

# Applying a Multi-criteria Group Decision-making Method in a Probabilistic Environment for Supplier Selection (Case study: Urban Railway in Iran)

Azam Modares<sup>a</sup>, Nasser Motahari Farimani<sup>a,\*</sup>, Vahideh Bafandegan Emroozi<sup>a</sup>

<sup>a</sup> Department of Management, Faculty of Economics and Administrative Sciences, Ferdowsi University of Mashhad, Mashhad, Iran.

Received 21 January 2022; Revised 17 February 2023; Accepted 19 February 2023

## Abstract

Supplier selection of urban train signaling equipment is one of the main problems in urban railways due to the serious impact of this equipment on the safety of travelers. The supplier's selection is a multi-criteria decision-making (MCDM) problem, in which the preferences over criteria are highly dependent on the opinions of decision-makers (DMs). The focus of the present study is to provide an appropriate model of multi-criteria group decision making in a probabilistic setting to effectively combine DMs' judgments. For this purpose, the Bayesian hierarchical model with the best-worst method (BWM) is used to determine the weights of criteria and the Vise Keriterijumska Optimizacija I Kompromisno Resenje (VIKOR) is used to prioritize suppliers. Bayesian BWM aggregates the opinions of all DMs at once, instead of averaging the individual opinions of the DMs that underlie MCDM methods. In this method, the probability of preference of one criterion over one another is calculated using Markov-chain Monte Carlo (MCMC), in addition to the weight of criteria, so that the confidence between pairs of criteria is revealed and ranking of criteria become more certain. Given that the average obtained confidence levels are 0.95, the validity of the Bayesian BWM results is confirmed. For different values of the VIKOR parameter, the third supplier will have the highest rating and the fifth supplier will have the lowest rating. The introduced multi-criteria decision model in this research will help the decision-makers of the urban railway company and other organizations with several suppliers to be able to select the best supplier by considering the relevant criteria.

**Keywords:** Group decision-making; Supplier selection; Bayesian BWM; VIKOR; Aggregating of DMs opinions.

## 1. Introduction

Nowadays, the selection of the most appropriate supplier as a strategic problem has been considered by many organizations (Kumar, Jain, & Kumar, 2014; Banaeian, Moblia, & Fahimnia, 2016). The nature of the decision-making to supplier selection is usually complex and lacks a clear structure. As the sensitivity of the product increases, the problem of decision-making becomes more complex and therefore requires more attention. Traditional approaches for supplier selection only considered economic criteria (Zhang, Zhang, & Lai, 2009; Goren, 2018; Liu, Eckert, Yannou-Le Bris, & Petit, 2019). To select the best supplier, a trade-off must be performed between conflicting qualitative and quantitative criteria (Burgess, Singh, & Koroglu, 2006; Cakravastia & Takahashi, 2004). The purchasing manager should always evaluate different criteria to select the best suppliers (Noci, 1997; Zhang, Zhang, & Lai, 2009). The overall goal of the supplier evaluation process is to identify the suppliers with the highest potential to meet the needs of the company at an acceptable cost and risk (Ecer & Pamucar, 2020; Büyüközkan & Çifçi, 2011). The problem of selecting signaling equipment suppliers is one of the fundamental and critical problems in the

the selection of the best suppliers guarantees traffic safety. Because any potential defect in the signaling system poses many dangers to the moving trains, purchasing management must make sufficient analysis to select its suppliers (Roozkhosh, Pooya, & Agarwal, 2022). Supplier selection of urban railway equipment is not an easy task and the complexity of this choice is because each supplier meets a part of the purchase criteria and selection of the best supplier requires a structured and systematic approach (Roozkhosh & Motahari Farimani, 2022; Ahadi, Mahpour, & Taraghi, 2018). Therefore, considering that urban railway equipment strongly affects the safety of traffic and plays an important role in preventing accidents, supplier selection of this equipment is very important. Any approach that is used for supplier selection of urban railway equipment must account for a variety of criteria such as reliability, production capability, technical capability, price, quality, after-sales service, time-on delivery, etc. Multi-criteria decision-making (MCDM) techniques help the decision-maker to assess all these criteria. Since there are many criteria involved in the problem of supplier's selection of urban railway, it is a MCDM in which the degree of data reliability, the number of decision-makers, and aggregate their opinion

\*Corresponding author Email address: n.motahari@um.ac.ir.

must be considered. The purpose of MCDM is to integrate objective survey data with the subjective judgments of experts to provide effective management of information in formulating optimal strategies (Hsu & Hu, 2007; Lo, et al., 2020; Hsu & Hu, 2009; Shahhoseini & Yousefinejad Attari, 2018).

Given that MCDM is based on expert judgment as a group, after identifying the criteria for ranking suppliers, it is necessary to systematically combine decision-makers' evaluations (Kumar Kar, 2014; Bafandegan Emroozzi & Fakoor, 2023; Alinezhad & Seif, 2020). In group decision-making methods, there are two approaches to aggregate opinions of decision makers that are based on pairwise comparison. In the first approach, all experts make decisions separately and at the same time, and then their opinions are integrated into one and the resulting aggregated pairwise comparisons are treated like a single decision maker problem (Forman & Peniwati, 1998; Blagojević, et al., 2016). In the second approach, the criteria are weighed separately by each expert and the final weights are obtained by the geometric or arithmetic averaging (Morais & Ahmeid, 2012). The averages are, however, sensitive to outliers and provide limited information according to the general preferences of the decision makers (Mohammadi & Rezaei, 2020). Therefore, to solve these problems, an approach must be used that can gather the opinions of experts at once and obtain the weights of the criteria as well as their confidence based on availability of information. In this study, the Bayesian BWM method, which is one of the most effective probabilistic and random approaches in group decisions, is used to calculate the weights of criteria for a group of decision makers (Roozkhosh & Kazemi, 2022). The BWM is flexible and effective and has two main advantages: the BWM produces the comparison relations such that fewer comparisons than in the pairwise comparison matrix in the AHP technique is needed, and the weights calculated by the BWM with practical cases are more consistent (Aboutorab, Saberi, Hussain, & Chang, 2018). In this approach, the weights of the criteria are obtained by a group of experts using the Bayesian hierarchical model, which is a probabilistic method for modeling uncertainty, and obviates the disadvantages mentioned in the two methods above.

Since in real-world problems the input information is not deterministic to the problem, information uncertainty with probable nature is considered and the optimal weights are obtained by using probabilistic distributions. In MCDM, when the weights of the criteria are obtained, a higher weight for a criterion indicates that it is superior to others. Nevertheless, the confidence of superiority of one criterion over another criterion cannot be determined only by comparing two numbers. This problem is more important when the weight vector shows the group preferences of decision makers (Mohammadi & Rezaei,

2020). Therefore, in this study, using Markov-chains Monte Carlo (MCMC), the confidence of the relation is measured among various criteria. Therefore, in addition to obtaining the weight of the criteria, the degree of superiority of the criteria over each other is also calculated. Then, using the VIKOR method, which ranks alternatives and specify the solution that is the closet to the ideal, the effect of each of the criteria on the supplier selection is determined.

While the problem of supplier selection has been widely used in previous studies, most research has used supplier selection using multi-criteria group decision-making methods that ultimately average their opinions. So far, in group decisions, the approach that can be used to collect the judgment of decision makers at once and simultaneously in a probabilistic environment has not been considered in the supplier's selection. In the literature, many MCDM approaches have been performed to evaluate the process of supplier selection. For example, Wang and Cai (2017) applied a group decision-making model using distance-based VIKOR to solve emergency supplier selection problems with heterogeneous information. Rezaei et al. (2016) proposed the BWM using environmental criteria incorporating and traditional business for supplier selection. Wan et al. (2017) proposed an integrated method consisting of ANP and ELECTRE II for supplier selection in the context of interval 2-tuple linguistic variables. Safaei Ghadikolaei and Valipour Parkouhi, (2017) developed a resilience method using fuzzy ANP and grey VIKOR methods for supplier selection. Liu et al. (2019) applied an integrated MCDM approach that combined MULTIMOORA and interval-valued intuitionistic uncertain linguistic sets for supplier selection in a group environment. In Bai et al. (2019), a grey-based group decision-making method using the BWM and TODIM was developed for supplier selection. Liu et al. (2019) used the alternative queuing method (AQM) and BWM to select suppliers under the interval-valued intuitionistic uncertain linguistic environment.

Sayadi, Heydari and Shahanaghi (2009) developed the VIKOR approach for criteria with some duration of the interval, and it can rank the suppliers based on obtained weights. Luthra et al. (2017) presented an integrated VIKOR and AHP model for evaluation of supplier selection due to the increased pressure from the government policies. Sanayei et al. (2010) studied on group decision-making process using the VIKOR approach for supplier selection in the automobile part manufacturing industry. Roostaei et al. (2012) to evaluate supplier selection problems extended the VIKOR method with IFS theory. Sanayei et al. (2010) developed the fuzzy VIKOR method to handle MCDM supplier selection problems, taking non-commensurable and conflicting criteria. Lo et al. (2018) developed a hybrid model of extended fuzzy TOPSIS, BMW for solving supplier selection problems. Lo et al. (2020) developed an integrated model for solving problems in supplier

selection and order allocation. Kazemitash et al. (2021) are concerned with the hybrid of the Best Worst method rough set theory to evaluate the supplier selection problem of biofuel companies. Banaeian et al. (2018) applied fuzzy set theory into TOPSIS, VIKOR, and GRA methods. The methods are then utilized to solve supplier selection studies for the agricultural industry. However, to date, it is clear that the Bayesian BWM with hesitant VIKOR model has not been studied in group MCDM. Probabilistic uncertainties in supplier selection have been investigated in very limited studies (Nepal & Yadav , 2015; Lei, Wei, & Gao, 2020; Li & Wang, 2017; Liu, Quan, Li, & Wang, 2019; Hosseini & Barker, 2016; Lei, Wei, & Gao, 2020).

The main contributions in this paper are as follows: The first contribution of this research is obtaining the confidence of the relation among various criteria in addition weight of criteria. Therefore, decision-makers will be more certain of the relation of two criteria if the confidence level between these is high. The second contribution is aggregating the opinions of DMs properly so that information is not lost using averaging. The weight of criteria is obtained based on the judgment of decision-makers at once and simultaneously. Also, studies have been conducted on the selection of suppliers in industrial, agricultural, food, etc., and supplier selection in service environments, especially urban railway not performed. Therefore, the purpose of this study, while identifying the criteria for selecting suppliers of urban railway equipment, is to select the best supplier according to these criteria so that the safety of urban railways can be achieved with a better confidence in these organizations.

The remainder of the paper is organized as follows. Section 2 describes the literature and theoretical foundations of the subject. Section 3 discusses the research process. The findings are presented in Section 4. The conclusion is given in Section 5.

## 2. Literature Review

### 2.1. Best-Worst method (BWM)

BWM is one of the MCDM methods based on pairwise comparisons, which was introduced by Rezaei in 2015 (Rezaei, 2015). In this method, the best and worst criteria are determined by the decision-maker and a paired comparison between each of these two criteria with other criteria is done (Bai and Sarkis, 2014; Liu, Xiao, Ji, Wang, and Tsai, 2018; Modares (c), et al., 2022). The following steps are performed to determine the weight of the criteria using the BWM method:

1. DM determines a set of criteria.
2. DM determines the best and worst criteria from the set of criteria.

3. DM makes pairwise comparisons between the best criterion with the other of the criteria ( $A_B$ ) and the other of the criteria with the worst criteria ( $A_W$ ). Eqs. (1) and (2) show the vectors of pairwise comparisons.

$$A_B = (a_{B1}, a_{B2}, \dots, a_{Bn}) \quad (1)$$

$$A_W = (a_{W1}, a_{W2}, \dots, a_{Wn})^T \quad (2)$$

where is  $a_{bj}$  pair comparison between the best criterion with criterion  $j$  and  $a_{wj}$  is pair comparison of criterion  $w$  with worst criterion. The vectors  $A_B$  and  $A_W$  show only the preferences of a decision-maker.

### 4. Obtaining optimal weights

The following linear model is solved to obtain the optimal weight. Optimal weights vector ( $w_1^*, w_2^*, \dots, w_n^*$ ) and  $\varepsilon$  are obtained by solving the model 3 (Noci, 1997; Rezaei, 2016).

$$\begin{aligned} & \min \varepsilon \\ & \text{s.t. } \frac{w_B}{w_j} - a_{Bj} \leq \varepsilon \quad \forall j \\ & \quad \frac{w_j}{w_w} - a_{jw} \leq \varepsilon \quad \forall j \\ & \quad \sum_{j=1}^n w_j = 1 \quad w_j \geq 0 \end{aligned} \quad (3)$$

### 2.2. Bayesian statistics

In the Bayesian technique, in addition to observations, basic information is also important and is considered in decision making. In this technique, unknowns are random variables and a probability function is obtained for unknowns. The Bayesian School requires an initial estimate of the researcher's information, which is expressed as a probability function and is called the prior distribution. Observations are then made and information about the unknowns is collected by the researcher, and using this new information, the initial probability function is updated and the posterior distribution is obtained. Eq. (4) Shows how to obtain the posterior distribution.

$$p(\theta|D) = p(D|\theta) \times p(\theta) \quad (4)$$

where  $p(\theta)$  represents the prior distribution,  $p(D|\theta)$  is the likelihood function, and  $p(\theta|D)$  is the posterior distribution (Goldstein, 2011).

### 2.3. Markov-chain monte carlo (MCMC)

MCMC is one of the most important stochastic processes used to estimate parameters such as mean, variance, and expected values and estimate the posterior distribution of Bayesian models. These methods are used for numerical approximations of multidimensional integrals. Calculating these integrals using analytical methods is very difficult and often impossible, so the MCMC method can use dependent simulations for posterior distribution. This method simulates Markov chains that contain samples of

the target distribution. Therefore, using these samples, it estimates the target distribution well. These methods are used to calculate Bayesian hierarchical models that must combine many unknown parameters. In this research, the Gibbs sampling method has been used as one of the MCMC methods. The integrals can be well approximated using the obtained sampling from MCMC. This method is designed for samples that the next sample depends on the existing sample, which is called the Markov chain. This leads the algorithms to produce the approximate value of the distribution with a large number of random variables (Gilks, 1995).

2.4. Bayesian BWM

This method was first introduced by Mohammadi and Rezaei (Mohammadi & Rezaei, 2020). In this method, the Bayesian hierarchical model is used to estimate weights in an original BWM framework. The inputs and parameters in Bayesian BWM are the same as the inputs to the original BWM, which are paired comparison vectors. In this method, from a probabilistic point of view, the criteria are considered random events. So, their weights are likelihood of their occurrence. Because the pairwise comparison vectors are integers by each decision maker, multinomial distributions are used to model criteria. In a multinomial distribution,  $A_w$  and  $A_B$  are preference vectors of all vectors over the worst vector and preference vector of best criteria over other criteria, respectively that show the number of occurrences of each event. The Dirichlet distribution is also used to model the weight vector.

The BWM-Bayesian method up to step 3 is similar to the BWM method described in Section 1-2. If we assume that exists  $X$  decision -makers, the vector of the total preference of the decision-makers is denoted by  $A_x^B$  and  $A_x^W$ . In this method,  $w^{1:x}$  which is the weight vector of the decision-makers, is first calculated and then  $w^{agg}$  is obtained from their average which shows the final cumulative weight vector. To obtain  $w^{1:x}$  and  $w^{agg}$  simultaneously, the Bayesian hierarchical model, which is shown in Figure 1, is used.

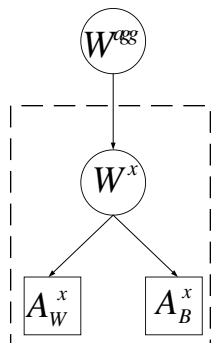


Fig .1. BWM Bayesian hierarchical model

Circles in Figure 1 are variables that need to be estimated, and squares represent the input variables that are the original BWM inputs. It is clear the value of  $w^x$  depends on  $w^{agg}$  and value of  $w^x$  is dependent on  $A_B^x$  and  $A_W^x$ , and  $w^{agg}$  is computed based on the weight of  $x$  decision makers shown by  $w^x$ . For performing each statistical inference, we need to write the joint probability distribution of all random variables given the available data. In this joint probability distribution,  $A_B^x$  and  $A_W^x$  are available data. To estimate and obtain  $w^{1:x}$  and  $w^{agg}$ , we must obtain the cumulative distribution that is given in Eq. (5).

$$p(w^{1:x} | w^{1:x}, A_W^{1:x}, A_B^{1:x}) \propto p(w^{agg}) \times \prod_{x=1}^X p(A_W^x | w^x) p(A_B^x | w^x) p(W^k | w^{agg}) \tag{5}$$

To estimate weights, we must obtain the probabilistic distributions of each element in Eq. (4).  $p(A_W^x | w^x)$  and  $p(W^k | w^{agg})$  are the probability mass function (PMF) of the probability distribution of multinomial for the vectors  $A_B^x$  and  $A_W^x$ , respectively.  $p(w^{agg})$  is the posterior distribution of  $w^{agg}$ . PMF for a given  $A_w$  is given in Eq. (6).

$$p(A_w | w) = \frac{(\sum_{j=1}^n a_{jw})!}{\prod_{j=1}^n a_{jw}} \prod_{j=1}^n w_j^{a_{jw}} \tag{6}$$

In multinomial distribution, the probability of event  $j$  is proportional to the number of occurrences of events to the total number of trials. Therefore, probability of the worst ( $w_B$ ) and the best criteria ( $\frac{1}{w_B}$ ), can write as Eq. (7).

$$w_w \propto \frac{a_{ww}}{\sum_{i=1}^n a_{iw}} = \frac{1}{\sum_{i=1}^n a_{iw}}, \quad \frac{1}{w_b} \propto \frac{a_{BB}}{\sum_{i=1}^n a_{Bi}} = \frac{1}{\sum_{i=1}^n a_{Bi}} \tag{7}$$

Eq. (8) shows the distribution of the first and second elements in Eq. (7). As it clear  $A_B$  produce the inverse of weight according to Eq. (7).

$$A_B^x | w^x \cong \text{multinomial} \left( \frac{1}{w^x} \right) \tag{8}$$

$$A_W^x | w^x \cong \text{multinomial} (w^x)$$

The Dirichlet distribution is used to model the weight vector in Eq. (9), where  $\gamma$  is the concentration parameter of the distribution that should be modeled using the gamma distribution and  $w^{agg}$  is the mean of Dirichlet distribution. Eq. (10) says that  $w^x$  must be in the proximity of  $w^{agg}$ . Also  $a$  and  $b$  in Eq. (10) are the shape parameters of the gamma distribution.

$$w^x | w^{agg} \cong \text{Dri}(\gamma \times w^{agg}) \quad X = 1, 2, \dots, X \tag{9}$$

$$\gamma \sim \text{gamma}(a, b) \quad (10)$$

Due to the complexity of the calculations for obtaining the posterior distribution using the multiplying likelihood (multinomial distributions) in the prior distribution (Dirichlet distribution), the MCMC is used to obtain the final weight. Having a posterior distribution, the confidence level of various relationships between criteria is obtained using the cardinal ranking. Credal ranking measure the extent to which a group of decision makers prefer one criterion over another criterion. Credal ordering for a pair of criteria  $c_i$  and  $c_j$  is calculated as follows:

$$O = (c_i, c_j, R, d) \quad (11)$$

where  $R$  shows the relationship between criteria  $c_i$  and  $c_j$ . Also  $d$  shows the confidence of the relation. Credal ordering is calculated for a total of pairs of criteria. The confidence that  $c_i$  is better than  $c_j$  is obtained as follows.

$$p(C_i > C_j) = \int I_{w_i^{agg} > w_j^{agg}} \times p(w^{agg}) \quad (12)$$

where  $p(w^{agg})$  is the posterior distribution and  $I$  is 1 if the subtitle condition is met, and zero otherwise. This integral is approximated by obtaining a sample obtained from MCMC. This relationship is calculated for all pairs of criteria (Mohammadi & Rezaei, 2020).

### 2.5. VIKOR method

VIKOR's method was developed by Serafim Opricovic in 1980 to solve decision problems with conflicting and disproportionate criteria (Opricovic, et al., 2004). This model is based on agreement planning. The steps of the VIKOR method are as follows:

*Phase 1:* In this method, first the normalization of the decision matrix is done as follows:

$$f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (13)$$

where  $f_{ij}$  are normalized numbers of the decision matrix.

*Phase 2:* Determine the best ( $f_j^*$ ) and worst values ( $f_j^\circ$ ) in all alternatives.

*Phase 3:*  $S_i$  and  $R_i$  that are usefulness and regret values of alternative  $j$  calculated based on Eqs. (14) and (15).

$$S_j = \frac{\sum_{j=1}^n w_j (f_j^* - f_{ij}^\circ)}{f_j^* - f_j^\circ} \quad \forall i \quad (14)$$

$$R_j = \frac{\max w_j (f_j^* - f_{ij}^\circ)}{f_j^* - f_j^\circ} \quad \forall i \quad (15)$$

where  $w_j$  is the weight of the criteria.

*Phase 3:* Calculating the value  $Q_j$ , which is the VIKOR index, from the following equation.

$$Q_i = v \frac{S_i - S^*}{S^\circ - S^*} + (1 - v) \frac{R_i - R^*}{R^\circ - R^*} \quad (16)$$

where  $v$  is the degree of agreement of the decision-makers.  $S^*, S^\circ, R^*$  and  $R^\circ$  in Eq. (15) are obtained by Eq. (17).

$$\begin{aligned} S^* &= \min(S_j) & S^\circ &= \max(S_j) \\ R^* &= \min(R_j) & R^\circ &= \max(R_j) \end{aligned} \quad (17)$$

*Phase 4:* Ranking the alternatives.

The alternatives are sorted from small to large according to the values of  $S$ ,  $Q$  and  $R$ . The final ranking is based on  $Q$  values (Opricovic & Tzeng, 2007).

### 3. Research Process

In this study, the suppliers of signaling equipment in the urban railway corporation of Mashhad, one of the most populous and busy cities of Iran have been considered. In the urban railway industry, speed, mobility, and ease of accessibility are very important (Bafandegan Emrooz, et al., 2022; Motahari Farimani, et al., 2022). Despite the required accuracy in purchasing signaling equipment, in this company, this problem is done solely based on the individual experience of the company managers and the use of scientific methods to select suppliers is not common. The most important goal of urban railway systems is to guide the train and control its security. A set of hardware devices that have been used to execute and control issued commands by the traffic control center, which is responsible for this task, is called signaling equipment. Therefore, considering the effect that equipment has on traffic safety, in this study, MCDM techniques have been used to select the best supplier. The company buys its equipment from 6 suppliers. Figure 2 shows the proposed framework for supplier selection. Figure 3 shows the step of research in more detail.

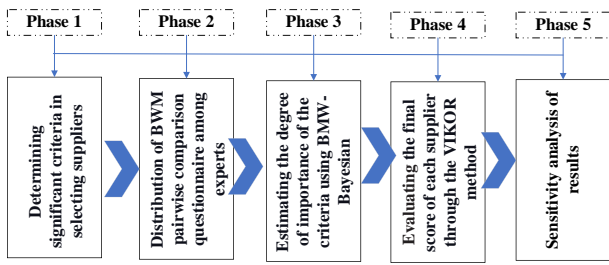


Fig. 2. The proposed framework for supplier selection

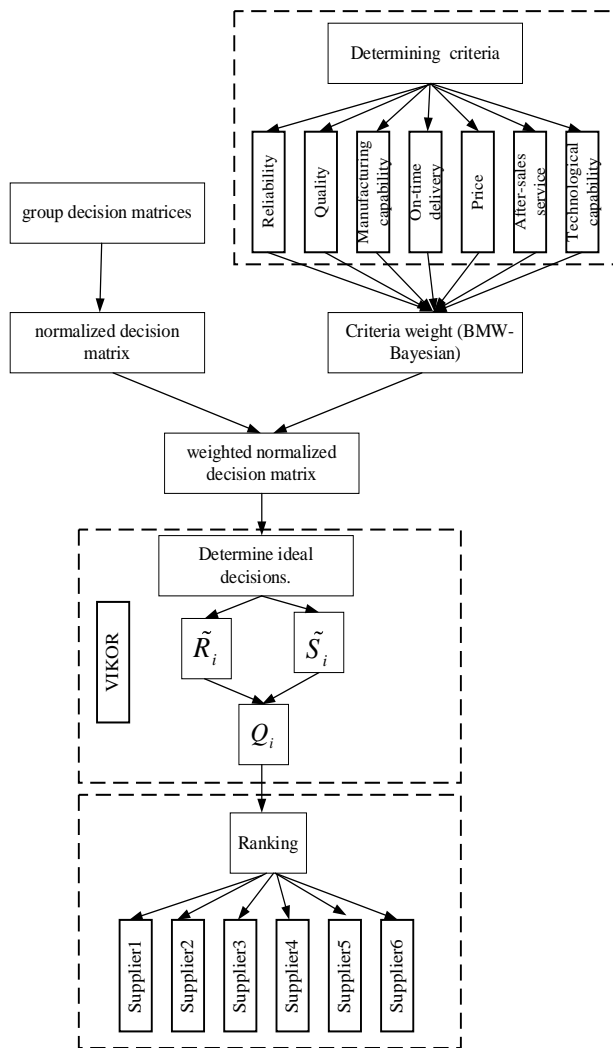


Fig. 3. General steps of research

### 3.1 Determining significant criteria in selecting suppliers

In selecting the best supplier, the first phase is to prepare a complete list of criteria related to the supplier's selection. One of the most problems in designing a model is determining criteria (Modares (a), et al., 2022). Because if sufficient accuracy is not taken at this problem, the criteria will not be selected comprehensively and as a result, the final model will not be evaluated accurately. Therefore, first, by studying the literature and using the results of previous studies, the criteria for evaluating suppliers are extracted. To extract the most effective criteria for the selection of supplier between the studied

criteria, the opinions of local experts are determined based on the strategy and needs of the organization. The total number of these people is announced after a negotiation with the company, which are about 15 people. After the necessary interviews and reviews, seven criteria of price, manufacturing capability, technical capability, on-time delivery, reliability, quality, and after-sale service of the supplier were are from the set criteria.

As mentioned, one of the criteria under consideration is manufacturing capability. To measure this, suppliers are evaluated in terms of competence, conditions, adequacy, and production capacity of facilities, machinery, equipment, and staff. For this purpose, the company under review has regular visits to its suppliers and evaluates this criterion. Technical capability is a measurement index of technological capability and supplier technology. This criterion gives higher rankings to suppliers who have higher research and development capabilities, more product innovation, and the use of up-to-date technologies. Quality criteria are also quality rankings based on the product quality review. They also pay attention to quality control activities, quality control certifications, quality improvement programs when evaluating suppliers. On-time delivery in this company is calculated from the number of delays of suppliers to the total number of orders. Reliability is the probability that the company's production processes will not be disrupted at a certain period (Modares, et al., 2021). After-sales service criteria are also considered based on 5-year records. The lower the price, the more point's suppliers get.

### 3.2. Distribution of BWM pairwise comparison questionnaire among experts

By placing the extracted criteria in the designed questionnaire, the required data are collected based on the BWM method. The questionnaires are related to pairwise comparisons with Saaty scales (1 to 9) to compare the best criterion with other criteria and the remaining criteria with the worst criterion, in the form of 15 questions that are distributed among the company's experts to collect data.

### 3.3. Estimating the degree of importance of the criteria using Bayesian BWM

After making pairwise comparisons between the best criterion and other criteria, as well as between the other criteria and the worst criterion, the criteria weights are obtained using Bayesian BWM. Considering that by comparing the numbers obtained for weight with certainty, it cannot be said which criterion has a higher rank, the probability of the preference of each criterion over the other criterion was calculated. If the probability of preference of criterion A to criterion B is more than 0.5, it can be said that criterion A is preferable to criterion

B. These probabilities were calculated using a posterior distribution using Markov-chain Monte Carlo (Modares (b), et al., 2022)

Table 1 shows the results of the MCMC, which shows the probabilities of the criteria preferences over each other. These probabilities are derived from Eq. (12). For example, to obtain the probability that criterion 1 is better than criterion 2, the integral in Eq. (18) must be calculated.

$$p(C_1 > C_2) = \int I_{w_i^{agg} > w_j^{agg}} \times p(w^{agg}) \quad (18)$$

Also, the sum of the probabilities according to the Eq. (19) is 1.

$$p(C_1 > C_2) + p(C_2 > C_1) = 1 \quad (19)$$

These integrals must be calculated for all pairs of criteria.

Table 1  
Probabilities of the criteria preferences over each other

C	1	2	3	4	5	6	7
1	0	0.87	0.9	0.999	0.6998	1	1
2	0.913	0	0.9994	1	0.9719	1	1
3	0.04	0.006	0	0.96	0.1073	1	0.9995
4	0.001	0	0.0373	0	0.012	0.9886	0.9420
5	0.3002	0.28	0.8926	0.9988	0	1	1
6	0	0	0	0.114	0	0	0.2430
7	0	0	0.0005	0.058	0	0.7570	0

Criteria abbreviated C

Figure 4, which is the software output of the MCMC method, also shows the probabilities of the criteria preferences. For example, the number 0.97 on the figure indicates that with a probability of 0.97 the quality criterion has more weight than the production capability criterion. The numbers 1 also indicate that with a confidence of 100%, we can say that the criteria are superior to each other. In general, the average of the criteria preferences was 0.95, which indicates the validity of the answers obtained from this method. High Confidence in results can provide more information for DMs to improve their decisions.

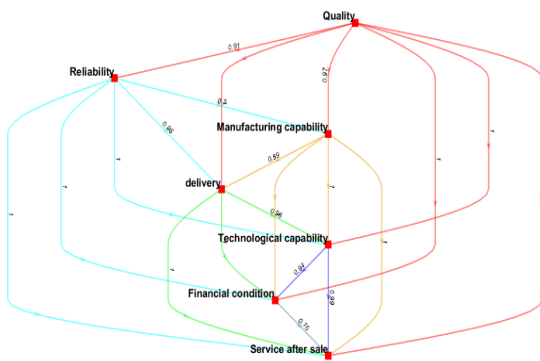


Fig. 4. The output of Credal ranking in supplier selection

Table 2 and Figure 5 show the results of the weight of the criteria for selecting suppliers using the Bayesian BWM method.

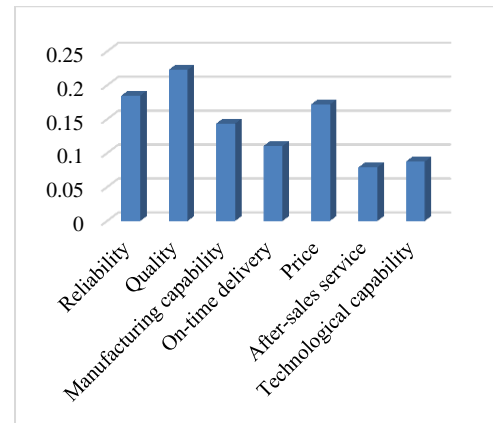


Fig. 5. Value of criteria weight

As it is clear, the quality criterion has the highest weight of 0.2228. The reliability criterion with a weight of 0.1848 has gained the second rank. After the reliability criteria, the criteria of price, production capability, on-time delivery, technical capability, after-sales service are ranked with weights of 0.1715, 0.1413, 0.1715, 0.1105, 0.0875, and 0.079, respectively. From the numbers in Figure 4, which show the probabilities of the criteria preferences relative to each other, it is clear that the total probabilities are greater than 0.5, and therefore the obtained weights and the ranking and preference of the criteria over each other can be trusted. As shown in Figure 4, the quality criterion with a probability of 0.91 is preferable to the reliability criterion. With a probability of 97% and 70%, it is preferable to the criterion of production and on-time delivery. It can also be said with confidence 100% that quality is superior to the criteria of technical capability, price, and after-sales service. Also, in general, the average probabilities and confidence coefficients of the whole pair of criteria are 0.95, which clarifies the validity of the results.

Table 2  
Detail of criteria weights

Criteria	Weight
Technological	0.0875
After-sales service	0.079
Price	0.1718
On-time	0.1105
Manufacturing capability	0.1436
Quality	0.2228
Reliability	0.1848
Service after sale	



3.4. Evaluating the final score of each supplier through the VIKOR method

To apply the VIKOR technique, first, the decision matrix is made and normalized according to the opinions of the experts, and then the weighted matrix is calculated according to the weights obtained from Bayesian BWM. Positive ideal and negative ideal values are determined for all criteria and by using these values, the parameters of the VIKOR technique are estimated and at the end, the suppliers are ranked based on values  $S, Q$  and  $R$ . Tables 3 and 4 show the normalized decision matrix obtained from the decision makers' survey and the weighted decision matrix of the criteria, respectively.

Table 3  
Normalized decision matrix

	C1	C2	C3	C4	C5	C6	C7
S1	0.1849	0.0243	0.0294	0.0172	0.0469	0.0234	0.0315
S2	0.1891	0.0340	0.0211	0.0287	0.0508	0.0141	0.0135
S3	0.1996	0.0437	0.0378	0.0402	0.0501	0.0327	0.0405
S4	0.1828	0.0248	0.0126	0.0172	0.0532	0.0327	0.0225
S5	0.1870	0.0049	0.0211	0.0057	0.0548	0.0421	0.0315
S6	0.1786	0.0243	0.0294	0.0517	0.0501	0.0047	0.0135

Table 4  
Weighted (normalized) decision matrix

	C1	C2	C3	C4	C5	C6	C7
S1	0.0342	0.0054	0.0042	0.0019	0.0081	0.0018	0.0028
S2	0.035	0.0076	0.003	0.0032	0.0087	0.0011	0.0012
S3	0.0369	0.0097	0.0054	0.0044	0.0086	0.0026	0.0035
S4	0.0338	0.0054	0.0018	0.0019	0.0091	0.0026	0.002
S5	0.0346	0.0011	0.003	0.0006	0.0094	0.0033	0.0028
S6	0.033	0.0054	0.0042	0.0057	0.0086	0.0004	0.0012

Table 5 shows the ideal and negative ideal values of the decision.

Table 5  
Ideal decisions

	C1	C2	C3	C4	C5	C6	C7
$F^+$	0.003	0.003	0.008	0.005	0.005	0.009	0.036
	5	3	1	7	4	7	9
$F^-$	0.001	0.000	0.009	0.000	0.001	0.001	0.033
	2	4	4	6	8	1	

Tables 6 and 7 show the Victor indexes and the ranking by the Victor indexes, respectively. According to the presented algorithm in the VIKOR method, the best alternative (supplier) should be the best in all three values ( $S, Q, R$ ) otherwise, the best alternative is the alternative that has the smallest  $Q$  and is recognized as the top rank in at least one of the groups  $R$  and  $S$ , and provided it is true.

Table 6  
VIKOR indexes

	S1	S2	S3	S4	S5	S6
$S$	0.5187	0.6653	0	0.9366	1	0.7414
$R$	0.3936	0.1753	0	0.5135	1	0.7534
$Q$	0.4686	0.4693	0	0.7673	1	0.7462

Table 7  
Ranking by VIKOR indexes

	S1	S2	S3	S4	S5	S6
$S$	2	3	1	5	6	4
$R$	3	2	1	4	6	5
$Q$	2	3	1	5	6	4

In the present study, considering the parameter  $v = 0.6$ , the S3 option has a better rank in all three values and is selected as the best alternative. The final ranking of the alternative is as follows. In this research, because group opinions are valued more,  $v = 0.6$  is considered (the more agreement and more valued group opinions, the value is considered).

3.5. Sensitivity analysis

In the present study, different values  $v$  have been used to analyze the sensitivity of the VIKOR technique. For the VIKOR index ( $v$ ), values in the range 0 – 1 with intervals of 0.1 are considered. The results are presented in Tables 8 and 9. The ranking of the alternatives according to the values of  $v$ , as shown in figures 6 and 7, shows that for different values of  $v$ , S3 is still the best and S5 is the worst alternative to select the supplier. When  $v \leq 0.5$  the ranking of the suppliers will be  $S3 > S2 > S1 > S4 > S6 > S5$  and when  $v > 0.5$  the ranking of the suppliers will be  $S3 > S1 > S2 > S6 > S4 > S5$ .

Table 8  
The  $Q$  values for different  $v$

	S1	S2	S3	S4	S5	S6
$v = 0.0$	0.3936	0.1753	0	0.5135	1	0.7534
$v = 0.1$	0.4061	0.2243	0	0.5558	1	0.7522
$v = 0.2$	0.4186	0.2733	0	0.5981	1	0.7510
$v = 0.3$	0.4311	0.3223	0	0.6404	1	0.7498
$v = 0.4$	0.4437	0.3713	0	0.6827	1	0.7486
$v = 0.5$	0.4561	0.4203	0	0.7250	1	0.7474
$v = 0.6$	0.4686	0.4693	0	0.7673	1	0.7462
$v = 0.7$	0.4812	0.5183	0	0.8097	1	0.7450
$v = 0.8$	0.4937	0.5673	0	0.8520	1	0.7438
$v = 0.9$	0.5062	0.6163	0	0.8942	1	0.7426
$v = 1.0$	0.5187	0.6653	0	0.9366	1	0.7414



Table 9  
Ranking for different  $v$

$v$	S1	S2	S3	S4	S5	S6
$v = 0.0$	3	2	1	4	6	5
$v = 0.1$	3	2	1	4	6	5
$v = 0.2$	3	2	1	4	6	5
$v = 0.3$	3	2	1	4	6	5
$v = 0.4$	3	2	1	4	6	5
$v = 0.5$	3	2	1	4	6	5
$v = 0.6$	2	3	1	5	6	4
$v = 0.7$	2	3	1	5	6	4
$v = 0.8$	2	3	1	5	6	4
$v = 0.9$	2	3	1	5	6	4
$v = 1.0$	2	3	1	5	6	4

As it is clear in Figures 6 and 7 since S3 is superior to another alternative in all indicators and values of  $S, Q, R$  is zero in S3. Similarly, S5 is more inefficient than all other suppliers in all respects, and for various values of  $v$  are 1 and the most undesirable alternative in terms of supplier selection. In the range of value  $V = 0.5 - 0.6$ , the intersection of charts S2 with S1 and S6 with S4 shows the change of priority and ranking of alternatives in the range = 0.5 - 0.6.

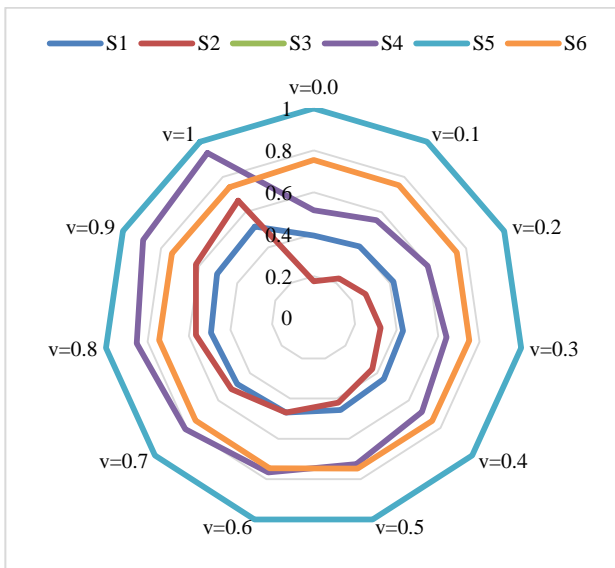


Fig. 6. Result of sensitivity analysis

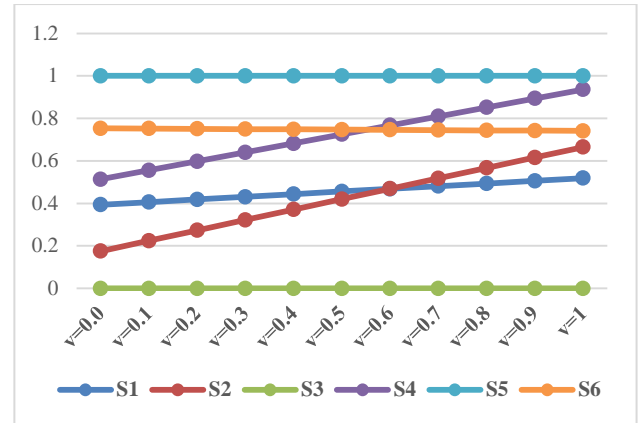


Fig. 7. Result of sensitivity analysis

#### 4. Conclusion

This study considers a multi-criteria group decision to evaluate and select suppliers, taking into account probabilistic uncertainty. Bayesian hierarchical approach with BWM was used to control the uncertainty in obtaining the final weights of the criteria. The Bayesian model determined the weight probabilistic distribution of each individual in group decision making and finally obtained a cumulative distribution for all decision makers' preferences. This method, unlike other MCDM methods that use averaging to aggregate DM opinions, gained the group's collective judgments at the same time. Given that the averaging of the DMs judgments is sensitive to outlier data, as well as the preferences of one person who has significant differences with others may affect the answer. Therefore, the obtained answer from the Bayesian model is more valid. Also, in this method, credal ranking, which shows the probability of superiority of the pair of criteria over each other, was used. Given that, in MCDM methods only the weight vector is obtained and the higher the number obtained, the higher the rank assigned to that criterion. However, by comparing the criteria with each other, it is not possible to say with complete certainty that one criterion is better than another. Decision-makers will be more certain of the relation of two criteria if the confidence level between these is high. In this study, except for three probabilities, the rest were above 90% and most of them were 1.

The lowest obtained probability is related to the preference of quality criteria to on-time delivery, which was obtained 70%. Although this probability is more than 50%, it is possible to rely on the result with confidence, but it is better to interpret and analyse the relationship between these two criteria more carefully. After obtaining the weight of the criteria, the weight of the suppliers was obtained using VIKOR. As it is clear from the results, the third supplier got the highest rank because it is superior in terms of all indicators. The results of the sensitivity analysis showed that different rankings are provided for different values of the VIKOR parameter, which is determined according to the degree of agreement of the expert group. For different values of the VIKOR

parameter, the third supplier will have the highest rating and the fifth supplier will have the lowest rating, and the ranking of other suppliers. The rank of the first, second, fourth, and sixth suppliers will change according to the agreement of the decision-making group, while there is no change in the ranking of the third and fifth suppliers because in terms of criteria are significantly different from other suppliers. The introduced multi-criteria decision model in this research will help the decision-makers of the urban railway company and other organizations with several suppliers to be able to select the best supplier by considering the relevant criteria. For future research, it is suggested that for each criterion, the related sub-criteria be identified to have a more comprehensive view of the problem, and other multi-criteria decision-making methods are used and the weight related to them is calculated. The fuzzy VIKOR approach can also be used to prioritize suppliers to bring the problem closer to the real world. Given that the choice of supplier of signaling equipment is widely related to risk criteria, in future research, the selection of suppliers can be done according to the identification of these criteria.

## References

- Aboutorab, H., Saberi, M., Hussain, O., & Chang, E. (2018). ZBWM: the z-number extension of best-worst method and its application for supplier development. *Expert Syst. Appl*, 107, 115-125.
- Ahadi, M., Mahpour, A., & Taraghi, V. (2018). A Combined Fuzzy Logic and Analytical Hierarchy Process Method for Optimal Selection and Locating of Pedestrian Crosswalks. *journal of optimization in industrial engineering*, 11(2), 79-89. doi:10.22094/JOIE.2017.458.0
- Alinezhad, A., & Seif, A. (2020). Application of Fuzzy Analytical Hierarchy Process and Quality Function Deployment Techniques for Supplier's Assessment. *journal of optimization in industrial engineering*, 13(2), 279-289. doi:10.22094/JOIE.2020.27.1
- Bafandegan Emrooz, V., & Fakoor, A. (2023). A new approach to human error assessment in financial service based on the modified CREAM and DANP. *Journal of Industrial and Systems Engineering*.
- Bafandegan Emrooz, V., Modares, A., & Mohemi, Z. (2022). Presenting a model for diagnosing the implementation of total quality management based on performance expansion model (Case: Simorgh Rail Transportation Company). *Road*. doi:10.22034/ROAD.2022.319629.2007
- Bai, C., & Sarkis, J. (2014). Determining and applying sustainable supplier key performance indicators. *Supply Chain Management: An International Journal*, 19(3), 275-291.
- Banaeian, N., Mobilia, H., & Fahimnia, B. (2016). Green supplier selection using fuzzy group decision making methods: A case study from the agri-food industry. *Computers & Operations Research*.
- Blagojevic, B., Srdjevic, B., Srdjevic, Z., & Zoranovic, T. (2016). Heuristic aggregation of individual judgment in AHP group decision making using simulated annealing algorithm. *Information Science*, 330, 260-273.
- Burgess, K., Singh, P., & Koroglu, R. (2006). Supply chain management: A structured literature review and implications for future research. *International Journal of Operations & Production Management*, 26(7), 703-729.
- Büyükoçkan, G., & Çifçi, G. (2011). A novel fuzzy multi-criteria decision framework for sustainable supplier selection with incomplete information. *Comput. Ind*, 62(2), 164-174.
- Cakravastia, A., & Takahashi, K. (2004). Integrated model for supplier selection and negotiation in a make-to-order environment. *International Journal of Production Research*, 42(21), 4457-4474.
- Ecer, F., & Pamucar, D. (2020). Sustainable supplier selection: A novel integrated fuzzy best worst method (F-BWM) and fuzzy CoCoSo with Bonferroni (CoCoSo'B) multi-criteria model. *Journal of Cleaner Production*, 266.
- Forman, E., & Peniwati, K. (1998). Aggregation in individual judgments and priorities with the analytic hierarchy process. *European journal of operation al research*, 108(1), 165-169.
- Gilks, W. (1995). *Markov Chain Monte Carlo in Practice*. London, UK: Informa UK Limited.
- Goldstein, D. (2011). Doing Bayesian Data Analysis: A Tutorial with R and BUGS, John K. Kruschke. Academic Press, Elsevier. *Journal of Economic Psychology*, 32(5), 724-725.
- Goren, H. (2018). A decision framework for sustainable supplier selection and order allocation with lost sales. *J. Clean. Prod*, 183, 1156-1169.
- Hosseini, M., & Barker, K. (2016). A Bayesian network model for resilience-based supplier selection. *International Journal of Production Economics*, 180, 68-87.
- Hsu, C., & Hu, A. (2009). Applying hazardous substance management to supplier selection using analytic network process. *J. Clean Prod*, 17(2), 255-264.
- Hsu, C.-W., & Hu, A. H. (2007). Application of analytic network process on supplier selection to hazardous substance management in green supply chain management. Proceedings of the IEEE International Conference on Industrial Engineering and Engineering Management.
- Kazemitash, N., Fazlollahab, H., & Abbaspour, M. (2021). Rough Best-Worst Method for Supplier Selection in Biofuel Companies based on Green criteria. *Operational Research in Engineering Sciences: Theory and Applications*, 4(2), 1-12.
- Kumar Kar, A. (2014). Revisiting the supplier selection problem: An integrated approach for group decision support. *Expert Systems with Applications*, 41, 2762-2771.

- Kumar, A., Jain, V., & Kumar, S. (2014). A comprehensive environment friendly approach for supplier selection. *Omega*, 42, 109–123.
- Lei, F., Wei, G., & Gao, H. (2020). TOPSIS Method for Developing Supplier Selection with Probabilistic Linguistic Information. *Int. J. Fuzzy Syst*, 22, 749–759.
- Li, J., & Wang, J. (2017). An Extended QUALIFLEX Method Under Probability Hesitant Fuzzy Environment for Selecting Green Suppliers. *Int. J. Fuzzy Syst*, 19, 1866-1879.
- Liu, A., Xiao, Y., Ji, X., Wang, K., & Tsai, S. (2018). A Novel Two-Stage Integrated Model for Supplier Selection of Green Fresh Product. *Sustainability*, 10(7), 2371.
- Liu, H., Quan, M., Li, Z., & Wang, Z. (2019). A new integrated MCDM model for sustainable supplier selection under interval-valued intuitionistic uncertain linguistic environment. *Information Sciences*, 486, 254-270.
- Liu, Y., Eckert, C., Yannou-Le Bris, G., & Petit, G. (2019). A fuzzy decision tool to evaluate the sustainable performance of suppliers in an agrifood value chain. *Comput. Ind. Eng.*, 127, 196-212.
- Lo, H., Liou, J., Huang, C., & Chuang, Y. (2020). A new soft computing approach for analyzing the influential relationships of critical infrastructures. *Int. J. Crit. Infrastruct. Prot.*, 28(100336).
- Luthra, S., Govindan, K., Kannan, D., Kumar Mangla, S., & Prakash Garg, C. (2017). An integrated framework for sustainable supplier selection and evaluation in supply chains. *Journal of Cleaner Production*, 140(3), 1686-1698.  
doi:10.1016/j.jclepro.2016.09.078
- Modares, A., Bafandegan Emroozi, V., & Mohemmi, Z. (2021). Evaluate and control the factors affecting the equipment reliability with the approach Dynamic systems simulation, Case study: Ghaen Cement Factory. *Journal of Quality Engineering and Management*, 11(2), 89-106.
- Modares, A., Motahari Farimani, N., & Bafandegan Emroozi, V. (2022). A vendor-managed inventory model based on optimal retailers selection and reliability of supply chain. *Journal of Industrial and Management Optimization*, 19(5), 3075-3106.  
doi:10.3934/jimo.2022078
- Modares, A., Motahari Farimani, N., & Bafandegan Emroozi, V. (2022). A new model to design the suppliers portfolio in newsvendor problem based on product reliability. *Journal of Industrial and Management optimization*. doi:10.3934/jimo.2022124
- Modares, A., Motahari Farimani, N., & Bafandegan Emroozi, V. (2022). Developing a Newsvendor Model based on the Relative Competence of Suppliers and Probable Group Decision-making. *Industrial Management Journal*, 14(1), 115-142.  
doi:10.22059/IMJ.2022.331988.1007872
- Mohammadi, M., & Rezaei, J. (2020). Bayesian best-worst method: A probabilistic group decision making model. *Omega*, 96(102075).
- Morais, D. C., & Ahmeid, A. T. (2012). Group decision making on water resources based on analysis of individuals ranking. *Omega*, 40(1), 42-52.
- Motahari Farimani, N., Ghanbarzade, J., & Modares, A. (2022). A New Approach for Pricing Based on Passengers' Satisfaction. *Transportation Journal*, 61(2), 123-150. doi:10.5325/transportationj.61.2.0123
- Nepal, B., & Yadav, O. (2015). Bayesian belief network-based framework for sourcing risk analysis during supplier selection. *International Journal of Production Research*, 53(20), 6114-6135.
- Noci, G. (1997). Designing 'green' vendor rating systems for the assessment of a supplier's environmental performance. *Eur. J. Purchasing Supply Manage*, 3(2), 103-114.
- Opricovic, S., & Tzeng, G. (2007). Extended VIKOR method in comparison with outranking methods. *European Journal of Operational Research*, 178, 514–529.
- Opricovic, S., & Tzeng, G. (2004). Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. *Eur. J. Oper. Res.*, 156, 445-455.
- Rezaei, J. (2015). Best-worst multi-criteria decision-making method. *Omega*, 53, 49-57.
- Rezaei, J. (2016). Best-worst multi-criteria decision-making method: Some properties and a linear model. *Omega*, 64, 126-130.
- Roostaei, R., Izadikhah, M., Hosseinzadeh Lotfi, F., & Rostamy-Malkhalifeh, M. (2012). A Multi-Criteria Intuitionistic Fuzzy Group Decision Making Method for Supplier Selection with VIKOR Method. *International Journal of Fuzzy System Applications*, 2(1), 1-17.  
doi:DOI:10.4018/IJFSA.2012010101
- Roorkhosh, P., & Kazemi, M. (2022). Application of Internet of Things in Green Supply Chain and Investigating the Effective Factors for Selecting a Green Supplier: A Case Study: Mashhad Rubber Factory. *Supply Chain Management Journal*, 24(75), 61-73.
- Roorkhosh, P., & Motahari Farimani, N. (2022). Designing a new model for the hub location-allocation problem with considering tardiness time and cost uncertainty. *International Journal of Management Science and Engineering Management*, 1-15.
- Roorkhosh, P., Pooya, A., & Agarwal, R. (2022). Blockchain acceptance rate prediction in the resilient supply chain with hybrid system dynamics and machine learning approach. *Operations Management Research*, 1-21.
- Sanayei, A., Mousavi, S., & Yazdankhah, A. (2010). Group decision-making process for supplier selection with Vikor under fuzzy environment. *Expert Syst Appl*, 37(1), 24-30.
- Sayadi, M. K., Heydari, M., & Shahanaghi, K. (2009). Extension of VIKOR method for decision making problem with interval numbers. *Applied Mathematical Modelling*, 33(5), 2257-2262.  
doi: 10.1016/j.apm.2008.06.002

- Shahhoseini, A., & Yousefinejad Attari, M. (2018). Hybrid Techniques of Multi-Criteria Decision-Making for Location of Automated Teller Machines (ATMs): Shahr Bank Branches in Tehran, 1st District Municipality. *Journal of optimization in industrial engineering*, 11(2), 139-148. doi:10.22094/JOIE.2017.706.1445
- Wan, S., Xu, G., & Dong, J. (2017). Supplier selection using ANP and ELECTRE II in interval 2-tuple linguistic environment. *Inf. Sci*, 385-386, 19-38.
- Wang, X., & Cai, J. (2017). A group decision-making model based on distance-based VIKOR with incomplete heterogeneous information and its application to emergency supplier selection. *Cybernetic*, 46, 501-529.
- Zhang, D., Zhang, J., & Lai, K.-K. (2009). An novel approach to supplier selection based on vague sets group decision. *Expert Systems with Applications*, 36, 9557-9563.

**This article can be cited:** Modares, A., Motahari Farimani, N., & Bafandegan Emroozi, V. (2023). Applying a Multi-criteria Group Decision-making Method in a Probabilistic Environment for Supplier Selection (Case study: Urban Railway in Iran). *Journal of Optimization in Industrial Engineering*, 16(1), 129-140. doi: 10.22094/joie.2023.1950386.1929

