measure the dependency between bivariate financial data.

3.1.Marginal model

Financial data have some stylized facts including fat tails, volatility clustering, leverage effects, long memory and co-movements in volatility. However, this paper will only discuss a few stylized facts mentioned above. Frequently, financial time series data experience fatter tails due to the value of kurtosis which is greater than three. Therefore, we consider four type of error distributions in this study namely normal distribution, skewed normal distribution, *t* distribution, and skewed *t* distribution to capture the stylized facts of fatter tails. Equations for each error distribution can be found in Jiang (2012), Ashour and Abdel-hameed (2010) and Hu and Kercheval (2006).

The marginal model consists of two components which are conditional mean and conditional variance. For estimating the first component, we use ARMA (p, q) model with equation

$$Y_t = \alpha_0 + \sum_{i=1}^{p} \varphi_i Y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \cdot$$

The Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) are referred in order to choose the appropriate value for lag p and lag q. Meanwhile, GARCH (p, q) and EGARCH (p, q) models with different type of error distributions are employed while estimating the conditional variance part.

3.1.1.GARCH Model

Bollerslev (1986) introduces GARCH model to overcome the weaknesses of ARCH model which is unable to capture the volatility clustering. The model specifications for GARCH (p, q) is:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \qquad \text{b} \qquad (1)$$

where σ_t^2 is the conditional variance at time *t*, $\alpha_i {\varepsilon_{t-i}}^2$ is the ARCH components, and $\beta_j {\sigma_{t-j}}^2$ is the GARCH components. For an accurate model, $\alpha_i + \beta_j < 1$ and all parameters in the variance equation must be positive. Due to the failure of Generalized ARCH (GARCH) model to deal with negativity, some scholars started to create other GARCH family model.

3.1.2 . EGARCH model

There are some advantages of EGARCH model as compared to GARCH model. Firstly, EGARCH model is able to capture the leverage effects which allow different response on conditional variance between negative and positive shocks. In addition, EGARCH model can tackle non-negative constraint issue on the coefficient since the conditional variance will always be positive. The mathematical equation for EGARCH (p, q) model as proposed by Nelson (1991) is:

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^{p} \beta_j \log(\sigma_{t-j}^2) + \sum_{j=1}^{q} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \qquad (2)$$

The conditional variance parameter is represented by σ_t^2. It is always positive due to the existence of logarithm even if any right-hand side parameter is negative. ω, β and α indicate the constant parameter, persistence in conditional volatility and symmetric effect respectively. In addition, γ determine the asymmetric or leverage effect. The model is symmetric if $\gamma = 0$ whereas $\gamma > 0$ and $\gamma < 0$ signify asymmetric model.

3.2.Copula model

In the second stage of IFM approach, the standardized residuals of selected GARCH and EGARCH model are extracted. Subsequently, the standardized residuals are transformed into pseudo observations by using the equation $(u_i, v_i) = [(\text{Rank}\, O_{1i}/n_{1i}+1), (\text{Rank}\, O_{2i}/n_{2i}+1)]$. Finally, the MATLAB software is used to estimate the dependency between bivariate data by considering four copula models. In contrast to past studies, this paper will employ static and time-varying copula approach. Generally, Copula model can be written as the following expression (Nelson, 2006):

$$H(x, y) = C[G_1(x), G_2(y)] \qquad (3)$$

H represents the bivariate distribution function of copula model, while x and y illustrate the marginal distributions of univariate series. From the above equation we can conclude that both copula and marginal models can be estimated separately. Thus, this make copula approach as a flexible tool for modelling the dependency.

3.2.1. Gaussian copula

The Gaussian copula is categorized under elliptical copula families. It has no tail dependence ($\lambda_U = \lambda_L = 0$) and the distribution is symmetric. It can be defined as the mathematical equation below:

$$C(u, v) = \int_{-\infty}^{\phi-1(u)} \int_{-\infty}^{\phi-1(v)} \frac{1}{2\pi\sqrt{1-\delta^2}} \exp\left\{ -\frac{x^2 - 2\delta xy + y^2}{2(1-\delta^2)} \right\} dxdy$$

(4)

$$C = \Phi_\delta(\Phi^{-1}(u), \Phi^{-1}(v)), -1 \le \delta \le 1 \qquad (5)$$

Parameter δ represents the linear correlation for the bivariate data. Φ and $\Phi^{-1}(.)$ illustrate the bivariate cumulative density function and the inverse function for univariate standard normal distribution respectively.

According to Patton (2006), the dynamic Gaussian copula can be written as:

$$\delta_t = \lambda\left(\omega + \beta\delta_{t-1} + \alpha\frac{1}{10}\sum_{j=1}^{10}[\Phi^{-1}(u_{t-j})\Phi^{-1}(v_{t-j})]\right) \quad (6)$$

The modified logistic transformation is represented by parameter λ, where $\lambda = (1-e^{-x})(1+e^{-x})^{-1}$. It is assumed that equation (6) follows ARMA (1,10) type process. The persistence effects are captured by coefficient $\beta\delta_{t-1}$.

3.2.2 Symmetric joe-clayton copula

Unlike Gaussian copula model, Symmetric Joe-Clayton (SJC) copula is used for asymmetric dependence and both upper and lower tail dependences. The SJC copula is an alteration version of the BB7 copula or Joe-Clayton copula. The mathematical expression for BB7 copula is:

$$C_{JC}(u,v|\tau^u,\tau^L) = 1-\left(1-\left\{[1-(1-u)^k]^{-\gamma}+[1-(1-v)^k]^{-\gamma}-1\right\}^{-\frac{1}{\gamma}}\right)^{\frac{1}{k}} \quad (7)$$

where

$$k = \frac{1}{\log_2(2-\tau^U)} \quad \text{for} \quad \tau^U \in (0,1) \quad (8)$$

and

$$\gamma = -\frac{1}{\log_2(\tau^L)} \quad \text{for} \quad \tau^L \in (0,1) \quad (9)$$

The SJC Copula is introduced by Patton (2006) and can be defined as:

$$C_{SJC}(u,v|\tau^U,\tau^L) = 0.5[C_{JC}(u,v|\tau^U,\tau^L)+C_{JC}(1-u,1-v|\tau^U,\tau^L)+u+v-1]$$

(10)

where τ^U and τ^L respectively represent the upper and lower tails.

The mathematical expression for both tails of SJC Copula when it evolves over time can be written as:

$$\tau^{U/L} = \tilde{\lambda}\left(\omega^{U/L} + \beta^{U/L}\tau_{t-1} + \alpha^{U/L}\frac{1}{10}\sum_{i=1}^{10}|u_{1,t-i}-u_{2,t-i}|\right) \quad (11)$$

The coefficient $\tilde{\lambda}$ represents the logistic transformation and can be expressed as $\tilde{\lambda} = \left((1+e^{-x})^{-1}\right)$.

3.2.3.Information criterion

In order to choose the best marginal and copula model, we refer to the estimated parameter and the value of information criterion namely Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Akaike (1974) defined the AIC equation as:

$$AIC = -2\log(L) + 2k \quad (12)$$

The number of parameters used in the statistical model is represent by k, whereas L indicates the likelihood function.

The Schwarz's Bayesian SBC or well known as the BIC is proposed by Schwarz (1978) and can be written as:

$$BIC = -2\log(L) + k(\log(n)) \quad (13)$$

where k illustrates number of parameters, whereas *n* signify number of observations. L denotes the maximum likelihood function and can be mathematically defined by:

$$L(k) = -\sum_{i=1}^{T}[\ln(h_i) + \varepsilon_i^2 / h_i] \quad (14)$$

where h_i and ε_i^2 signify the value of conditional variance and squared residuals respectively.

4. Results and Findings

This section will discuss thoroughly on the empirical findings of the study. The sub section of this paper contains the descriptive statistic for the series of Islamic (FBMHS) and conventional (KLCI), the best marginal distribution for univariate data, and the dependence estimated for KLCI-FBMHS pair.

4.1. Preliminary analysis

In order to know the characteristics of sample data, we calculate the summary statistics, stationary test, and normality test for each series. The summary statistics for price series is not stated here but it is available upon request. In brief, the result shows that the Augmented Dickey Fuller (ADF) test have insignificant value for both closing price series. Therefore, the closing prices are converted into returns series due to non-stationary issues. Table 1 presents the findings of summary statistics for the return series of KLCI and FBMHS index.

Table 1
Summary statistics for KLCI and FBMHS series.

Stock	**KLCI**	**FBMHS**
Mean	-0.00010	-0.00015
Median	-0.00032	-0.00031
SD	0.00731	0.00785
CV	-7141.824	19.81684
Minimum value	-0.04259	-0.04537
Maximum value	0.09979	0.11090
Skewness	1.12324	1.17344
Kurtosis	18.82007	21.68352
ADF Test (*p*-value)	0.01	0.01
JB Test	29734.00**	41294.00**

***The values are significant at 1% level*

The negative mean and median for both series signify that both Islamic and conventional indices have negative returns. Besides, the average value of KLCI index is slightly higher than FBMHS index which explain that the conventional series is riskier than its Islamic counterparts. By referring to the value of coefficient of variation (CV) and standard deviation (SD) for FBMHS and KLCI, it can be concluded that the Syariah index is slightly more volatile than the conventional index. The findings are reinforced by the larger difference between the maximum and minimum value for FBMHS index as compared to the KLCI index.

Table 1 also illustrates that the value of Kurtosis for both indices is greater than 12. This explained that all series have leptokurtic distribution. Thus, the stylized facts of heavier tails are proven for our data set. Next, the skewness has a value greater than zero which implies that both series are positively skewed. The significant value of Jarque-Bera (JB) test supports the evidence of non-normality for both distributions. Finally, the null hypothesis of ADF test is rejected and implies that both series are stationary.

4.2.Marginal models

We use time series model since the Malaysia indices are not independently and identically distributed (*iid*). Table 2 shows the parameter estimates, standard errors (in brackets), *p*-values of diagnostic test for the preferred marginal model, and the value of AIC and BIC. The ARMA(1,0)-GARCH(1,1) model and ARMA(1,0)-EGARCH(1,1) model with t distributions are considered to be well-fitted for both series.

In terms of ARMA model specification, the AR(1) shows a good fit for Malaysia indices indicating that the previous day's return affects the current return. The lag 1 and lag 0 are chosen by referring to ACF and PACF, significant value of parameter estimation, and the lowest value of AIC and BIC.

To measure the volatility of the return series, the hybrid ARMA(p,q)-GARCH(P,Q) models are employed. Under GARCH model, all parameters are significant at 1 percent level except for omega (ω) coefficient and the stationary assumption still holds since β_1 is positive and less than one. In addition, both series have high persistence volatility since the sum of alpha (α_1) and beta (β_1) are close to value one.

EGARCH model is also considered for this study due to the inability of GARCH model to capture the asymmetric behavior and the leverage effect. The ARMA(1)-EGARCH(1,1) in Table 2 show that the leverage effects (γ_1) are positive for both series, implying that positive past events greatly influence the volatility of future stock, compared to negative past events. Besides, the significant of γ_1 coefficient illustrate the existence of asymmetric effects for KLCI and FBMHS returns series. Findings also shows that the parameters estimated for both series are significant.

Next, diagnostic tests are examined to ensure that the specification of marginal models with various error distributions are correct. The Ljung-Box Q-statistics with lag 10, 15, and 20 and the Langrage Multiplier (LM) test respectively detects autocorrelation and heteroscedasticity problems in the residuals. Insignificant values for both tests imply that the selected models are appropriate to capture the volatility of returns series. Generally, the findings reveal that AR(1)-GARCH(1,1)-student-*t* and AR(1)-EGARCH(1,1)-student-*t* are well-fitted to both indices. However, the lowest information criterion explained that the ARMA-EGARCH model outperformed the ARMA-GARCH model for KLCI and FBMHS series.

Table 2
Results for selected ARMA-GARCH and ARMA-EGARCH model.

	ARMA (1,0) – GARCH(1,1)		**ARMA (1,0) – EGARCH(1,1)**	
Stock	**KLCI**	**FBMHS**	**KLCI**	**FBMHS**
Parameter estimation				
μ	-0.000327** (0.000105)	- 0.000299** (0.000105)	-0.000439** (0.000101)	-0.000360** (0.000105)
ϕ_1	0.106982** (0.019333)	0.102163** (0.018587)	0.114004** (0.018543)	0.108712** (0.017554)
ω	0.000001 (0.000001)	0.000001 (0.00001)	-0.115286** (0.003262)	-0.115480** (0.001738)
α_1	0.112530** (0.021084)	0.102861** 0.015241	-0.102112** (0.013197)	-0.078022** (0.013698)
β_1	0.873323** (0.019846)	0.888216** (0.012784)	0.988465** (0.000446)	0.988432** (0.000361)
γ_1	- -	- -	0.170203** (0.001456)	0.177265** (0.005645)
ν	6.184515** (0.688408)	4.706431** (0.374103)	6.716744** (0.810864)	5.019226** (0.504493)
Diagnostic tests				
Q (10)	0.6211	0.7651	0.6203	0.8010
Q (15)	0.8528	0.7922	0.8301	0.7988
Q (20)	0.7336	0.8957	0.6188	0.8895
Q^2 (10)	0.3757	0.5516	0.1231	0.4671
Q^2 (15)	0.4777	0.6597	0.1969	0.5403
Q^2 (20)	0.4021	0.7054	0.1986	0.5977
LM test	0.3848	0.7475	0.2595	0.5223
Information Criterion				
AIC	-7.4130	-7.3391	-7.4378	-7.3546
BIC	-7.4002	-7.3263	-7.4229	-7.3397

***The values are significant at 1% level*

4.3 Dependence estimations

The residuals for selected marginal model are extracted and converted into pseudo samples. Then, we used pseudo sample to estimate the dependency by using static and time-varying copula approach. The overall dependence of KLCI-FBMHS returns will be explained by the Gaussian copula. Meanwhile, the SJC copula is used to describe the tail dependence of the bivariate data.

4.3.1.Static copula

Table 3 illustrates the findings for the static copulas namely the Gaussian and the SJC copula for different marginal models (ARMA-GARCH and ARMA-EGARCH).

Table 3
Summary results for static copulas

GARCH type	**ARMA - GARCH**	**ARMA – EGARCH**
Gaussian Copula		
δ	0.9029	0.9009
AIC	-4720.4516	-4664.6704
BIC	-4720.4495	-4664.6683
Symmetric Joe Clayton Copula		
τ^U	0.7612	0.7651
τ^L	0.7441	0.7366
AIC	-4621.7832	-4562.7403
BIC	-4621.7789	-4562.7361

The value of estimated parameter for both input model of Gaussian copula is close to 1. This indicates that the KLCI index has strong correlation with its Islamic counterpart. Regardless of whether the ARMA-GARCH or the ARMA-EGARCH is chosen as the input model, the upper tail coefficient of the SJC copula is slightly greater than the lower tail coefficient. This scenario describes the co-movement among extreme gains during booming period. The finding is in line with the results of leverage effect from the finding of marginal models in previous sub section. The information criterion (AIC and BIC) reveal that the Gaussian copula is more appropriate to present the time-invariant dependency between KLCI-FBMHS pair throughout the study period regardless any kind of marginal model used. This output is consistent with past article written by Aminuddin, Razak, and Ismail (2018).

4.3.2.Time-varying copula

Next, we used the time-varying Gaussian copula and the time-varying SJC copula to explain the time-varying dependency of Islamic-conventional pair. The results of both copulas with ARMA-GARCH and ARMA-EGARCH as an input model will be discussed in this section. The coefficients α (α^U or α^L) and β (β^U or β^L) are used to measure the time distinctions of dependency between KLCI and FBMHS index. On the other hand, ω (ω^U or ω^L) is used to measure the dependence level of bivariate data.

The value of β coefficients for Gaussian copula are relatively lower than α coefficients implying that most of the sample period are close to white noise. In contrast to the Gaussian copula, the SJC copula has larger value of α compared to its persistence coefficient β for both upper and lower tail when the ARMA-EGARCH is selected as the marginal model. This finding explained that the dependence structure has a slight change for both tails over the study period. However, ARMA-GARCH models have zero value for parameters of α^U, α^L, β^U, and β^L.

Table 4
Summary results for time-varying copulas

GARCH-type	**ARMA - GARCH**	**ARMA - EGARCH**
Time-varying Gaussian Copula		
ω	4.178	4.9997
α	0.518	0.4659
β	-1.8133	-2.7038
AIC	-4761.2490	-4697.3242
BIC	-4761.2426	-4697.3178
Time-varying Symmetric Joe Clayton Copula		
ω^U	1.1594	1.1790
α^U	0	-0.0010
β^U	0	-4.41e-05
ω^L	1.0674	1.0316
α^L	0	-0.0006
β^L	0	0.0033
AIC	-4621.6938	-4562.7418
BIC	-4621.6810	-4562.7291

Table 4 also shows that the value of all parameters ω^U are higher than ω^L. This reveals that the correlation between both indices are slightly weaker during the crisis period compared to the normal period. Overall, the time-varying Gaussian copula model is chosen as the best dependence model for the KLCI-FBMHS pair due to the lowest value of AIC and BIC. Hence, both marginal models used have similar dependence structure, but the value of parameter estimations are slightly different.

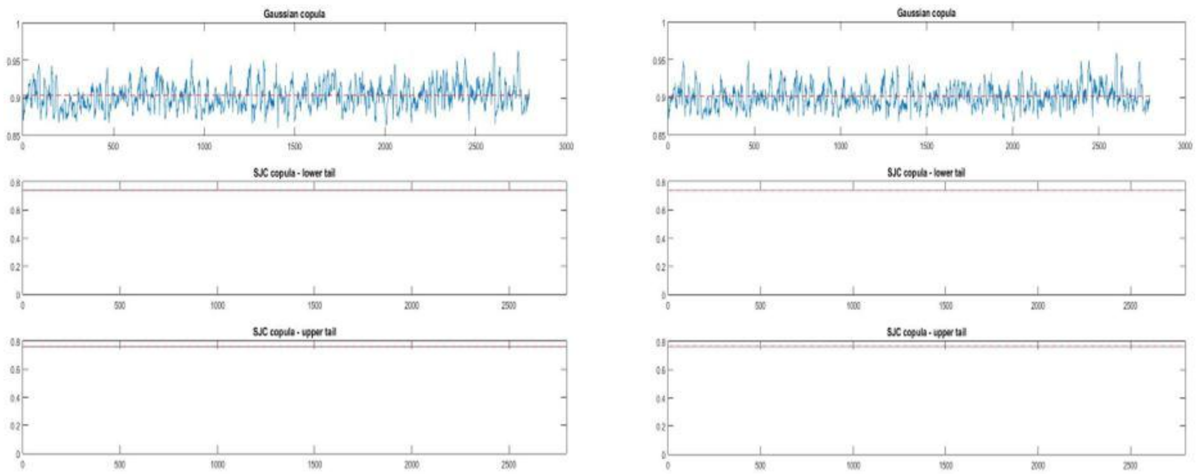

Fig. 1. Graph dependency for KLCI-FBMHS pair using a) ARMA-GARCH models and b) ARMA-EGARCH models

Figure 1 illustrates graphs dependency of time-varying copula for Islamic-conventional pair involving parameters δ_t (Gaussian), τ^U (SJC upper tail), and τ^L (SJC lower tail). The parameter values δ_t range between 0.87 and 0.96 for the ARMA-GARCH models. The output for the ARMA-EGARCH models is slightly different where the parameter values range between 0.86 and 0.96. In addition, the parameter values for the lower tail are constant at 0.74 for both marginal models. The ARMA-EGARCH models also experiences a constant value of τ^U (0.74) throughout the study period. For data sets with ARMA-GARCH as an input models, the estimates of upper tail parameters result in a range between 0.76 and 0.77.

Overall, the time-varying Gaussian copula model is chosen as the best dependence model for the KLCI and FBMHS index due to the lowest value of AIC and BIC. As a conclusion, both marginal models (ARMA-GARCH and ARMA-EGARCH) have similar dependence structure, but the value of parameter estimations are slightly different.

5.Conclusion

As a conclusion, the Islamic and conventional stock markets have non-normal distributions which are positively skewed, fat tails and leptokurtic. This findings is consistent with a study by Razak, Ismail, and Aridi (2016) which stated that the Islamic and conventional series have identical characteristics. In term of marginal model, AR (1) is chosen as the best model for conditional mean. Meanwhile, GARCH(1,1) and EGARCH(1,1) are selected as the well-fitted model for conditional variance part. Based on the lowest value of information criterion, AR(1)-EGARCH(1,1) with *t* distributions is selected as the best model for KLCI-FBMHS pair. Generally, the KLCI-FBMHS pair experience positive correlation, strong relationship, and the correlation are stronger during the stable period compared to the crisis period. Therefore, the diversification between KLCI-FBMHS is advisable during normal time. In term of dynamic dependency, the time-varying Gaussian copula is chosen as an appropriate model for Malaysia stock markets data. The results are consistent regardless of whether ARMA-GARCH or ARMA-EGARCH is used as the marginal model. Hence, we can conclude that different marginal models have similar dependence structure. However, the value of parameter estimations is slightly different. Thus, inaccurate marginal model will lead to inaccurate results for copula model.

This paper is only limited to one set of bivariate data, two GARCH-type models, four types of error distributions and four types of copulas. Future study is recommended to use more data set, various GARCH-type models with six error distributions including Generalized error distributions (GED) and skewed GED and other copula families. Instead of using daily closing price as a sample data, tick by tick data or minutes data of the Malaysia stock markets need to be considered as well.

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