

Robust Adaptive H_∞ Controller Based on Hybrid Genetic Wavelet Kernel Principal Component for Nonlinear Uncertain Systems

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ABSTRACT:

In this paper, two adaptive H_∞ control schemes based on a genetic wavelet kernel support vector machine (SVM) and a hybrid genetic wavelet kernel SVM is presented for nonlinear uncertain systems. In these methods, wavelet kernel SVM is employed to establish the adaptive controller and an on-line learning rule for the weighting vector and bias is obtained. The H_∞ control technique is combined with adaptive control algorithm and wavelet support vector machine to achieve the desired attenuation on the tracking error caused by wavelet-SVM approximation error and external disturbances. The most important characteristic of this strategy is its intrinsic robustness and its ability to treat the nonlinear behavior of the system. The results of simulation show that this SVM online algorithm controller is very effective and the SVM controller can achieve a satisfactory performance.

KEYWORDS: Genetic Algorithm (GA), Wavelet Support Vector Machines, Hybrid Wavelet and RBF Support Vector Machines, adaptive control, H_∞ Control, Nonlinear Uncertain System.

1. INTRODUCTION

Many significant results in the control design of nonlinear systems have been obtained in the recent decades. Applications of these approaches are restricted because they rely on the exact knowledge of the plant nonlinearities. Different approximators based on the adaptive control techniques have been used in the past decades to free some exact model limitations. Some approximators such as neural networks, fuzzy systems and wavelet functions have been employed due to their ability to handle the nonlinear behaviour of the systems and their inherent robustness in presence of uncertainties and external disturbances. These approximators have been widely used to present a model of unknown nonlinear system for the controller design [1-6].

Recently wavelet has been introduced as a powerful tool for approximation [7], [8]. So, it is valuable for us to investigate the problem of whether a better performance could be achieved in the control of nonlinear systems if we combine the wavelet technique with controllers. In [1], an adaptive wavelet-neural-network (WNN)-based H_∞ position tracking controller has been proposed that has combined the capability of neural networks for on-line learning ability and the

capability of wavelet decomposition for identification ability. In [9], a wavelet adaptive backstepping control (WABC) has been proposed for a class of nonlinear systems. This control scheme has combined the advantages of wavelet neural network identification, adaptive backstepping control, and L2 robust control techniques. In [10], a novel adaptive fuzzy wavelet neural sliding mode controller (AFWN-SMC) has been presented for a class of uncertain nonlinear systems. In the proposed scheme, composed of an Adaptive Fuzzy Wavelet Neural Controller (AFWNC) to construct an equivalent term of SMC and an Adaptive Proportional-Integral (A-PI) controller has been employed as switching control term of SMC. The present adaptive control techniques, such as neural network suffers from some bugs such as trapping in local minimum and lack of generalization ability. Researchers started to search more efficient methods due to these problems.

Support vector machines (SVM) proposed by Vapnik [11], [12] have been established as a general approximation tool for system modelling and identification. SVM uses a device called kernel mapping to map the nonlinear data in input space to a higher dimensional feature space where the data becomes linearly separable. There are many kinds of

kernels that can be used, such as the Gaussian (RBF), Polynomial, Sigmoid and Wavelet kernel. Wavelet kernel is a kind of multidimensional wavelet function that can approximate nonlinear functions. Wavelet kernel support vector machines can converge to minimum error with better sparsity [13-17]. In [18], the multiwavelet SVM has been proposed and applied to do regression and pattern recognition. So, wavelet SVM is used to find the optimal approximation in the space spanned by multidimensional wavelet kernels.

Although using approximation techniques represent a model of unknown nonlinear dynamic systems, but there exist approximation errors between the real system and approximated model. So, to overcome the approximation errors and external disturbances problems, several robust control approaches have been proposed. H^∞ optimization control theory is a very important and powerful tool for designing robust controllers and it has been widely used to attenuate the influence of approximation errors and external disturbances on the tracking error.

In this paper the robust tracking control for a class of nonlinear SISO systems with plant uncertainties and external disturbances is investigated. Two approximation methods, wavelet kernel SVM and a hybrid wavelet kernel SVM based on adaptive H^∞ control scheme, are proposed to attenuate the on-line tracking error caused by external disturbances and plant uncertainties. In the proposed control system, wavelet-SVM techniques are adapted to approximate unknown nonlinear dynamics in the plant and then an adaptive controller is presented.

The rest of this paper is organized as follows. Section 2 is assigned to introduce the problem formulation. In section 3, the wavelet support vector machine is represented. In section 4, the robust adaptive H^∞ controller based Wavelet Kernel principal component is presented. In section 5, the simulation results show the performance of the controllers and finally, a conclusion is provided in Section 6.

2. PROBLEM FORMULATION

Consider the nonlinear n -th order SISO system as the form:

$$\dot{x}^{(n)} = f(x, \dot{x}, \dots, x^{(n-1)}) + g(x, \dot{x}, \dots, x^{(n-1)})u + d \quad (1)$$

$$y = x$$

Where f and g are the uncertain but the bounded nonlinear functions,

$x = [x, \dot{x}, \dots, x^{(n-1)}]^T = [x_1, x_2, \dots, x_n]^T \in R^n$ is a vector of states which is assumed to be available, $u \in R, y \in R$ are the input and output of the system, respectively. D denotes a bounded external disturbance signal of the system.

The control problem is to force the output of the system y to track the given reference signal y_m . The output tracking error is denoted as:

$$e = y_m - y \quad (2)$$

The considered control goal in this paper is to design an adaptive H^∞ controller based on wavelet for nonlinear SISO system (1) under plant uncertainties and external disturbances such that the following conditions are satisfied:

(1) The boundedness of all variables of the closed-loop system is guaranteed.

(2) The following H^∞ tracking for the overall system is achieved [4]:

$$\int_0^T \underline{e}^T Q \underline{e} dt \leq \underline{e}^T(0) P \underline{e}(0) + \frac{1}{\gamma} \tilde{W}^T(0) \tilde{W}(0) + \rho^2 \int_0^T \omega^T \omega dt \quad (3)$$

$$\forall T \in [0, \infty) \quad \omega \in L_2[0, T]$$

where $\underline{e} = (e, \dot{e}, \dots, e^{(n-1)})^T$, $Q = Q^T \geq 0$ and $P = P^T \geq 0$, ω denotes the sum of matching errors caused by wavelet-SVM approximation errors and external disturbances. W is the parameter vector of wavelet-SVM, \tilde{W} is the wavelet-SVM parameter estimation error, $\gamma > 0$ is the learning rate of wavelet-SVM and $\rho > 0$ is a prescribed attenuation level.

Remark: while the system starts with initial conditions $\underline{e}(0) = 0, \tilde{W}(0) = 0$, then the H^∞ performance in (3) can be reduced as:

$$\sup_{\omega \in L_2[0, T]} \frac{\|\underline{e}\|_2}{\|\omega\|_2} \leq \rho \quad (4)$$

Where $\|\underline{e}\|_2^2 = \int_0^T \underline{e}^T \underline{e} dt$ and $\|\omega\|_2^2 = \int_0^T \omega^T \omega dt$. The above expression means that the L_2 -gain from ω to the tracking error \underline{e} must be equal to or less than ρ .

At the first stage, an adaptive wavelet-SVM algorithm with a parameter update law is employed to learn the behavior of nonlinearities. At the second stage, a robust controller is used to guarantee the desired H^∞ tracking performance in (3).

3. WAVELET-SVM AND ITS PARAMETER SELECTION

In this section genetic wavelet is discussed. GA wavelet combines the genetic algorithm and wavelet SVM technique.

3.1. Wavelet-SVM Regression

In this section, SVM regression is discussed. Suppose the training data are:

$$\{(x_k, y_k) | k = 1, 2, \dots, l\} \quad (5)$$

where $x \in R^n$ and $y \in R$, where R^n represents input space and R represents output space. The output of an SVM is written as:

$$f(x) = w^T \Phi(x) + b \quad (6)$$

where $\Phi(x): R^n \rightarrow R^{nh}$ is a nonlinear mapping from the input data x into a so-called high dimensional space, $w \in R^{nh}$ is the weight vector and $b \in R$ is the bias term. Model (6) is obtained by solving the following optimization problem:

$$\min \frac{1}{2} \|w\|^2 + C \sum_{i=1}^n |y_i - f(x_i, w)|_\varepsilon \quad (7)$$

Where $\varepsilon > 0$ is a small positive number and $C > 0$ is a regularization item. The second term is defined as:

$$|y - f(x, w)|_\varepsilon = \begin{cases} 0, & \text{if } |y - f(x, w)| < \varepsilon \\ |y - f(x, w)| - \varepsilon, & \text{otherwise} \end{cases} \quad (8)$$

By using Lagrange multiplier techniques, the dual of this optimization problem is:

$$W(\alpha^{(*)}) = -\varepsilon \sum_{i=1}^l (\alpha_i^* + \alpha_i) + \sum_{i=1}^l (\alpha_i^* - \alpha_i) y_i - \frac{1}{2} \sum_{i=1}^l \sum_{j=1}^l (\alpha_i^* - \alpha_i) (\alpha_j^* - \alpha_j) K(x_i, x_j) \quad (9)$$

Subject to:

$$\sum_{i=1}^l (\alpha_i^* - \alpha_i) = 0 \quad (10)$$

$$\alpha_i^{(*)} \in [0, C]$$

Then the approximation function takes the form:

$$f(x) = \sum_{i=1}^l (\alpha_i^* - \alpha_i) K(x, x_i) + b \quad (11)$$

Where $K(x_i, x_j)$ is a given function defined as kernel function of SVM and α_i is the Lagrangian coefficient.

In this paper, the wavelet technique has been combined with SVMs to construct WSVMs. The wavelet kernel is a kind of multidimensional wavelet function that can approximate arbitrary nonlinear functions. In the first control scheme, the wavelet kernel

$$K(x, x') = \prod_{i=1}^N h\left(\frac{x_i - x'_i}{a}\right) = \prod_{i=1}^N \left(\cos\left(1.75 \times \frac{(x_i - x'_i)}{a}\right) \exp\left(-\frac{\|x_i - x'_i\|^2}{2a^2}\right) \right) \text{ is chosen}$$

as the kernel function [15], [19]. Then in the second control scheme, the mixed wavelet RBF kernel

$$K(x, x') = \left(2 \sin c\left(2 \frac{(x_i - x'_i)}{2a}\right) - \sin c\left(\frac{(x_i - x'_i)}{2a}\right) \right) \exp\left(-\frac{\|x_i - x'_i\|^2}{2\sigma^2}\right)$$

is chosen as the kernel function [16]. So, it is valuable for us to study the problem of whether a better performance could be achieved if we combine the wavelet SVM technique with controllers.

3.2. Parameter selection based on GA algorithm

The GA algorithm has been proven as a powerful tool

for solving optimal for a given optimization problem. In this subsection, a binary GA algorithm is used to find an optimal choice of the kernel width σ and the regularization parameter C . To execute the genetic algorithm, a cost function should be defined in the beginning. The mean square error of the approximated system is chosen as a cost function defined by:

$$MSE = \frac{1}{N} \sum_{i=1}^N (f(x(i)) - \hat{f}(x(i)))^2 = \frac{1}{N} \sum_{i=1}^N e^2(i) \quad (12)$$

Where N is the number of given sampling steps.

The binary GA works with bits and it begins by defining a chromosome or an array of variable values to be optimized. It provides better searching capability compared to the traditional gradient method because the gradient method searches for a problem solution only from a single direction, while GA algorithm is from multiple directions due to its cross over and mutation operations [20]. a simple GA works as follows:

1. Start with a randomly generated population of n l -bit chromosomes (candidate solutions to a problem).
2. Calculate the fitness $f(x)$ of each chromosome x in the population.
3. Repeat the following steps until n offspring have been created:

a. Select a pair of parent chromosomes from the current population, the probability of selection being an increasing function of fitness. Selection is done "with replacement," meaning that the same chromosome can be selected more than once to become a parent

b. With probability pc (the "crossover probability" or "crossover rate"), cross over the pair at a randomly chosen point (chosen with uniform probability) to form two offspring. If no crossover takes place, form two offspring that are exact copies of their respective parents. (Note that here the crossover rate is defined to be the probability that two parents will cross over in a single point. There are also "multi-point crossover" versions of the GA in which the crossover rate for a pair of parents is the number of points at which a crossover takes place.)

c. Mutate the two offspring at each locus with probability pm (the mutation probability or mutation rate), and place the resulting chromosomes in the new population. If n is odd, one new population member can be discarded at random.

4. Replace the current population with the new population.

5. Go to step 2. [21].

4. ADAPTIVE H_∞ CONTROLLER DESIGN AND STABILITY ANALYSIS BASED ON WAVELET-SVM

In this section, the adaptive H_∞ controller is designed for nonlinear SISO system (1) under plant uncertainties and external disturbances. This control scheme

guarantees boundedness of all the variables of closed loop system and output tracking of desired trajectory $y_m(t)$ in the presence of model uncertainty and external disturbance.

First, let $\underline{k} = [k_n, \dots, k_1]^T \in R^n$ such that all roots of the polynomial $h(s) = s^n + k_1 s^{n-1} + \dots + k_n$ are in the open left-hand complex plane. If the functions $f(\underline{x})$ and $g(\underline{x})$ are known and $d = 0$, the control law:

$$u^* = \frac{1}{g(\underline{x})} (-f(\underline{x}) + y_m^{(n)} + k^T e) \quad (13)$$

applied to the system (1) can result in the following asymptotically error for dynamic system:

$$e^{(n)} + k_1 e^{(n-1)} + \dots + k_n e = 0 \quad (14)$$

Which implies that the tracking error converge to zero: $\lim_{t \rightarrow \infty} e(t) = 0$ (15)

In practice, system is uncertain and $d \neq 0$ or its dynamic functions $f(\underline{x})$ and $g(\underline{x})$ are generally unknown, then the optimal control u^* cannot be applicable. To overcome this problem, the signal control u would be planned which uses an adaptive control $\hat{u}(x|W)$ based on wavelet-SVM in order to approximate this optimal control and a H_∞ robust control v to attenuate the effect on the tracking error caused by wavelet-SVM approximation errors, external disturbances and model uncertainties. Fig. 1 depicts the block diagram of the control system.

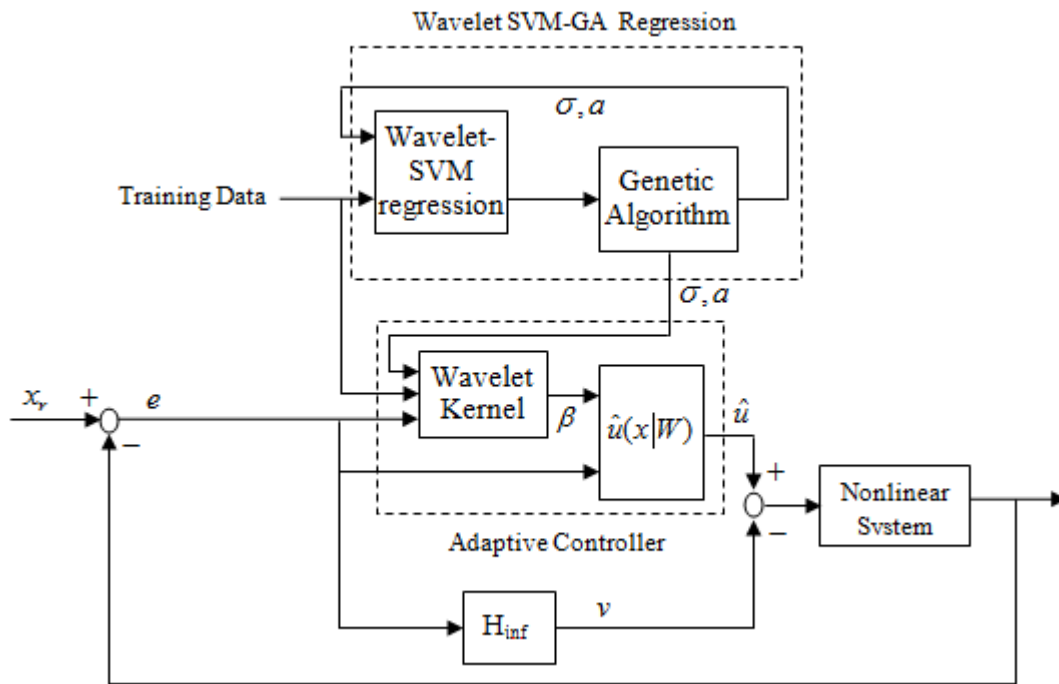


Fig. 1. The system block diagram of the control scheme

Consider the nonlinear system (1) with given nonlinear functions $f(x), g(x)$. Then, the control input u is chosen as:

$$u = \hat{u}(x|W) - v \quad (16)$$

By applying (16) to (1) and some straightforward manipulation, the tracking error dynamic equation is obtained as follows:

$$\dot{e} = A e + B v + B[u^*(x) - \hat{u}(x|W)] - B \frac{d}{g(x)} \quad (17)$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -k_n & -k_{n-1} & -k_{n-2} & \dots & -k_2 & -k_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ g(x) \end{bmatrix}$$

As mentioned before, Wavelet-SVM is employed to approximate the optimal control. The output of the wavelet-SVM approximator is defined as follows:

$$\hat{u}(x|W) = W^T \beta \quad (18)$$

where $\beta(x) = [1, K(x_1, x), \dots, K(x_N, x)]^T$ and

$$W = [w_1 \ w_2 \ \dots \ w_{N+1}]^T.$$

The wavelet-SVM approximator is valid under the

following assumption:

Assumption: let x belongs to a compact set $M_x = \{x \in R^n : \|x\| \leq m_x\}$ and m_x is a designed parameter. It is known that optimal parameter vector W lies in a convex region

$$M_W = \{W \in R^h : \|W\| \leq m_w\} \quad (19)$$

Where m_w is constant.

The optimal weight vector W^* is defined as follows:

$$W^* = \arg \min_{W \in M_W} [\sup_{x \in M_x} |\hat{u}(x|W) - u^*|] \quad (20)$$

Where M_W, M_x denote the sets of suitable bounds on W, x , respectively. Also the minimum approximation error is defined as follows:

$$\omega_e = u^*(x) - \hat{u}(x|W^*) \quad (21)$$

Substituting (18), (21) into (17), the tracking error equation (17) can be rewritten as:

$$\begin{aligned} \dot{e} = & Ae - B\{[u^*(x) + \hat{u}(x|W^*)] \\ & + [\hat{u}(x|W^*) - \hat{u}(x|W)]\} + Bv - B \frac{d}{g(x)} \end{aligned} \quad (22)$$

or, equivalently

$$\dot{e} = Ae - B\tilde{W}^T \beta + Bv + B\omega \quad (23)$$

where $\omega = \omega_e - d/g(x)$, $\tilde{W} = W - W^*$ is the adaptation error of the estimation parameter W .

$$\dot{W} = \gamma e^T PB\beta \quad (24)$$

Theorem [22]: for the nonlinear system (1) if the adaptive control law based on wavelet-SVM is chosen as (16) with the adaptive control law as (23) and the H_∞ robust control law as follows:

$$v = -\frac{1}{r} B^T P e \quad (25)$$

Where $r > 0$ is a design parameter and $P = P^T \geq 0$ satisfying the following Riccati equation:

$$PA + A^T P + Q - \frac{2}{r} PBB^T P + \frac{1}{\rho^2} PBB^T P = 0 \quad (26)$$

Where $r \leq 2\rho^2$. Then the H_∞ tracking performance from the external disturbance to the tracking error is achieved for a prescribed attenuation level ρ .

Proof: the Lyapunov function is chosen as follows:

$$V = \frac{1}{2} e^T P e + \frac{1}{2\gamma} \tilde{W}^T \tilde{W} \quad (27)$$

By the fact $\dot{W} = \dot{\tilde{W}}$, time derivative of this function is as follows:

$$\dot{V} = \frac{1}{2} \dot{e}^T P e + \frac{1}{2} e^T P \dot{e} + \frac{1}{\gamma} \dot{\tilde{W}}^T \tilde{W} \quad (28)$$

Substituting (23), (24), (25) into (28), we have:

$$\begin{aligned} \dot{V} = & \frac{1}{2} \dot{e}^T P e + \frac{1}{2} e^T P \dot{e} + \frac{1}{\gamma} \dot{\tilde{W}}^T \tilde{W} \\ = & \frac{1}{2} [e^T A^T P e - \frac{1}{r} e^T P^T B B^T P e - \beta \tilde{W}^T P e \\ & + \omega B^T P e + e^T P A e - e^T P B \tilde{W}^T \beta \\ & - \frac{1}{r} e^T P B B^T P e + e^T P B \omega + \frac{1}{\gamma} (2\tilde{W}^T \dot{\tilde{W}})] \\ = & \frac{1}{2} e^T (A^T P + P A - \frac{2}{r} P B B^T P) e + \frac{1}{2} e^T P B \omega \\ & + \frac{1}{2} \omega^T B^T P e - \tilde{W}^T (e^T P B \beta - \frac{1}{\gamma} \dot{\tilde{W}}) \end{aligned} \quad (29)$$

From the adaptive law (24) and the Riccati equation (26), we achieve:

$$\begin{aligned} \dot{V} = & -\frac{1}{2} e^T Q e - \frac{1}{2\rho^2} e^T P B B^T P e \\ & + \frac{1}{2} (e^T P B \omega + \omega^T B^T P e) \\ = & -\frac{1}{2} e^T Q e - \frac{1}{2} \left(\frac{1}{\rho} B^T P e - \rho \omega \right)^T \left(\frac{1}{\rho} B^T P e - \rho \omega \right) \\ & + \frac{1}{2} \rho^2 \omega^T \omega \leq -\frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 \omega^T \omega \end{aligned} \quad (30)$$

Then, we integrate (30) in $t = [0, T]$ and it yields:

$$V(T) - V(0) \leq -\frac{1}{2} \int_0^T e^T Q e dt + \frac{1}{2} \rho^2 \int_0^T \omega^T \omega dt \quad (31)$$

Since $V(T) \geq 0$, from (27), the following inequality is obtained from (31):

$$\begin{aligned} \frac{1}{2} \int_0^T e^T Q e dt \leq & \frac{1}{2} e^T(0) P e(0) + \frac{1}{2\gamma} \tilde{W}^T(0) \tilde{W}(0) \\ & + \frac{1}{2} \rho^2 \int_0^T \omega^T \omega dt \end{aligned} \quad (32)$$

This implies that for the nonlinear system with the external disturbances and the uncertain parameters described by (1), when the robust adaptive control law (16) with the parameter adaptive law (24) is applied to the tracking control, all signals of the closed-loop system are uniformly bounded and the output tracking e satisfies the H_∞ tracking performance.

5. SIMULATION EXAMPLES

The wavelet-SVM and the mixed wavelet RBF-SVM are adapted to establish the adaptive controller. Then in order to find the effectiveness and superiority of the proposed control schemes, the results are compared with the obtained results by a scheme of adaptive control based on least squares support vector machines (LS-SVM) [23].

Example 1- Duffing forced oscillation system is defined as follows [24]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.1x_2 - x_1^3 + 12\cos(t) + u + d \end{aligned} \quad (33)$$

$$y = x_1$$

where $f = -0.1x_2 - x_1^3 + 12\cos(t)$, $b = 1$.

The control objective is to maintain the system to track the desired reference signal $y_d = \sin(t)$ in the presence of a square wave disturbance with the amplitude ± 1 and the period 1 for the initial conditions $x_1(0) = x_2(0) = 0$. The feedback gain matrix is chosen as $k = [k_2, k_1]^T = [1, 2]^T$.

To get the training data, the Gaussian noise with zero mean and standard deviation 1 is selected as the input. Using binary GA algorithm with estimated generalization error as the cost function, the optimal set of (C, a) is obtained as $(5.6693, 9.1342)$ for the wavelet kernel function and the optimal set of (C, a, σ) is obtained as $(7.3501, 4.5611, 5.2403)$ for the wavelet-RBF kernel function.

The H_∞ designing parameters are obtained as follows: Let the positive-definite matrix $Q = 10I_{2 \times 2}$ and the given prescribed attenuation levels $\rho = 0.1, 0.05$ and $r = 0.02, 0.005$.

After solving the Riccati equation (26), we have:

$$P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$$

The proposed control scheme is applied to the system (33), and the output tracking error, input control and output response are presented. The simulation results are illustrated in Fig. 2 ($\rho = 0.1$) and Fig. 3 ($\rho = 0.05$)

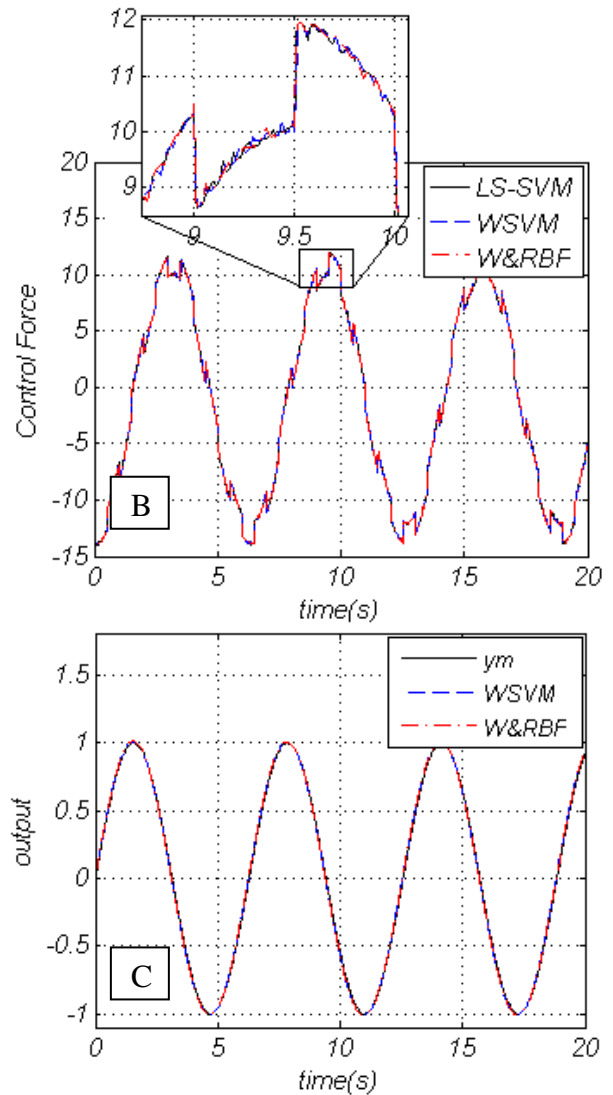
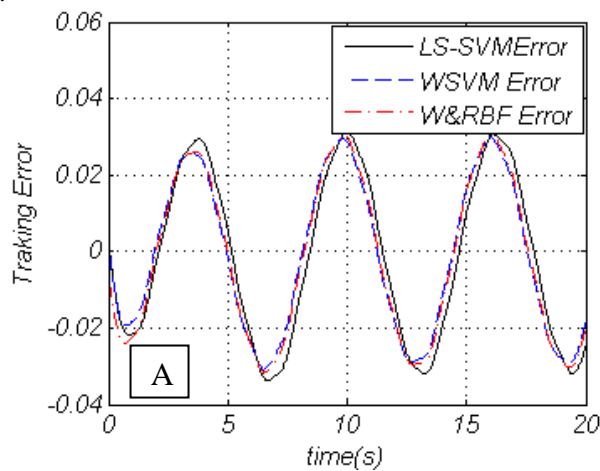
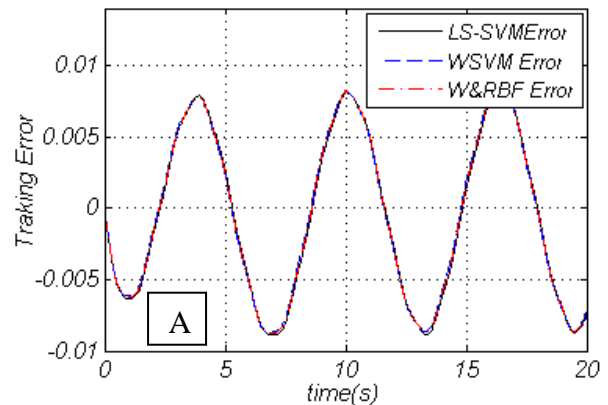


Fig. 2. Simulation results with $\rho = 0.1$: A. the tracking error, B. the control input, C. output response.



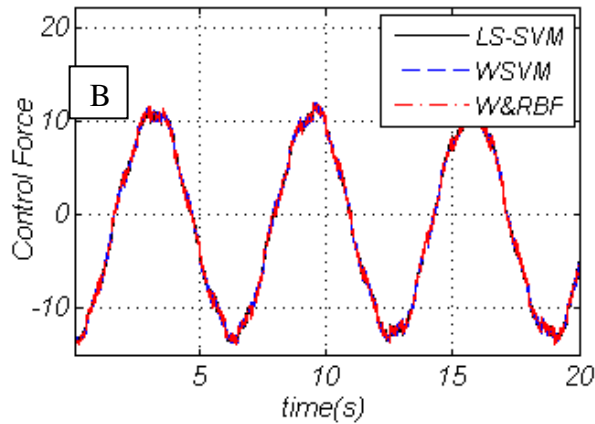


Fig. 3. Simulation results with $\rho = 0.05$: A. the tracking error, B. the control input.

It has been concluded through simulation results that under the smaller value of attenuation level ρ , the tracking error is reduced while the value of control input is bigger. Furthermore, the value of the control signal and a quantitative comparison of tracking error, mean square error (MSE), obtained by three approaches, have been reported in Table. 1. According to this table, the value of the control input and the mean square error (MSE) are decreased under the two proposed control schemes compared with the adaptive control based on least squares support vector machines (LS-SVM).

Example 2- in this Example, we investigate the performance of the inverted pendulum system under the proposed control scheme.

The inverted pendulum system is considered as the following equation:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g \sin(x_1) - (mLx_2 \sin(x_1) \cos(x_1)) / (m_c + m)}{L[4/3 - m \cos^2(x_1) / (m_c + m)]} \\ &+ \frac{\cos(x_1) / (m_c + m)}{L[4/3 - m \cos^2(x_1) / (m_c + m)]} u + d \end{aligned} \quad (34)$$

$$y = x_1$$

Where $x_1 = \theta$ and $x_2 = \dot{\theta}$ denote the angular position and velocity of the pole, respectively. $g = 9.8m/s^2$ is the acceleration due to gravity. m_c and m denote the mass of the cart and the mass of the pole respectively. l is the half length of the pole and the external disturbance d is a square wave with the amplitude ± 1 and the period 1 for the initial condition

$x_1(0) = x_2(0) = 0$. Assume that $k = [k_2, k_1]^T$ is the same as in Example 1.

To get the training data, the Gaussian noise with zero mean and standard deviation 1 is selected as the input. Using binary GA algorithm with estimated generalization error as the cost function, the optimal set of (C, a) is obtained as $(4.8446, 3.8323)$ for the wavelet kernel function and the optimal set of (C, a, σ) is obtained as $(1.29609, 3.2009, 8.4529)$ for the wavelet-RBF kernel function.

Assume that Q is the same as in Example 1. Consider also the given prescribed attenuation level $\rho = 0.1, 0.5$ and $r = 0.02, 0.005$. Then the solution of the Riccati equation is the same as before.

The proposed control scheme is applied to the system (34), and the output tracking error, input control and output response are presented. The simulation results are shown in Fig.4 ($\rho = 0.1$) and Fig. 5 ($\rho = 0.05$).

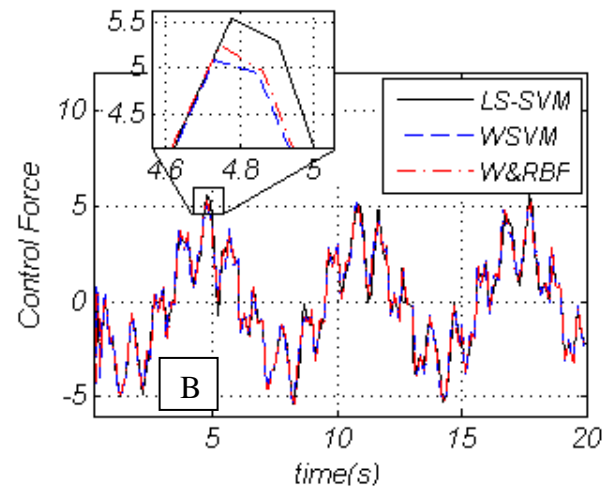
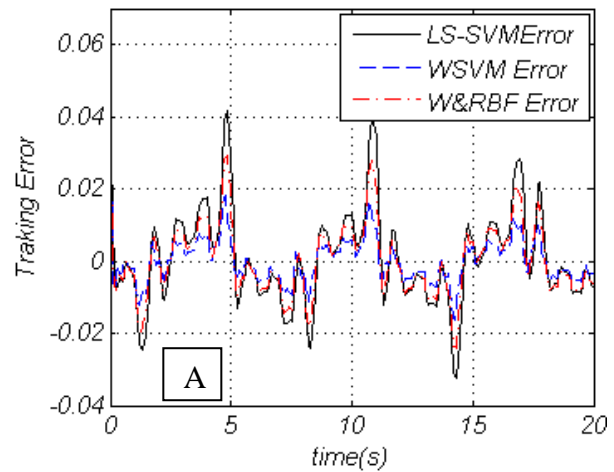


Table 1. The values of the control input and MSE criterion of three approaches

ρ	Criteria	Wavelet-SVM	Wavelet & RBF-SVM	LS-SVM [22]
$\rho = 0.05$	$\int u^2(t)dt$	2.2773×10^3	2.2794×10^3	2.2803×10^3
	$MSE(\sum_i E(e_i^2))$	3.3347×10^{-5}	3.3995×10^{-5}	3.9284×10^{-5}
$\rho = 0.1$	$\int u^2(t)dt$	1.6928×10^3	1.692×10^3	1.6932×10^3
	$MSE(\sum_i E(e_i^2))$	5.0016×10^{-5}	5.2643×10^{-5}	5.415×10^{-5}

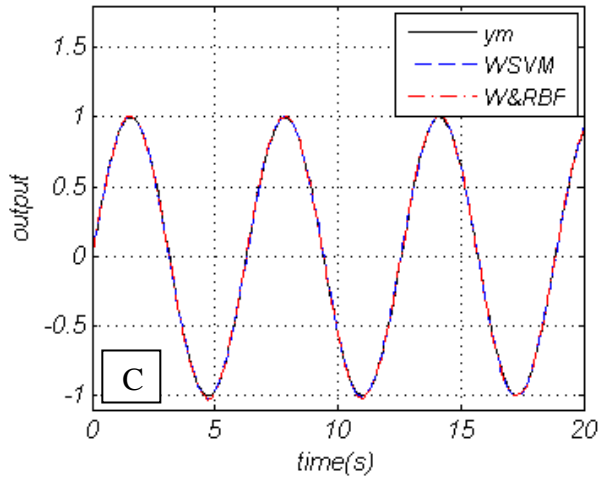


Fig. 4. Simulation results with $\rho = 0.1$: A. the tracking error, B. the control input, C. output response.

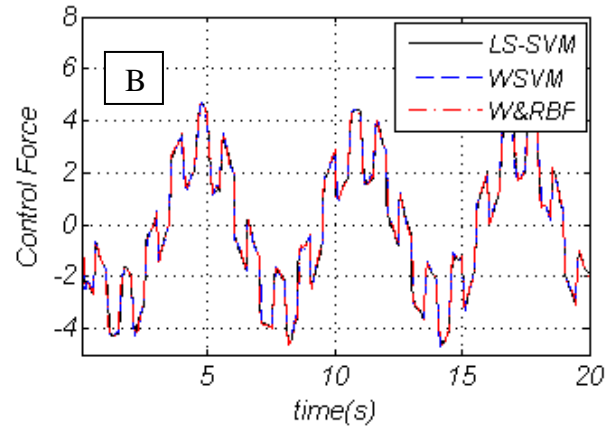
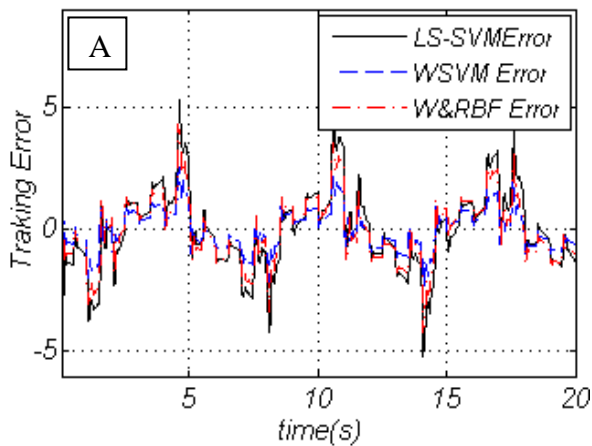


Fig. 5. Simulation results with $\rho = 0.05$: A. the tracking error, B. the control input.

Once again, the simulation results show that under the smaller value of attenuation level ρ the tracking error is reduced while the value of control input is bigger. Moreover, the value of the control signal and mean square error (MSE) of three methods are reported in Table 2. Table 2 shows that the mean square error (MSE) and the value of the control input are decreased significantly using the two proposed control schemes compared with the adaptive control based on least squares support vector machines (LS-SVM).

Table 2. The values of the control input and MSE criterion for three approach

ρ	Criteria	Wavelet-SVM	Wavelet & RBF-SVM	LS-SVM [22]
$\rho = 0.05$	$\int u^2(t)dt$	170.034	169.5067	171.8786
	$MSE(\sum_i E(e_i^2))$	7.4668×10^{-5}	8.8376×10^{-5}	1.0504×10^{-4}
$\rho = 0.1$	$\int u^2(t)dt$	166.2227	165.462	167.3584
	$MSE(\sum_i E(e_i^2))$	9.62923×10^{-5}	1.1737×10^{-4}	1.5912×10^{-4}

6. CONCLUSION

In this paper, an adaptive H_∞ control scheme based on the wavelet-SVM and the mixed wavelet RBF-SVM is proposed for a class of uncertain nonlinear systems. First, wavelet-SVM techniques are used to establish the adaptive controller to learn the behaviours of uncertain nonlinear dynamic. Then, the effect on the tracking error caused by wavelet-SVM approximation errors and external disturbances is compensated by the robust H_∞ controller. The obtained simulation results show that the proposed control scheme is quite effective in the control of uncertain nonlinear systems.

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